Evaluating the Impact of Vocational Training and Housing Assistance Programs on the Spread and Control of Homelessness in Los Angeles, California

Mohammed Alharbi[1,8], Jackson Carpenter[2], Vincent Jones[3], Jennifer Sánchez[4], Cesar Montalvo[2], Asma Azizi[2], Viswanathan Arunachalam[5], Carlos M. Hernández-Suárez[6], Christopher Kribs[1], José Flores[7], and Anuj Mubayi[2]

[1]University of Texas at Arlington

[2]Arizona State University

[3]University of Pennsylvania

[4]Yachay Tech University

[5]Universidad Nacional de Colombia

[6]Universidad de Colima

[7]University of South Dakota

[8]University of Jeddah

July 2019

Abstract

Homelessness is a major socioeconomic issue in the United States with more than half a million individuals homeless on any given night in 2018 (U.S. Department of Housing and Urban Development). The second highest number of homeless was reported from Los Angeles County with approximately 53,000 individuals, 10% of the national total. Several factors associated with lifetime homelessness include poor family functioning, socioeconomic disparity, isolation from social network communities, mental health issues, and addiction problems. Recent interventions such as rapid rehousing, educational programs, job skills training, and supportive health services have either been ineffective or have had limited scope over time. In this study, we evaluate the impact of two interventions, such as, vocational training and housing assistance programs on the spread and control of homelessness in Los Angeles, using a system of ordinary differential equations and data from Los Angeles County. Our results show that housing assistance programs are more effective in a short period of time, however, in order to achieve sustainability in the long term it is necessary to incorporate vocational training programs. The likelihood of the spreading of homelessness is influenced by social interactions and the interventions incorporated. Also, the environmental influence was found to be dominant when the rate of interventions is extremely low.

Contents

1 Introduction

On any given night in 2018, more than half a million people experienced homelessness in the United States, and 35% of these people did not spent their night in shelters. Further, although the total number of homeless people in the United States has dropped 15% from 2007 to 2018, there was a slight increase of 0.7% in the homeless rate over the last year. [14] These statistics indicate that, despite an overall increase in taxpayers' money throughout a relatively stable period of economic growth, more effort is required to combat the problem of homelessness. Homelessness is defined by the Department of Housing and Urban Development (HUD) as lacking a fixed, regular, and adequate nighttime residence. Furthermore, the factors that make a person vulnerable to homelessness are living under the poverty line, that is, making less than $12,000 per year per person, and being over the age of 18. In addition, homelessness is the result of a variety of factors, including poverty, lack of network support, poor health, mental and physical disabilities, unemployment, etc [12]. Moreover, socioeconomic factors, such as living in impoverished communities (having an income less than $50,000 [7]), play fundamental roles in the homelessness process by generating a cycle of poverty, resulting in a homeless person's inability to afford housing or rent. For our research, using data from the U.S. Census Bureau, we focus specifically on lack of social networks via unemployment and poverty as the principal risk factors for the transition into homelessness.

Los Angeles, California, (L.A.) is one of the most populous counties in the United States with a population of about 10.1 million people [6]. According to the U.S Department of Housing and Urban Development , $53,000$ L.A. County residents were homeless on any given night in 2018. For year 2018, about 15% of Los Angeles County residents live in poverty [6], in contrast to 12.3% percent across the entire United States [5]. L.A. County's lowest-income renters spend 71% of income on rent, leaving little left for food or other basic needs. Furthermore, renters in L.A. County need to earn $46.15/hour, more than four times the local minimum wage, to afford the median monthly rent of $2,400/month [22]. In 2018, the public investment targeting homelessness in Los Angeles, surpassed $619 million [8].

To best resolve the homeless problem, we will focus on two intervention strategies with respect to two of the largest factors contributing to homelessness: high housing costs and unemployment. In Los Angeles, housing costs have been on the rise for the last eight years with average prices going from $370,000 in 2012 to \$700,000 in 2019 [2]. These combined problems of not earning enough to both pay for housing and unemployment are the focus of the two interventions strategies: temporary housing assistance and vocational training programs.

We will follow the vocational training scheme offered by The Roberts Enterprise Development Fund (REDF) program, an established non-profit organization with extensive work on employment social enterprises. Within the program, participants are trained to build soft, vocational, or technical skills. At the same time, they seek to develop basic skills through regular classes. The program provides people who are currently jobless and above the age of 18 years with an opportunity to rejoin the workforce [1]. The REDF works with Social Enterprises (SEs), certain job partners that understand the stresses of the workforce and provide services that alleviate these stresses. These services range from strengthening workplace skills to financial literacy classes and even to mental health counseling. According to literature from the REDF [24], individuals in SE programs are more economically self-sufficient; the percentage employed after an SE job increased from 18% to 51%. In addition, income increased by \$570 (268 percent) for all SE workers, from \$213 before the SE job began to \$783 one year later.

The scheme that we will be following to incorporate housing assistance programs will be temporary housing as given by the Continuum of Care (CoC) program [20]. This is a program provided by state and local governments to quickly rehouse homeless individuals and families while minimizing the trauma and dislocation costs to homeless individuals. The program assists individuals and families experiencing homelessness with move into temporary housing if they live under the poverty line or near a disaster area. It is important to note that this temporary housing will only be covered for a maximum of 24 months [20]. In a study by Quentin T. Wodon [27] on an optimal control model for providing shelters or low income housing for the homeless, shelters were found to be short-term safety nets whereas low income housing were more akin to long-term investments in an economic sense. A steady-state saddle point is found, meaning that the prioritization of shelter beds and low income housing may not have as clear an effect as expected. In our study, we will still focus on temporary housing as another form of long-term investment that more effectively removes homeless individuals from the streets and provides stable living conditions.

Lacey et al. [16] focuses on the impact of council housing on the control of homelessness in the UK. The authors want to find a way to prioritize the focus on the availability of council (government assistance) housing for the homeless population, but consider very few dynamical concepts. The population is stratified into three groups based on whether they are in the private or council housing and seeking transfer accommodations. The model described keeps the populations in each state constant, and the transition between compartments is given by constant rate, giving a simple linear system. With numerical approximations, the authors determined, that an increase in the constant rate of homeless into council stock caused the population of homeless to increase dramatically. However, reducing the constant rate of homeless into council stock has little effect on the sizes of other categories on the waiting list. The research paper is the only attempt to capture the homeless dynamics even though does not provide a consistent model to study homelessness highlighting the non-linear dimensions of transitions of this social issue.

The vast majority of the literature focused on homelessness has been from a statistical and empirical perspective. Elliott and Krivo [9] attempted to determine the most crucial structural determinants of homelessness, rather than looking at personal characteristics of homeless individuals, as had been done previously. They found that lack of low-cost housing and high poverty rates are some of the leading factors for homelessness. With respect to high poverty, people must choose between paying for rent and other basic necessities such as food, clothing, or medical care. The results of this study show that there is a negative correlation between sensibility and low-cost housing, with a significance of $p < 0.01$. With respect to poverty levels, the results were inconclusive about the connection between poverty and homelessness.

In another study, Fusaro et al. [11] investigated the prevalence of homelessness among the Baby Boomer generation in the United States with respect to demographic characteristics (racial as well as ethnic disparities). The results of common investigations of the homeless population by the federal government include point-in-time (PIT) counts from a single night in January showed that 6% of all American adults from the Baby Boomer generation had experienced at least one episode of homelessness. The most recent PIT count of homeless individuals was approximately 550,000 people [14], making up some 0.17% of the entire U.S. population, a drastic difference from the 6% calculated from this study for overall lifetime prevalence of homelessness.

The focus of our study is to evaluate the impact of temporary housing assistance and vocational training programs on the spread and control homelessness in Los Angeles, especially under the presence of disparities in socioeconomic conditions. We also attempt to estimate parameters of a dynamical model and identify the critical outcomes of each of the two interventions over a long term period. To study our research objectives, we develop and analyze a compartmental model with a deterministic system of differential equations and a continuous-time Markov chain (CTMC) model in order to understand the dynamic characteristics of our homelessness and its interventions.

The paper is laid out as follows. First, the model's development is described along with assumptions and parameters. Second, the mathematical analysis is detailed with four cases of the general model to highlight the effect of the different interventions in the system. Third section presents numerical results, including parameter estimation, numerical simulations, influence of intervention strategies on homeless population, sensitivity analysis, and the influence of intervention success rates on homeless population. Finally, discussion presents the main results of the study.

2 Methodology

2.1 Model Development

Individuals are defined as either *vulnerable* or *disaffiliated* before entering the homeless class. We define vulnerable as residents of L.A. County who are over the age of 18 and living under the poverty line, that is, the annual income is less than $12,000 per year per one individual [19] or near a disaster area [3]. The disaffiliated class is comprised of those individuals from the vulnerable class that have lost their job. Charles Grigsby [12] defined disaffiliation as a process of increasing detachment from traditional institutions and social roles. Thus, in the case of this study, unemployment is one form of disaffiliation from the traditional workforce. Disaffiliated individuals are more likely to be influenced to become homeless. Grigsby argues that recently disaffiliated people cope with their distress by *re*-affiliating with homeless individuals.

Further, the novelty of this study is the consideration of the influence of the environment on a disaffiliated individual, implemented in a mathematical model. Disaffiliated people lose their job and struggle paying the rent, moreover, the environment could influence their transition into a homeless stage. A similar approach can be found in the model presented by Richard Levins in 1969 [17]. His model describes the time-dependent changes in the population of a species in a certain area and how it varies depending on the extinction and colonization of sub-populations [18]. In our model, the environmental influence is represented by the fraction of the city in which people live in high poverty stages, which we define as a compartment representing an indirect influence affecting the transition from a disaffiliated to a homeless state. Colonization and extinction rates would be represented by the increase and reduction of the high poverty areas, respectively.

The Levins Model is an example of patch occupancy metapopulations models, which in our case are associated with high poverty community areas. In order to adopt the Levins Model, it is necessary to consider the following assumptions: (1) the suitable habitat occurs in infinitely many patches that are equally large and of the same quality; (2) the patches have only two possible states, occupied or empty, the area is occupied by a high poverty community, not considering higher socioeconomic levels, (3) local extinctions and colonizations occur independently in different patches, and (4) all local populations are equally connected to other populations and patches [15].

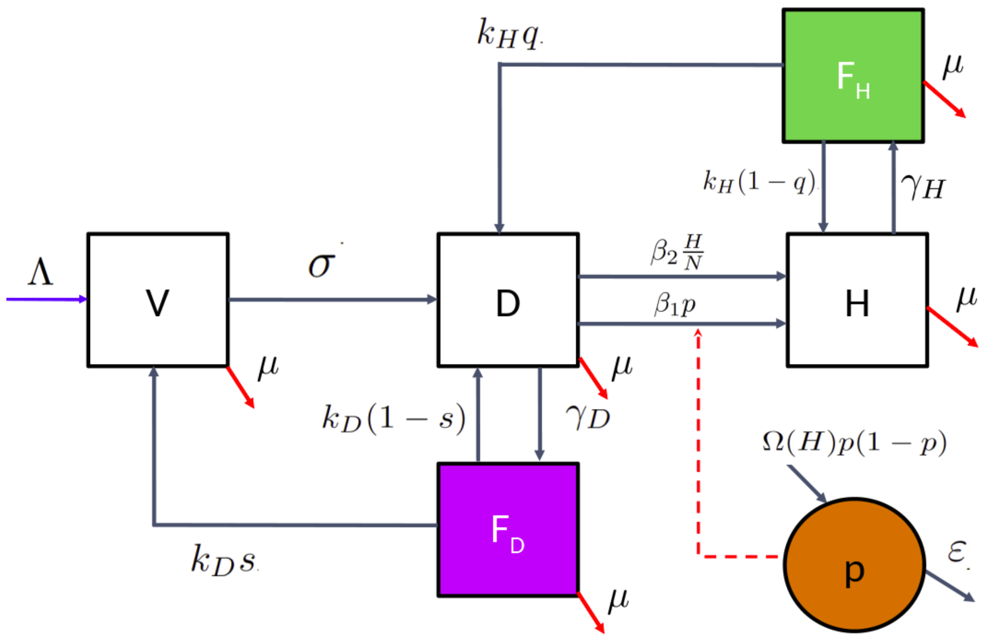

Figure 1: Compartmental Diagram of the model

Tables 1 and 2 give the standard definitions of the state variables and transition parameters used in the model (See Figure 1).

Variable	Definition
$V(t)$	Individuals in L.A. County over 18 years old, living under the poverty line or living under near a disaster area.
$D(t)$	Individuals from $V(t)$ who have lost their job.
$H(t)$	Individuals from $D(t)$ who have lost their house.
$F_D(t)$	Individuals from $D(t)$ who are in vocational training under the REDF program.
$F_H(t)$	Individuals from $H(t)$ who are in temporary housing under the CoC program.
Some parameters as β_1 and β_2 $N(t)$	Total population at-risk of becoming homeless.

Table 1: State variable definitions.

Param.	Definition	Units
μ	Per capita death natural rate.	years^{-1}
Λ	Recruitment rate from at-risk of becoming homeless population into vulnerable class.	$\frac{\text{person}}{\text{year}}$
σ	Per capita transition rate from the vulnerable to disaffiliated class.	years^{-1}
β_1	Per capita rate of influence to disaffiliated, due to environment $(p(t))$.	years^{-1}
β_2	Per capita rate of influence to disaffiliated, due to social interactions $(H(t))$.	years^{-1}
ϵ	Per capita reduction rate of high poverty communities area in L.A. County.	years^{-1}
γ_D	Per capita rate of disaffiliated enrolling in vocational training.	years^{-1}
γ_H	Per capita rate of disaffiliated enrolling in housing assistance programs.	years^{-1}
k_D	Per capita rate at which individuals leave vocational training programs.	years^{-1}
k_H	Per capita rate at which individuals leave housing assistance programs.	years^{-1}
s	Proportion of individuals that obtain a job after a vocational training program.	
q	Proportion of individuals that obtain a house after a housing assistance program.	
c	Intrinsic growth rate of the high poverty area.	(person years)$^{-1}$

Table 2: Parameter definitions and units.

2.1.1 Assumptions

Before presenting the system of equations, we establish the following assumptions that govern the model:

A1: Λ is the constant recruitment rate of the total population into the system, and everybody dies with the same per capita death rate μ.

A2: There is no transition from the homeless to disaffiliated compartment, and individuals must obtain a temporary house (F_H) before permanent housing.

A3: To transit back to vulnerable from the disaffiliated compartment, the individual must go through a vocational training program (F_D).

A4: There is sufficient funding available for temporary housing and vocational training programs for homeless and at-risk homeless population.

A5: There is a chance that individuals accessing the temporary housing and vocational training programs may return to becoming homeless or disaffiliated.

A6: All the individuals in disaffiliated and homeless state will eventually get involved in the vocational training or housing assistance program, respectively.

A7: All homeless individuals are unemployed.

A8: Similar to the work done by Grigsby et al. [12], we assume that the presence of homeless community affects the transition from disaffiliated to homeless, not as an epidemic, but due to disaffiliated population are more prone to be drawn into new communities, since these offer them the bonds they have lost thought unemployment.

A9: The environmental role influencing disaffiliated to become homeless is modelled according to the Levins Model assumptions.

The dynamics of the system are represented by the equations (1a) - (2). The model was developed based on nonlinear interactions, both direct and indirect, in order to explore the principal factors that push a vulnerable individual into a homeless state and to identify the most effective strategies to move this population out of homelessness. The time scale will be given in years, with one year as one unit of time for the deterministic analysis. The previous assumptions and definitions result in the following system of nonlinear differential equations:

$$\frac{dV}{dt} = \Lambda - \sigma V - \mu V + k_D s F_D \tag{1a}$$

$$\frac{dD}{dt} = \sigma V - \beta(H,p)D - \mu D - \gamma_D D + k_H q F_H + k_D(1-s)F_D \tag{1b}$$

$$\frac{dH}{dt} = \beta(H,p)D - \mu H - \gamma_H H + k_H(1-q)F_H \tag{1c}$$

$$\frac{dF_H}{dt} = \gamma_H H - k_H F_H - \mu F_H \tag{1d}$$

$$\frac{dF_D}{dt} = \gamma_D D - k_D F_D - \mu F_D \tag{1e}$$

$$\frac{dp}{dt} = \Omega(H)p(1-p) - \varepsilon p \tag{1f}$$

Because we assume a non-constant population, we obtain that $N = V + D + H + F_H + F_D$ and

$$\frac{dN}{dt} = \Lambda - \mu N \tag{2}$$

We define $\beta(H,p) := \beta_1 p + \beta_2 \frac{H}{N}$. The $\beta_1 p(t)$ term incorporates the influence of high poverty communities, whereas $\beta_2 \frac{H(t)}{N(t)}$ incorporates the influence of homeless on disaffiliated individuals. For $\Omega(H)$, we have a linear definition, $\Omega(H) := cH$, where c is the intrinsic growth rate of the high poverty communities with respect to incoming homeless individuals. For simplicity, this linear definition of $\Omega(H)$ assumes that as the homeless population of L.A. increases, so do the high poverty communities of L.A. County. However, with this assumption, we have that the intrinsic growth rate of high poverty communities can grow without bound.

Following the deterministic analysis, we complete stochastic modelling with a Continuous Time Markov Chain process (CTMC) and the Gillespie algorithm via MATLAB. Using our rates and transitions from our deterministic equations, we define probabilities of moving from class to class in the stochastic model. We can randomly generate events to see how the populations of individuals and impoverished city patches change over time. The main reason for this work is to be able to compare the deterministic and stochastic results in order to better understand the effectiveness of temporary housing and vocational training programs. Additionally, we will run various numerical simulations with our deterministic and stochastic models to view different scenarios of expected values and dispersion under which, we may be able to provide an answer to our research question.

3 Mathematical Analysis

To provide an appropriate social meaning of the model (1), it is necessary to prove that the population is bounded from above, suggesting that the number of individuals will not grow to infinity as time tends to infinity. This assumption can be verified by solving the differential equation for the total population. For $N(t)$, we obtain the following:

$$N(t) = \frac{\Lambda}{\mu} + e^{-\mu t}\left(N_0 - \frac{\Lambda}{\mu}\right),$$

where N_0 is the initial total population of the system. Then, taking the limit as $t \to \infty$:

$$\lim_{t\to\infty} N(t) = \frac{\Lambda}{\mu}.$$

Thus, $N(t)$ is always bounded. Additionally, the system is well-defined since we will never have negative population and it is bounded to a certain steady state. These facts make the model well-posed with respect to dynamics of homelessness and its interaction with the environment with respect to the natural world. Now we provide the mathematical analysis of four different sub-models of Model (1) to identify the effect of additional elements such as interventions, starting from the base model until the entire model is explained.

3.1 Model with no Intervention $VDH - P$

We begin with a simplified model that does not include intervention strategies, but rather rates of recovery from homelessness to disaffiliation and disaffiliation to vulnerability. We assume that individuals cannot stay homeless for their entire life and that there exist mechanisms that induce the movement from the homeless state to other states. The simplified version of the model is illustrated in Figure (2). The system has been

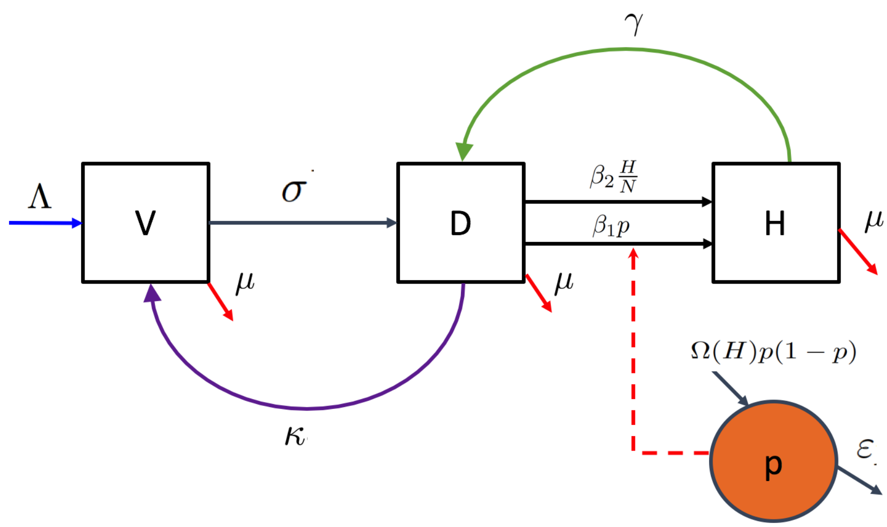

Figure 2: Case 1: Model diagram with no intervention.

normalized in order to work with proportions of the population in classes. Hence, we use v, d, h, f_H, f_D, and p as fractions of the population instead of our original state variables in this case (see Appendix A.2). The system of equations for this simplified system is as follows:

$$\dot{v} = \mu + \kappa d - (\sigma + \mu)v \tag{3a}$$

$$\dot{d} = \sigma v + \gamma h - (\kappa + \mu)d - \beta_1 pd - \beta_2 dh \tag{3b}$$

$$\dot{h} = \beta_1 pd + \beta_2 dh - (\gamma + \mu)h \tag{3c}$$

$$\dot{p} = chp(1-p) - \epsilon p. \tag{3d}$$

Model (3) presents three equilibria. The homeless-free equilibrium (HFE) suggests that there is a community in which, there are neither homeless individuals nor parts of the city under poverty. Additionally, there are two endemic equilibria, one when there is no influence of the environmental component: an environment-free equilibrium (EFE), that is $p^* = 0$ and when it reaches a steady state, as in the case of the Levins model result, $p^* = 1 - \frac{\epsilon}{ch^*}$. Equilibria are summarized as follows:

$$\begin{cases} HFE : (v^*, d^*, h^*, p^*) = (\frac{\kappa+\mu}{\kappa+\mu+\sigma}, \frac{\sigma}{\kappa+\mu+\sigma}, 0, 0) \\ EFE : (v^*, d^*, h^*, p^*) = (\frac{\gamma\kappa+(\beta_2+\kappa)\mu}{\beta_2(\mu+\sigma)}, \frac{\gamma+\mu}{\beta_2}, (1-\frac{1}{\mathcal{R}_0^*}) \cdot \frac{\sigma}{\mu+\sigma}, 0) \\ \begin{cases} Endemic : (v^*, d^*, p^*) = (\mu + \frac{\kappa(\gamma+\mu)h^*}{\beta_1(1-\frac{\epsilon}{ch^*})+\beta_2 h^*}, \frac{(\gamma+\mu)h^*}{\beta_1(1-\frac{\epsilon}{ch^*})+\beta_2 h^*}, 1-\frac{\epsilon}{ch^*}) \\ c\beta_2(2\gamma-\mu)h^{*3} + c\left[(\gamma+\mu)(\kappa\sigma-\kappa-\mu)+\sigma\beta_2+\mu\beta_1\right]h^{*2} + \left[\beta_1(c\mu\sigma-\gamma\epsilon)+\epsilon\beta_2(\gamma+\mu)\right]h^* - \sigma\mu\beta_1\epsilon = 0 \end{cases} \end{cases}$$

The EFE exists if and only if $R_0^* = \frac{\sigma}{\sigma+\mu+\kappa} \cdot \frac{\beta_2}{\mu+\gamma} > 1$, and endemic equilibrium exists if and only if $h^* > \frac{\epsilon}{c}$. For stability, we compute the Jacobian of the system and further we evaluate it at every equilibrium point as follows. Eigenvalues are $\left\{-\epsilon, -\mu, -\kappa-\mu-\sigma, \frac{\beta_2\sigma}{\kappa+\mu+\sigma} - \gamma - \mu\right\}$ and hence, HFE is locally asymptotically stable if and only if $\frac{\beta_2\sigma}{\kappa+\mu+\sigma} - \gamma - \mu < 0$, that is, $\mathcal{R}_0^* = \frac{\sigma}{\sigma+\mu+\kappa} \cdot \frac{\beta_2}{\mu+\gamma} < 1$. We define $\mathcal{R}_0^*$ as the basic reproductive number for model (3).

The $\mathcal{R}_0^*$ is the general reproductive number when considering the possibilities of homeless becoming disaffiliated (at rate γ) and disaffiliated becoming vulnerable (at rate κ). Without incorporating any intervention strategies, that is when $\gamma = \kappa = 0$, this reproductive number is given by

$$\mathcal{R}_0 = \frac{\beta_2\sigma}{\mu(\sigma+\mu)} = \frac{\sigma}{\sigma+\mu} \cdot \frac{1}{\mu} \cdot \beta_2$$

As we observe $\mathcal{R}_0$ is a product of β_2/μ (the inflow over the outflow of the homeless compartment) and $\frac{\sigma}{\sigma+\mu}$, which is the proportion of the population that goes through the vulnerable class and into the disaffiliated class, i.e. losing their job. Before entering homelessness, an individual must go through the disaffiliation of unemployment and not die, hence relating to the fact that $\frac{\sigma}{\sigma+\mu} < 1$.

3.2 Temporary Housing Intervention $(VDHP - F_H)$

In this section the housing assistance intervention is incorporated. The corresponding model diagram is as follows:

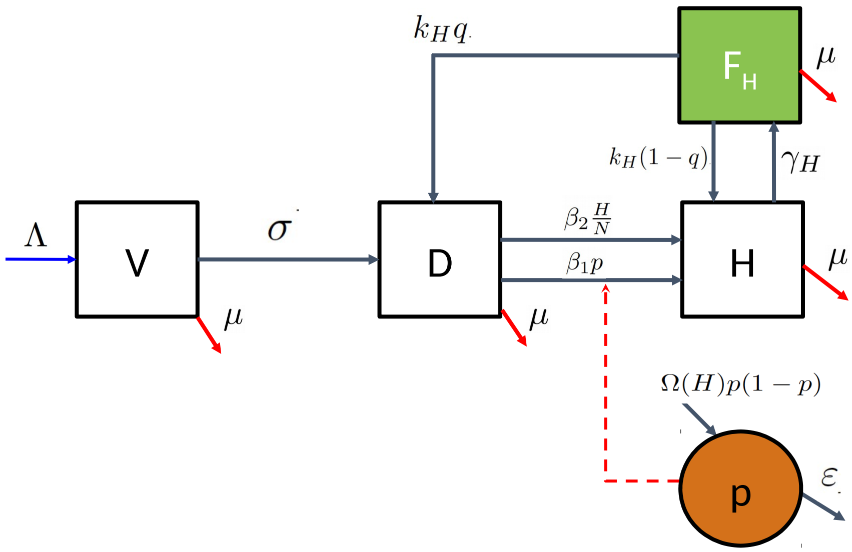

Figure 3: Temporary Housing intervention model diagram.

Then, the system of equations will be as follows:

$$\dot{v} = \mu - (\sigma + \mu)v \tag{4a}$$

$$\dot{d} = \sigma v + k_H q f_H - \mu d - \beta_1 p d - \beta_2 h d \tag{4b}$$

$$\dot{h} = \beta_1 p d + \beta_2 h d - \gamma_H h + (1-q) k_H f_H - \mu h \tag{4c}$$

$$\dot{f_H} = \gamma_H h - k_H q f_H - (1-q) k_H f_H - \mu f_H \tag{4d}$$

$$\dot{p} = chp(1-p) - \epsilon p \tag{4e}$$

Similar to Model (3), model (4) presents 3 equilibria listed below:

$$\begin{cases} HFE : (v^*, d^*, h^*, f_H^*, p^*) = (\frac{\mu}{\mu+\sigma}, \frac{\sigma}{\mu+\sigma}, 0, 0, 0) \\ EFE : (v^*, d^*, h^*, f_H^*, p^*) = (\frac{\mu}{\mu+\sigma}, \frac{\sigma}{\sigma+\mu} \cdot \frac{1}{\mathcal{R}_H}, (1-\frac{1}{\mathcal{R}_H}) \cdot \frac{k_H+\mu}{\gamma_H+k_H+\mu} \cdot \frac{\sigma}{\sigma+\mu}, (1-\frac{1}{\mathcal{R}_H}) \cdot \frac{\gamma_H}{\gamma_H+k_H+\mu} \cdot \frac{\sigma}{\sigma+\mu}, 0) \\ \begin{cases} Endemic : (v^*, d^*, f_H^*, p^*) = (\frac{\mu}{\mu+\sigma}, \frac{\sigma}{\mu+\sigma} - \frac{h^*(\gamma_H+k_H+\mu)}{k_H+\mu}, \frac{h^*\gamma_H}{k_H+\mu}, 1-\frac{\epsilon}{ch^*}) \\ -m_2 h^{*3} + (\beta_2 c m_2 + \beta_2 m_1 + (1-q)k_H m_3)h^{*2} + \beta_2(m_1 c - \epsilon m_2)h^* - \beta_2 c m_1 = 0 \\ m_1 = \frac{\sigma}{\mu+\sigma};\ m_2 = \frac{\mu+\gamma_H+k_H}{\mu+k_H};\ m_3 = \frac{\gamma_H}{k_H};\ m_4 = \gamma_H + \mu \end{cases} \end{cases}$$

where HFE (homeless free equilibrium) is defined as the case where the homeless is not present in the population, EFE (environment free equilibrium) is in the absence of environmental influence. The EFE exists if and only if $\mathcal{R}_H > 1$, and endemic equilibrium exists if and only if $h^* > \frac{\epsilon}{c}$.

For stability, we compute the Jacobian of Model (4), which has eigenvalues

$$\left\{-\epsilon, -\mu, -k_H - \mu, -\mu - \sigma, \frac{\beta_2\sigma - \gamma_H\mu - \gamma_H\sigma - \mu^2 - \mu\sigma}{\mu+\sigma}\right\}.$$

Hence, HFE is locally asymptotically stable if and only if $\frac{\beta_2\sigma}{\mu+\sigma} - \gamma_H - \mu < 0$, that is, if and only if $\frac{\beta_2}{\gamma_H+\mu} \cdot \frac{\sigma}{\mu+\sigma} < 1$. The control reproductive number for Model (4) is calculated using next generation matrix approach:

$$\mathcal{R}_H = \frac{\beta_2(k_H+\mu)}{(\mu(\gamma_H+k_H+\mu) + \gamma_H k_H q)} \cdot \frac{\sigma}{\sigma+\mu}$$

We simplify this reproductive number in terms of $\mathcal{R}_0$ for better understanding:

$$\mathcal{R}_H = \mathcal{R}_0 \cdot c_H,$$

where

$$c_H = \frac{k_H+\mu}{k_H+\mu+\gamma_H+\frac{\gamma_H k_H q}{\mu}}.$$

c_H represents the proportion of individuals who enter the housing assistance program and successfully obtain permanent housing (k_H, γ_H, and q). As the $\gamma_H + \gamma_H k_H q/\mu$ term increases, i.e. the rate of a successful cycle through the entire housing programs increases, or the proportion of success of the housing programs increases, $\mathcal{R}_H$ will decrease. This will minimize the spread of homelessness in an epidemiological sense.

3.3 Vocational Training Intervention $(VDHP - F_D)$

In this section the vocational training intervention is incorporated. The corresponding model diagram is as follows:

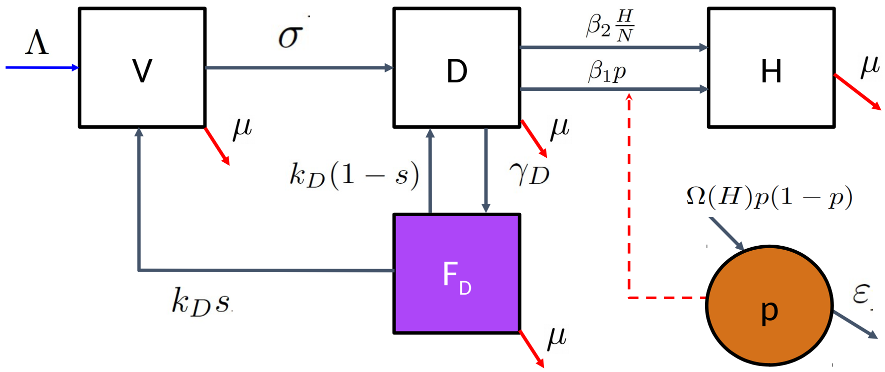

Figure 4: Vocational training intervention model diagram.

Then, the system of equations will be as follows:

$$\begin{aligned}
\dot{v} &= \mu - (\sigma + \mu)v + sk_D f_D \\
\dot{d} &= \sigma v - \gamma_D d + (1-s)k_D f_D - \mu d - \beta_1 pd - \beta_2 hd \\
\dot{h} &= \beta_1 pd + \beta_2 hd - \mu h \\
\dot{f_D} &= \gamma_D d - k_D s f_D - (1-s)k_D f_D - \mu f_D \\
\dot{p} &= chp(1-p) - \epsilon p
\end{aligned} \tag{5}$$

Doing some algebra we found the following equilibria for the Model (5):

$$\begin{cases} HFE:(v^*,d^*,h^*,f_D^*,p^*) = (\frac{\mu(\gamma_D+\mu)+k_D(\mu+\gamma_D s)}{(\gamma_D+\mu)(\mu+\sigma)+k_D(\mu+\gamma_D s+\sigma)}, \frac{\sigma(k_D+\mu)}{(\gamma_D+\mu)(\mu+\sigma)+k_D(\mu+\gamma_D s+\sigma)}, 0, \frac{\gamma_D\sigma}{(\gamma_D+\mu)(\mu+\sigma)+k_D(\mu+\gamma_D s+\sigma)}, 0) \\ EFE:(v^*,d^*,h^*,f_D^*,p^*) = (\frac{\beta_2\mu(k_D+\mu)+\mu s\gamma_D k_D}{\beta_2(\mu+\sigma)(k_D+\mu)}, \frac{\mu}{\beta_2}, (1-\frac{1}{\mathcal{R}_D})\cdot\frac{\sigma}{\sigma+\mu}, \frac{\gamma_D\mu}{(\mu+k_D)\beta_2}, 0) \\ \begin{cases} Endemic:(v^*,d^*,f_D^*,p^*) = (\frac{\gamma_D k_D s}{(k_D+\mu)(\mu+\sigma)}\frac{\mu h^*}{\beta_2} - \frac{\beta_1}{\beta_2 h^*}(1-\frac{\epsilon}{ch^*}) + \frac{\mu}{\mu+\sigma}, \frac{\mu h^*}{\beta_2} - \frac{\beta_1}{\beta_2 h^*}(1-\frac{\epsilon}{ch^*}), \frac{\gamma_D(\beta_1+\mu)}{\beta_2 h^*(k_D+\mu)}\left(\frac{\epsilon}{ch^*}-1\right), \\ \qquad 1-\frac{\epsilon}{ch^*}) \\ c_4h^{*5} + (\sigma c_1 - (\beta_1+\gamma_D+\mu c_5)c_4)h^{*4} + (c_3 - (c_2 + \frac{\beta_1\epsilon c_4}{c} + \beta_2 c_2)h^{*3} + (\frac{\epsilon c_2}{c}(1-\beta_2) \\ -c_2(\beta_1+\gamma_D+\mu c_5) - c_4c_5)h^{*2} - \frac{\epsilon c_2}{c}(2\beta_1+\gamma_D+c_5(\mu-\frac{c_6}{c_2}))h^* - \frac{\epsilon^2 c_2}{c^2} = 0 \\ c_1 = \frac{\gamma_D k_D s\mu}{(k_D+'mu)(\mu+\sigma)\beta_2}; c_2 = \frac{\beta_1}{\beta_2}; c_3 = \frac{\mu}{\mu+\sigma}; c_4 = \frac{\mu}{\beta_2}; c_5 = (1-s)k_D; c_6 = \frac{\gamma_D(\beta_1+\mu)}{\beta_2(k_D+\mu)}; \end{cases} \end{cases}$$

Because all parameters are positive, the HFE always exists, with the existence of EFE dependent on $\mathcal{R}_D = \frac{\beta_2\sigma(k_D+\mu)}{\mu((\gamma_D+\mu)(\sigma+\mu)+k_D(\mu+\gamma_D s+\sigma))}$. In fact EFE exists if and only if $\mathcal{R}_D > 1$. We call $\mathcal{R}_D$ as the control reproductive number for Model (5). In order to determine the stability of the system we compute the general Jacobian matrix. The computed eigenvalues of this matrix are negative, if and only if

$$\beta_2\sigma(k_D+\mu) < \mu((\gamma_D+\mu)(\sigma+\mu)+k_D(\mu+\gamma_D s+\sigma))$$

or equivalently, $\mathcal{R}_D = \frac{\beta_2\sigma(k_D+\mu)}{\mu((\gamma_D+\mu)(\sigma+\mu)+k_D(\mu+\gamma_D s+\sigma))} < 1$. We can rewrite $\mathcal{R}_D$ as

$$\mathcal{R}_D = \mathcal{R}_0 \cdot c_D$$

where

$$c_D := \frac{k_D+\mu}{k_D+\mu+\gamma_D+\frac{k_D\gamma_D s}{\sigma+\mu}}.$$

c_D term can be interpreted as: the proportion of individuals who enter intervention programs and successfully exit them. The γ_D and $\frac{k_D\gamma_D s}{\sigma+\mu}$ terms represent a successful cycle through the vocational training program.

3.4 Full Model ($VDHP - F_H F_D$)

Now we analyse our original model, Model (1), represented in Figure 1. Similar to previous reduced models, the full model (1) has three equilibria calculated as follows:

$$
\begin{cases}
HFE: (v^*, d^*, h^*, f_H^*, f_D^*, p^*) = \left(\frac{\mu(\gamma_D+\mu)+k_d(\mu+\gamma_D s)}{(\gamma_D+\mu)(\mu+\sigma)+k_d(\mu+\gamma_D s+\sigma)}, \frac{\sigma(k_D+\mu)}{(\gamma_D+\mu)(\mu+\sigma)+k_d(\mu+\gamma_D s+\sigma)}, 0, 0, \frac{\gamma_D \sigma}{(\gamma_D+\mu)(\mu+\sigma)+k_d(\mu+\gamma_D s+\sigma)}, 0\right) \\
\begin{cases}
EFE: (v^*, d^*, h^*, f_H^*, f_D^*, p^*) = \left(\frac{\mu}{\sigma+\mu} + \frac{k_D s}{\sigma+\mu} \cdot \frac{\gamma_D}{k_D+\mu} d^*, \frac{c_1}{\beta_2}, \frac{c_2}{c_3} d^* - \frac{\sigma\mu}{c_3(\sigma+\mu)}, \frac{\gamma_H}{k_H+\mu}\left(\frac{c_2}{c_3} d^* - \frac{\sigma\mu}{c_3(\sigma+\mu)}\right), \frac{\gamma_D}{k_D+\mu} d^*, 0\right) \\
\qquad c_1 = \gamma_H + \mu - \frac{\gamma_H k_H (1-q)}{k_H+\mu};\ c_2 = \gamma_D + \mu - \sigma \cdot \frac{k_D s}{\sigma+\mu} \cdot \frac{\gamma_D}{k_D+\mu} - \frac{\gamma_D k_D (1-s)}{k_D+\mu};\ c_3 = \frac{\gamma_H k_H q}{k_H+\mu}. \\
EE: (v^*, d^*, f_H^*, f_D^*, p^*) = \left(\frac{\mu(k_d+\mu)+\gamma_D k_D s}{(k_D+\mu)(\sigma+\mu)} \cdot \frac{c_1 h^*}{F(h^*)}, \frac{c_1 h^*}{F(h^*)}, \frac{\gamma_H}{k_H+\mu} h^*, \frac{\gamma_D}{k_D+\mu} \cdot \frac{c_1 h^*}{F(h^*)}, 1 - \frac{\epsilon}{ch^*}\right), \text{ where} \\
\qquad c_1 := \frac{(\gamma_H+\mu)(\gamma_D+\mu)-\gamma_H k_H (1-q)}{k_H+\mu}, F(h^*) := \beta_1\left(1 - \frac{\epsilon}{ch^*}\right) + \beta_2 h^*, \text{ and } h^* \text{ is a root} \\
\qquad \text{of the polynomial } a_1 h^2 + a_2 h + a_3, \text{ where} \\
\qquad a_1 = \frac{\beta_2 \gamma_H k_H q}{k_H+\mu} - \beta_2 c_1 \\
\qquad a_2 = \frac{\beta_1 \gamma_H k_H q}{k_H+\mu} - \frac{c_1(\mu(\mu(\beta_1+\gamma_D+\mu)+\sigma(\beta_1+\gamma_D))+\beta_1 k_D \sigma + k_D \mu(\beta_1+\mu+\gamma_D s))}{(k_D+\mu)(\mu+\sigma)} \\
\qquad a_3 = \frac{\beta_1 \epsilon (c_1(k_H+\mu) - \gamma_H k_H q)}{c(k_H+\mu)}.
\end{cases}
\end{cases}
$$

EFE exists if and only if the following conditions hold:

1. $\frac{(\gamma_D+\mu)(k_D+\mu)}{\frac{\sigma}{\sigma+\mu} k_D \gamma_D s + k_D \gamma_D (1-s)} > 1$
2. $\frac{(\gamma_H+\mu)(k_H+\mu)}{\gamma_H k_H (1-q)} > 1$
3. $\frac{c_1 c_2 (\sigma+\mu)}{\sigma\mu\beta_2} > 1.$

Due to complexity of the system, stability of the equilibria has to be incorporated numerically (see Appendix A.1). Using next generation approach we calculated the control reproductive number of the Model (1) as

$$\mathcal{R}_{HD} = \frac{\beta_2 \sigma (k_H+\mu)(k_D+\mu)}{(\mu(\gamma_H+k_H+\mu)+\gamma_H k_H q)((\gamma_D+\mu)(\sigma+\mu)+k_D(\mu+\gamma_D s+\sigma))},$$

which can be written as

$$\mathcal{R}_{\mathcal{HD}} = \mathcal{R}_0 \cdot c_H \cdot c_D,$$

that is, we were able to simplify $\mathcal{R}_{\mathcal{HD}}$ in terms of our other two intervention strategies independently. Thus, the two intervention strategies, when incorporated together, both work to decrease the basic reproductive number $\mathcal{R}_0$ with their respective mechanisms as described in the previous subsections.

We conclude Section 3 by summarizing stability analysis of all different models in Table (3). Generally, our analysis shows that in order for a homeless outbreak to die out, the corresponding reproductive numbers need to be less than one, otherwise, endemic equilibria will exist and homelessness will stay in the population.

	Equilibria	Stability
Model (1)	HFE: $(v^*, d^*, h^*, f_H^*, f_D^*, p^*) = (\frac{\mu(\gamma_D+\mu)+k_D(\mu+\gamma_D s)}{(\gamma_D+\mu)(\mu+\sigma)+k_D(\mu+\gamma_D s+\sigma)}, \frac{\sigma(k_D+\mu)}{(\gamma_D+\mu)(\mu+\sigma)+k_D(\mu+\gamma_D s+\sigma)}, 0, 0, \frac{\gamma_D\sigma}{(\gamma_D+\mu)(\mu+\sigma)+k_D(\mu+\gamma_D s+\sigma)}, 0)$; exists always	Stable (num.)
	EFE: $(v^*, d^*, h^*, f_H^*, f_D^*, p^*) = (\frac{\mu}{\sigma+\mu} + \frac{k_D s}{\sigma+\mu} \cdot \frac{\gamma_D}{k_D+\mu} d^*, \frac{c_1}{\beta_2}, \frac{c_2}{c_3} d^* - \frac{\sigma\mu}{c_3(\sigma+\mu)}, \frac{\gamma_H}{k_H+\mu} h^*, \frac{\gamma_D}{k_D+\mu} d^*, 0)$ where h^*, d^*, c_1, c_2, and c_3 are defined and exist as given in Section 3.4.	Unstable (num.)
	EE: $(v^*, d^*, f_H^*, f_D^*, p^*) = (\frac{\mu(k_d+\mu)+\gamma_D k_D s}{(k_D+\mu)(\sigma+\mu)} \cdot \frac{c_1 h^*}{F(h^*)}, \frac{c_1 h^*}{F(h^*)}, \frac{\gamma_H}{k_H+\mu} h^*, \frac{\gamma_D}{k_D+\mu} \cdot \frac{c_1 h^*}{F(h^*)}, 1 - \frac{\epsilon}{ch^*})$, where c_1 and $F(h^*)$ are given in Section 3.4, and h^* is a root of the polynomial $a_1 h^2 + a_2 h + a_3$ as defined in Section 3.4 as well.	Stable (num.)
Model (3)	HFE: $(v^*, d^*, h^*, p^*) = (\frac{\kappa+\mu}{\kappa+\mu+\sigma}, \frac{\sigma}{\kappa+\mu+\sigma}, 0, 0)$; exists always	Stable iff $\mathcal{R}_0^* < 1$
	EFE: $(v^*, d^*, h^*, p^*) = (\frac{\gamma\kappa+(\beta_2+\kappa)\mu}{\beta_2(\mu+\sigma)}, \frac{\gamma+\mu}{\beta_2}, \frac{\beta_2\sigma-(\gamma+\mu)(\kappa+\mu+\sigma)}{\beta(\mu+\sigma)}, 0)$; exists iff $\mathcal{R}_0^* > 1$	Unstable (num.)
	EE: $(v^*, d^*, p^*) = (\mu + \frac{\kappa(\gamma+\mu)h^*}{\beta_1(1-\frac{\epsilon}{ch^*})+\beta_2 h^*}, \frac{(\gamma+\mu)h^*}{\beta_1(1-\frac{\epsilon}{ch^*})+\beta_2 h^*}, 1 - \frac{\epsilon}{ch^*})$ where h^* is root of $a_1 h^3 + a_2 h^2 + a_3 h + a_4$ as defined in Section 3.1; exists iff $h^* > \frac{\epsilon}{c}$	Stable (num.)
Model (4)	HFE: $(v^*, d^*, h^*, f_H^*, p^*) = (\frac{\mu}{\mu+\sigma}, \frac{\sigma}{\mu+\sigma}, 0, 0, 0)$; exists always	Stable iff $\mathcal{R}_H < 1$
	EFE: $(v^*, d^*, h^*, f_H^*, p^*) = (\frac{\mu}{\mu+\sigma}, (1 - \frac{1}{R_H}) \cdot \frac{k_H+\mu}{\gamma_H+k_H+\mu} \cdot \frac{\sigma}{\sigma+\mu}) \frac{\mu(\gamma_H+k_H+\mu)+\gamma_H k_H q}{\beta_2(k_H+\mu)}, (1 - \frac{1}{R_H}) \cdot \frac{\gamma_H}{\gamma_H+k_H+\mu} \cdot \frac{\sigma}{\sigma+\mu}, 0\Big)$; exists $\Longleftrightarrow \mathcal{R}_H > 1$	Unstable (num.)
	EE: $(v^*, d^*, h^*, f_H^*, p^*) = ((\frac{\mu}{\mu+\sigma}, \frac{\sigma}{\mu+\sigma} - \frac{\gamma_H+k_H+\mu}{k_H+\mu} h^*, h^*, \frac{h^*\gamma_H}{k_H+\mu}, 1 - \frac{\epsilon}{ch^*}$ where h^* is a root of the polynomial $b_1 h^3 + b_2 h^2 + b_3 h + b_4$ as defined in Section 3.2	DNE (num.)
Model (5)	HFE: $(v^*, d^*, h^*, f_D^*, p^*) = (\frac{\mu(\gamma_D+\mu)+k_D(\mu+\gamma_D s)}{(\gamma_D+\mu)(\mu+\sigma)+k_D(\mu+\gamma_D s+\sigma)}, \frac{\sigma(k_D+\mu)}{(\gamma_D+\mu)(\mu+\sigma)+k_D(\mu+\gamma_D s+\sigma)}, 0, \frac{\gamma_D\sigma}{(\gamma_D+\mu)(\mu+\sigma)+k_D(\mu+\gamma_D s+\sigma)}, 0)$	Stable iff $\mathrm{R}_D < 1$
	EFE: $(v^*, d^*, h^*, f_D^*, p^*) = (\frac{\beta_2\mu(k_D+\mu)+\mu s\gamma_D k_D}{\beta_2(\mu+\sigma)(k_D+\mu)}, \frac{\mu}{\beta_2}, (1 - \frac{1}{\mathcal{R}_D}) \cdot \frac{\sigma}{\sigma+\mu}, \frac{\gamma_D\mu}{(\mu+k_D)\beta_2}, 0)$	Unstable (num.)
	EE: $(v^*, d^*, h^*, f_D^*, p^*) = (h^*, \frac{\mu h}{\beta_2} - \frac{\beta_1}{\beta_2 h}(1 - \frac{\epsilon}{ch}), \frac{\gamma_D k_D s}{(k_D+\mu)(\mu+\sigma)} \frac{\mu h}{\beta_2} - \frac{\beta_1}{\beta_2 h}(1 - \frac{\epsilon}{ch}) + \frac{\mu}{\mu+\sigma}, \frac{\gamma_D(\beta_1+\mu)}{\beta_2 h(k_D+\mu)}(\frac{\epsilon}{ch} - 1), 1 - \frac{\epsilon}{ch})$ where h^* is a root of the polynomial $p(h) = c_1 h^3 + c_2 h^2 + c_3 h$ as in Section 3.3.	Unstable (num.)

Table 3: **Summary of Analysis:** For stability values that are computed numerically (num.), see Appendix (A.1).

4 Numerical Results

4.1 Parameter Estimation

In order to analyze the performance of the numerical simulations, we provide some values for each parameter. Some of them could be taken directly from the literature (census, reports, scientific journals) while others were estimated according to the best fit to the data found. In the case of Λ, the value could be estimated taking the amount of people that enter into the vulnerable stage in one year. For instance, this would be the number of people per year whose income goes under the poverty line or that live near a natural disaster area, as defined above for our vulnerable population. For the estimation of the μ parameter, we took the time an individual spends in each compartment before going out of the system. This would be the inverse of the average life expectancy of an standard individual in the U.S. In order to estimate σ we took the amount of people going unemployed in a year, given that previously they were in a low wage job, σV. Then, in order to obtain just the value of σ, we divided that value by the amount of people in the vulnerable stage in that year.

The data used for this estimations come from the U.S. census from the years 2018 and 2017. We took the poverty rate, homeless population, unemployed population and total population per state of U.S in 2018 and then took the difference with respect to 2017, in order to obtain the change with respect to one year, that is our unit of time. In order to estimate parameters as β_1 and β_2 we need to do some assumptions, since some of the data needed can not be currently found in the census. First we are assuming that the high poverty areas grow at the same rate as people fall under the poverty line. Second, we consider that the influence produced by β_1 (environmental) and β_2 (social interactions) are independent, so the presence of one of them do not have an effect on the other. We take the following simplified formula for the estimation of the parameters:

$$\frac{\Delta H}{\Delta t} \approx \beta_1 p D + \beta_2 \frac{H}{N} D \tag{6}$$

First, to estimate β_1 we just take the first term of the equation 6 that is: $\frac{\Delta H}{\Delta t} \approx \beta_1 p D$, since we are assuming that each influence is independent on each other. Then have to proceed as it follows:

- Estimate p, that is the change of the poverty rate per year.
- Estimate D, that is the number of unemployed individuals per state.
- Estimate $\frac{\Delta H}{\Delta t}$ that is the change of homeless individuals per state per year.

We multiply our estimates for p and D and make a linear regression with respect to $\frac{\Delta H}{\Delta t}$. Then, β_1 will be the slope of that linear regression 5(a). For β_2 we follow a similar approach, considering just the second term of the equation 6 that is $\frac{\Delta H}{\Delta t} \approx \beta_2 \frac{H}{N} D$. Then we proceed as it follows:

- Compute the proportion of homeless population over the total population per state ($\frac{H}{N}$).
- Estimate D, that is the number of unemployed individuals per state.

We multiply our estimates for $\frac{H}{N}$ times D and make a linear regression with respect to $\frac{\Delta H}{\Delta t}$. Then,$\beta_2$ will be the slope of that linear regression 5(b).

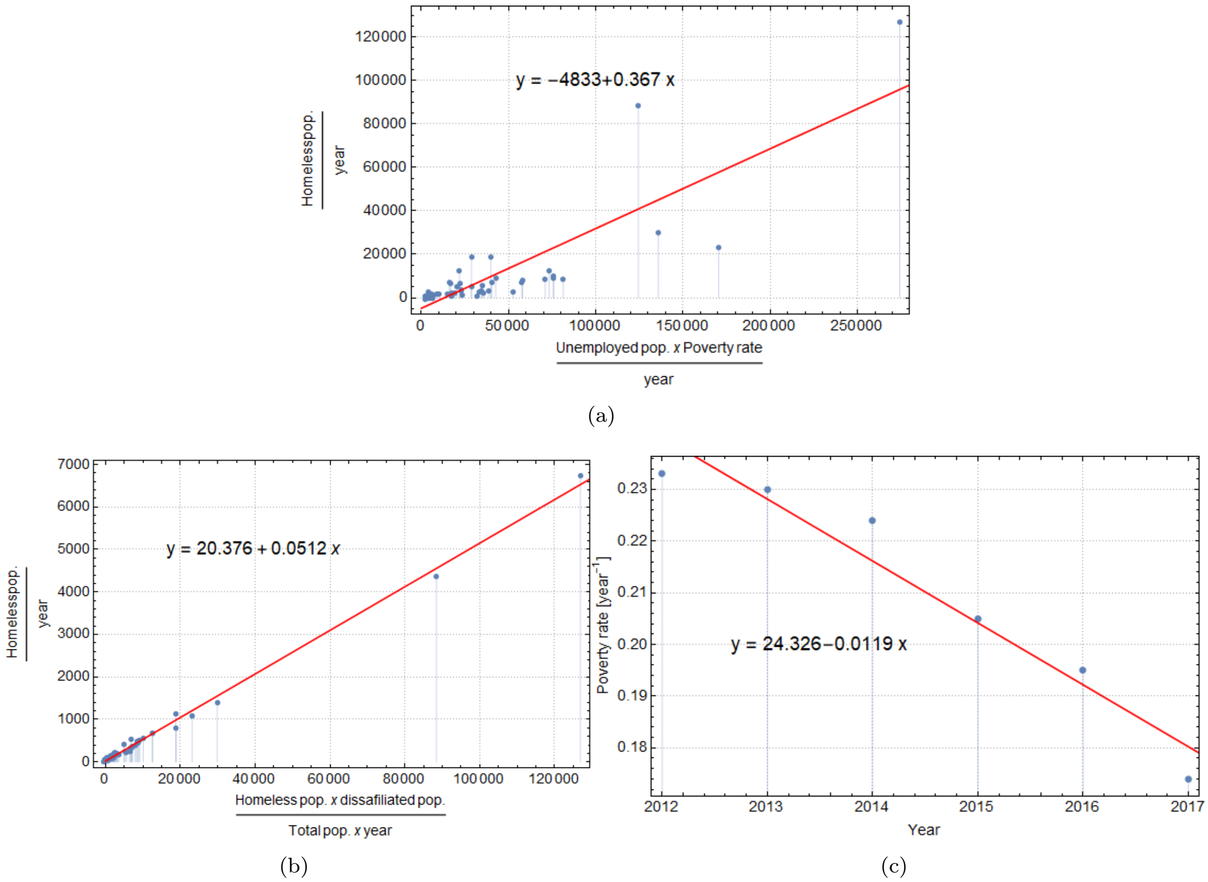

Figure 5: A set of three subfigures: (a) β_1 estimation using linear fitting; (b) β_2 estimation using linear fitting; (c) ϵ estimation using linear fitting;

In the case of ϵ and c, the parameters related to our high poverty communities, we found data on the poverty proportions of L.A. county from 2012 to 2017 [6] We fit a linear regression and find the reduction rate of poverty, ϵ, and intrinsic growth rate c. Because the poverty information from the census is total net change of poverty, we calculate the slope values over 2016 and 2017, along with the current poverty levels at those years to arrive at a system of equations in terms of $r := cH$ and ϵ, in terms of our $\frac{dp}{/}dt$ equation. Once we have these values for ϵ and r, we divide r by the total number of homeless to arrive at c.

For the estimation of γ_H and γ_D, we took the inverse of the average time that an individual remains in the homeless and disaffiliated state before going into the temporary housing assistance or vocational training programs, respectively. Similarly, for k_D and k_H, we took the inverse of the average time an individual remains in the temporary housing and vocational training programs, before moving to the disaffiliated and vulnerable stage, respectively. The estimation of q and s was obtained in the same literature as k_D and k_H, since each program has statistics about the percentage of individuals that finish the program with their own permanent house or job. Analogously, $(1-q)$ and $(1-s)$ represent the proportion of individual that failed and went back into the homeless and disaffiliated states. A summary of these values can be seen in Table 3.

Parameter	Estimate	Reference
μ	$1/78.6 \approx 0.013$ year^{-1}	[23]
Λ	37000 people $\cdot$ year^{-1}	[6]
σ	$7000/28000 \approx 0.25.$ year^{-1}	[6]
β_1	0.367 year^{-1}	estimated
β_2	0.051 year^{-1}	estimated
ϵ	0.012 year^{-1}	estimated
γ_D	$1/0.5 = 2$ year^{-1}	[13]
γ_H	$12/7 \approx 1.71$ year^{-1}	[10]
k_D	$1/0.25 = 4$ year^{-1}	[24]
k_H	$1/2 = 0.5$ year^{-1}	[20]
q	0.65	[21]
s	0.56	[24]

Table 3: Parameter estimations.

4.2 Numerical Simulations

Here we present our result of simulations performed via both deterministic and stochastic continuous time Markov chain (CTMC) approaches. We mention that state p is continuous, therefore, in order to keep this state variables discrete, we discretize the variable $p(t)$ into N_p patches instead of thinking about $p(t)$ as a fraction of L.A. County. Therefore, we define the random vector of six discrete states, based on the deterministic model:

$$X(t) = [V(t), D(t), H(t), F_H(t), F_D(t), p(t)]$$

The transition probability function for all the events in Model (1) is explained in Appendix A.4. For this numerical simulation, note that we simplified our intrinsic growth rate $\Omega(H)$ to $\frac{H(t)}{N(t)}$ instead of $cH(t)$ as in the deterministic equation. For the simulations, we arbitrarily take a population of $N = 500$ individuals and discretize the city into $N_p = 500$ patches for Levins model considerations as well. All of the simulations start at a balanced equilibrium obtained with the model baseline parameters in Table 3, unless stated otherwise. For the stochastic simulations we will perform an ensemble of runs by seeding the same initial condition.

4.2.1 Influence of Intervention Strategies on Homeless Population

To answer our research question, we first consider the general effect that the proposed intervention strategies have on the homeless population of L.A. County through numerical simulation of the time series and plotting the deterministic solution numerically for different models defined in Section 3. We select initial condition as $(V(0), D(0), H(0), F_H(0), F_D(0), p(0) = (400, 50, 50, 0, 0, 50)$.

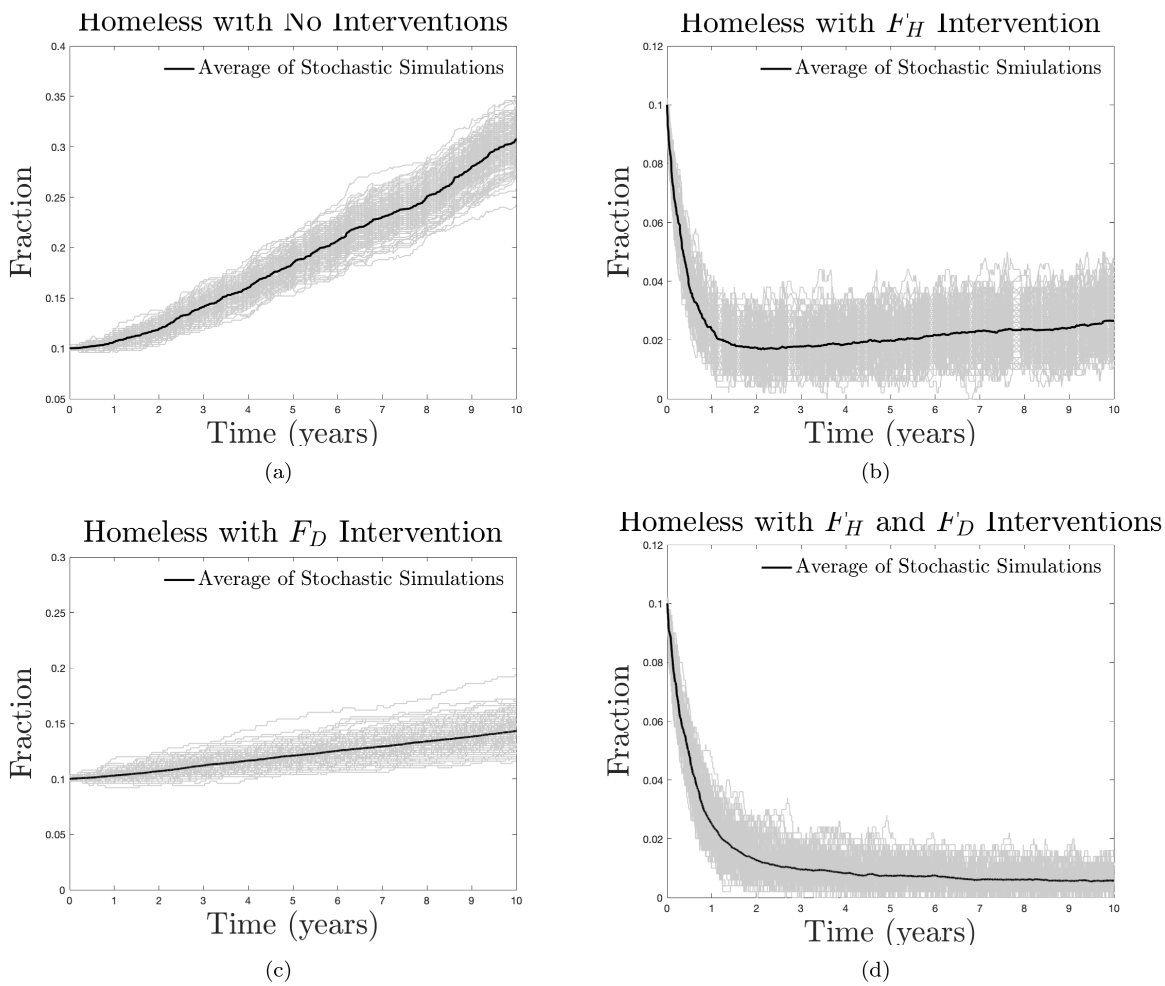

Figure 6: Time-series of fraction of homeless individuals: The light areas are the result for 200 different stochastic simulations the dark curve is the mean value of those simulations.

In Figures 6 and 7, we present the time series of the fraction of homeless population with different combinations of our intervention strategies along with distributions of endemic homeless prevalence for the 200 simulations. With no interventions, the prevalence of homelessness tends to anywhere from 28% to 36% of the entire vulnerable population after 10 years, according to our distributions, within 2 standard deviations of the mean. These results makes mathematical sense because in our special case with no interventions, individuals cannot leave the homeless class. Including only the vocational training intervention, we have a marked decrease in the prevalence of homelessness after 10 years: some 12% to 17% of the vulnerable population is homeless. However, this is still an undesirable increase of the homeless population. With the inclusion of the temporary housing intervention program, we have a decrease in the prevalence of homelessness. After 10 years and beyond, 0 to 6% of the population is homeless. In fact, after some two years with temporary

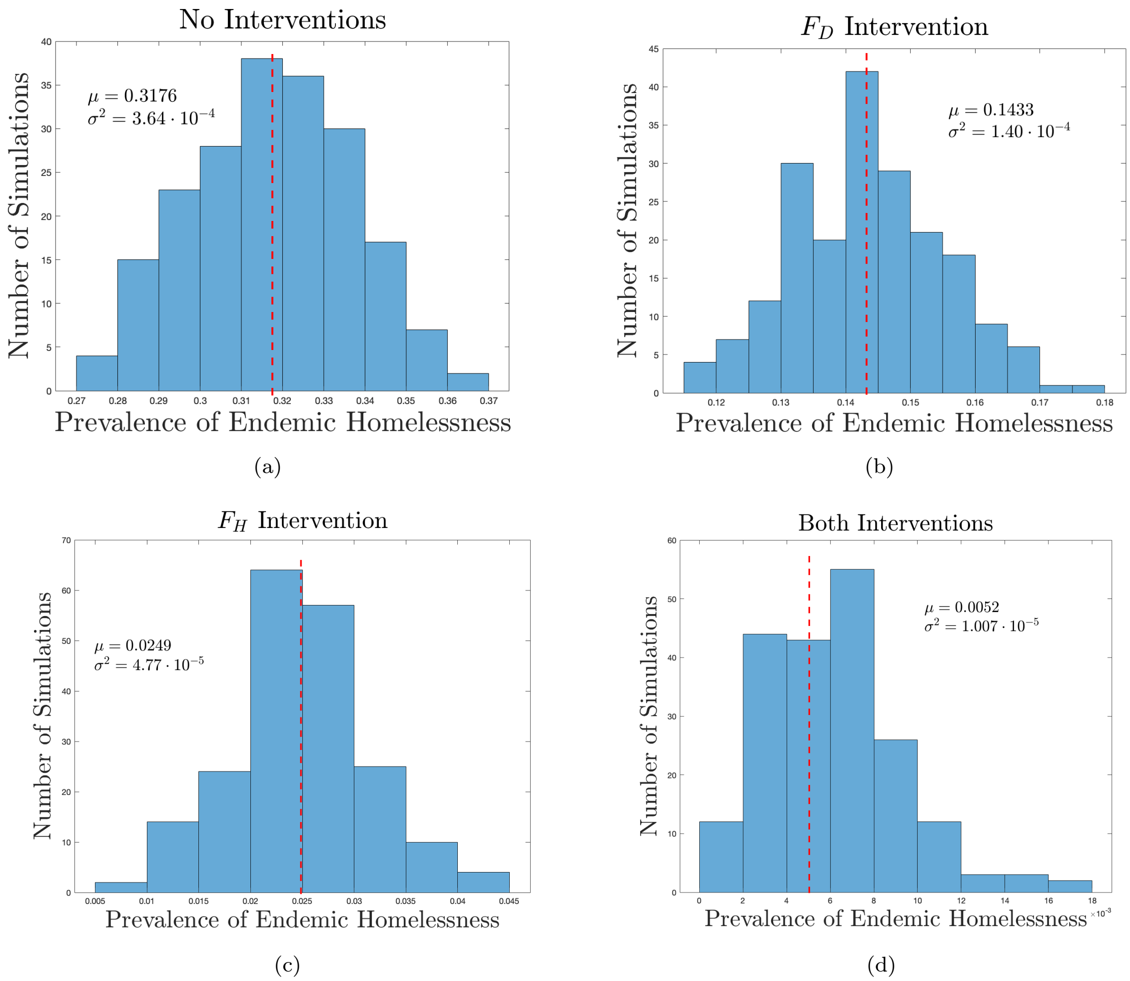

Figure 7: Distributions of endemic homeless prevalence

housing assistance programs, the homeless population reaches a minimum of approximately 2%. Furthermore, when both intervention strategies are included, the homeless population reaches a low prevalence of 0 to 1% after 10 years. Clearly, this would be the case with both intervention strategies in place. Surprisingly, there is a quick initial decrease in homeless prevalence when the temporary housing program is in use. The major conclusion from this stochastic work is that to treat the problem of homelessness in the short term, housing assistance programs are the most effective. On the other hand, to reduce homeless prevalence in the long term, disaffiliated individuals need to be provided with a steady income, a sustainable approach to ending homelessness. We examine this further in the subsequent section with sensitivity analysis.

4.2.2 Sensitivity Analysis

Sensitivity analysis can be defined as how the uncertainty in the output of a mathematical model or system can be divided and allocated to different sources of uncertainty in its parameters, [25]. In order to determine the role that each parameter plays in the dynamics of the system, it is necessary to do sensitivity analysis considering the parameters involved in it. In this subsection we conduct a sensitivity of reproductive numbers and quasi-stationary state for homeless people with respect to all involved parameters.

Table (5) gives the relative sensitivity indices for the change in the reproductive numbers with respect to the model parameters at the baseline parameter values. If the parameter p changes by $x\%$, then $\mathcal{R}$ will change by $x\mathcal{S}_p^{\mathcal{R}}\%$, where $\mathcal{R}$ here stands for a general reproductive number. For no intervention Model (3), or when the resources are invested in vocational training programs only–Model (5)–the most sensitive positive parameter is the per capita rate of transition from disaffiliated to homeless via homelessness influence (β_2), followed by rate of transition from vulnerable to disaffiliated (σ). These results indicate that in the absence of any intervention strategies, the most effective way to control homeless spread is through decreasing β_2, which corresponds to an increase in the time a person stays unemployed under the influence of homeless environment. The next most effective one under the same conditions would be decreasing σ, which is increasing the amount of time people remain in the vulnerable stage before becoming disaffiliated via unemployment. Another effective strategy for Model (5) is increasing γ_D , that is, increasing recruitment into a vocational training program. However, when the resources are invested in temporary housing programs only– Model (4)–the parameter σ plays a more significant role than β_2. Also, decreasing the time that an individual remains homeless before going to a housing assistance program has a non-negligible impact, γ_H. Finally, for the full model of combining both programs, the most sensitive positive parameter is β_2, followed by k_H. The most sensitive negative parameter is γ_H, followed by k_D and γ_D. This shows us that in a system where both strategies are combined, the most effective parameter is β_2, which is increasing the time a person remains disaffiliated when it is being influenced by homelessness. Otherwise, we decrease k_H, or in other words, increase the time a person remains in the housing program before going to disaffiliated state.

Quantity of Interest	Parameter of Interest	Sensitivity Index
Basic Reproductive Number $\mathcal{R}_0$	μ	-1.01022
	β_2	1
	σ	0.987175
Control Reproductive Number $\mathcal{R}_c$ In presence of F_H	β_2	1
	γ_H	-0.996887
	σ	-0.0148842
	k_H	0.0111039
	μ	0.987175
Control Reproductive Number $\mathcal{R}_c$ In presence of F_D	μ	-1.01022
	β_2	1
	γ_D	-0.825879
	σ	0.751997
	k_D	0.0841029
Control Reproductive Number $\mathcal{R}_c$ In presence of F_D and F_H	β_2	1
	γ_H	-0.988593
	k_H	0.98763
	k_D	-0.912658
	γ_D	-0.825879
	σ	-0.248003
	μ	-0.0124967

Table 5: Relative sensitivity index of Reproductive Numbers

In addition, we conduct sensitivity of quasi-stationary state of homeless population for four different presented models, Figure (8). For the no intervention system when there is no environmental influence ($p = 0$) (Model (3)), we have that the most sensitive positive parameter is the per capita rate of transition from disaffiliated to vulnerable (κ). The most sensitive negative parameter is the rate of transition from disaffiliated to homeless (β_2), followed by the rate of transition from vulnerable to disaffiliated (σ). This result tells us that, in a system where no interventions strategies are implemented the most effective parameter to reduce endemic homelessness is increasing the time that a person remain in disaffiliated stage before going to the vulnerable state.

For the case where there is no intervention but the environmental influence is present ($p \neq 0$) (Figure 8(b)), we have that the most sensitive positive parameter is the per capita rate of transition from disaffiliated to vulnerable (κ). The most sensitive negative parameter is the per capita death rate (μ). In this case since the homeless endemic equilibrium expression is dependent of the disaffiliated population we set up $D = 100$. This result tells us that the most effective strategy is increasing κ, that is decreasing the time a person remains disaffiliated before going to the vulnerable state.

For the case where we only have the temporary housing assistance intervention and no environmental influence ($p = 0$) (Figure 8(c)), the most sensitive positive parameter is the per capita rate of transition from a housing program to the disaffiliated stage (k_H), followed by the rate of transition from homelessness to housing program(γ_H). The most sensitive negative parameter is the per capita rate of transition from disaffiliated to homeless state. This result tells us that the most effective strategy is decreasing k_H that is increasing the time a person remain in the housing assistance program before going to disaffiliated state.

For the case where we only have the vocational training intervention and no environmental influence ($p = 0$) (Figure 8(d)), the most sensitive positive parameter is the per capita death rate (μ), followed by the rate of transition from disaffiliated to vocational training program (γ_D). The most sensitive negative parameter is the per capita rate of transition from disaffiliated to homeless (β_2), followed by the rate of transition from vulnerable to disaffiliated (σ). This result tells us that the most effective strategy is decreasing (γ_D) that is decreasing the time a person remain in the disaffiliated state before going to a vocational training program.

Overall, σ and β_2 are most effective at preventing new homelessness incidence, while k_H and γ_D are most effective for reducing endemic homelessness.

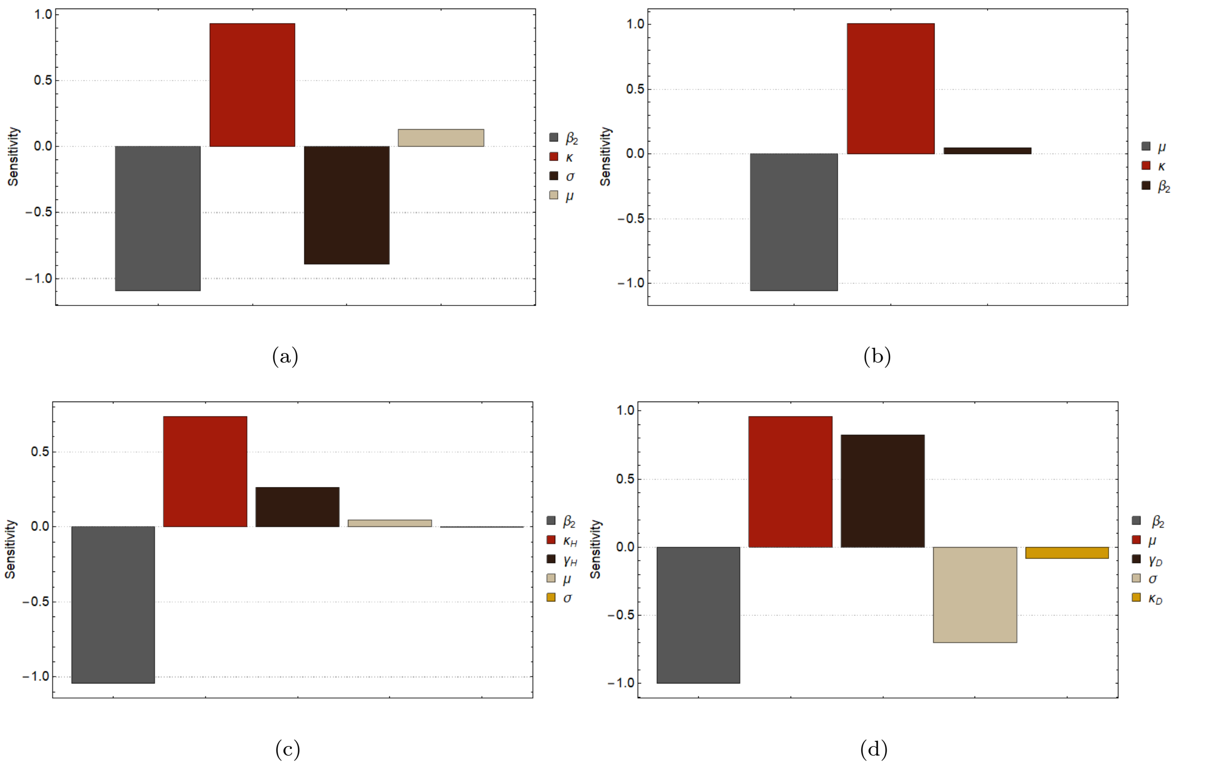

Figure 8: Sensitivity analysis of the Homeless Endemic Equilibrium points: (a) No interventions homeless equilibrium with $P = 0$ $(VDHP)$; (b) no interventions homeless equilibrium with $P \neq 0$ $(VDHP)$ and $D = 100$; (d) homeless equilibrium with vocational training intervention $(VDHP - F_D)$ and, (c) homeless equilibrium with temporary housing intervention interventions $(VDHP - F_H)$.

Besides one-dimensional sensitivity analysis, we examine the effects of different combinations of parameters on homelessness at the quasi-stationary state, Figure(9). To begin with, we have our primary parameters in the transition from disaffiliation to homelessness, the influence of an impoverished environment (β_1) and the influence of social interactions between disaffiliated and homeless individuals (β_2). The left panel of Figure (9) suggests that the influence of an impoverished environment (β_1) is almost negligible in comparison with the influence of social interactions of disaffiliated with homeless individuals (β_2). However, in the right panel, we observe an optimal point for k_H while changing k_D. That is increasing the time of housing assistance monotonically does not necessarily reduces the number of homeless people. We are guessing this trend is because of competition between the two programs, housing assistance and vocational training; more vocational training may attract more individuals from F_H and then D compartment. Therefore, more people lose the risk of being homeless.

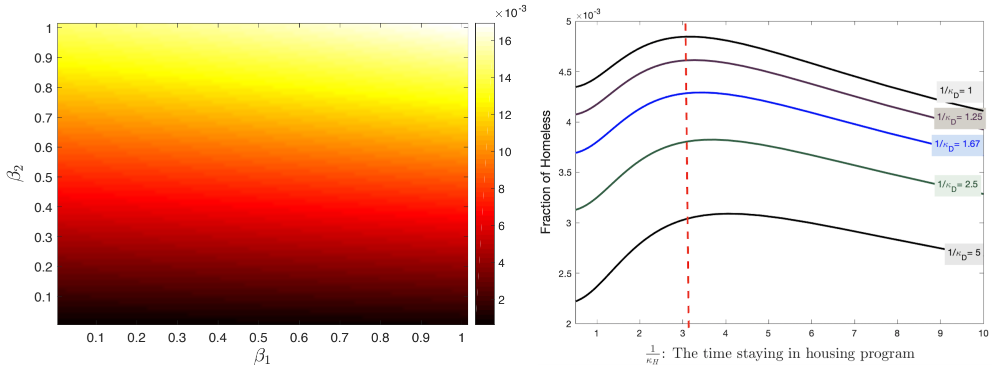

Figure 9: (Left) **Homelessness v.s β_1 v.s β_2:** the influence of an impoverished environment (β_2) is almost negligible in comparison with the influence of social interactions β_2 (Right) **Homelessness v.s k_H for different k_D values:** Increasing the time of housing assistance does not reduce the number of homeless

4.2.3 Influence of Intervention Success Rates on Homeless Population and Cost Analysis

In Figure 10, as stated earlier, s and q stand for the proportion of individuals successfully moving from F_D to V (the vocational training class to vulnerable) and F_H to D (temporary housing to disaffiliated) respectively. For each of the line graphs, the assumption is that one variable is controlled for, or held constant, while the variable labelled in the x-axis is graphed. Thus, in the case of $q = 0$, the fraction of homelessness after 10 years is .035 and decreases steadily to .05 when $q = 1$, all while s is held constant. Further, the optimal number of individuals to decrease homelessness is determined by the maximum slope of the graph. Thus, for the graph of q, it appears as though the optimal number of individuals to reduce homelessness is between 5-10 percent of those in temporary housing. Similarly, for the graph of s, the optimal ratio of individuals may be at 10 percent of those in the vocational training class. Further, since both graphs appear to plateau at around .4 to .5, we can conclude that the cutoff for the effectiveness of each type of intervention is around 40 percent of the designated population. In addition, as the proportion of individuals increases for both values, it appears as though both types of interventions exhibit diminishing marginal reductions, with each graph showing horizontal asymptotes.

In terms of cost analysis, Los Angeles Homeless Services Authority estimates that the total cost to house the individuals and youth homeless population in L.A. County per day is approximately $194,100, which translates to $21 per person per day [8]. On the other hand, [26] estimates the complete cost structure of providing job assistance taking into consideration fixed and variable costs to maintain a vocational training program. The estimate of this study is $1.37 dollars per student for three large cities in the US in 1968 which, corresponds to $11.80 per student per day once they have been adjusted to inflation, using information from the Federal Reserve Bank. Other estimates about the cost to provide job assistance and training is

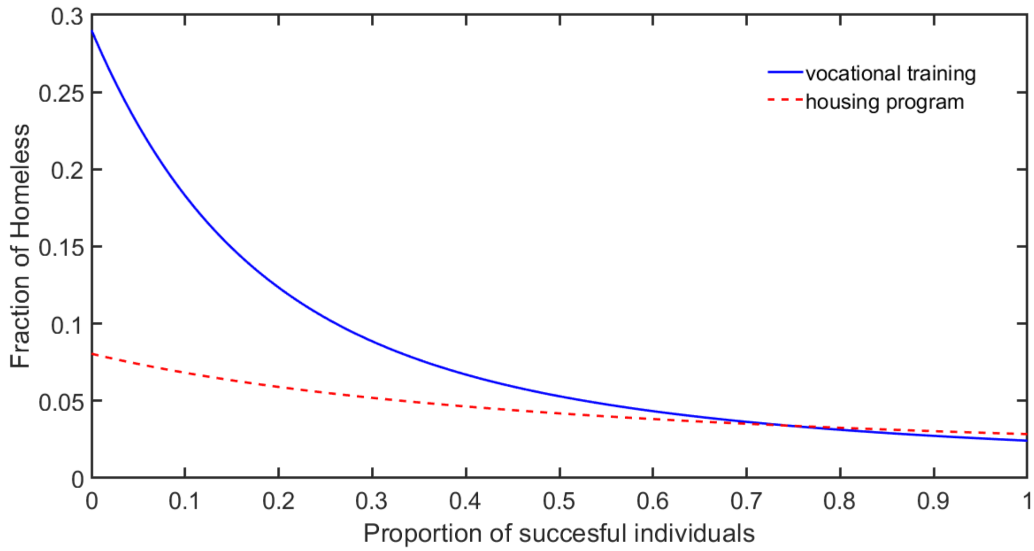

Figure 10: Fraction of homeless individuals vs. the successful proportion of individuals after a housing assistance (s) or a vocational training program (q).

approximated from the statistics of the SEs of the REDF program, started in 2011 in San Francisco which are similar to the mentioned cost.

5 Discussion

Homelessness still poses a heavy social burden for cities like L.A. County, where more than fifty thousand individuals were identified as homeless during 2018 [8]. We developed a compartmental model, in order to study the effectiveness of temporary housing assistance and vocational training programs in controlling the dynamics of homelessness in L.A. County, assuming an impoverished environment. We consider both a deterministic as well as a stochastic approach to analyze the sequence of events including different combinations of the two discussed intervention strategies.

We constructed a stochastic model to capture the variation of results, considering both demographic and environmental stochasticity. The results suggest that there should be a focus of efforts on temporary housing programs in L.A. County in the short term (one to two years) but that vocational training is necessary for sustainable reduction of homeless prevalence in the long term (10+ years). Although vocational training programs do lead to additional reduction in the prevalence of homelessness, they pale in comparison to the benefits that temporary housing brings. Housing assistance programs, as mentioned by Wodon [27], are seen as a short term investment versus long term investments such as vocational training programs for the control of homeless prevalence. Providing housing will remove individuals from the streets, but only after providing disaffiliated individuals with a source of income will the problem be best treated in the long term.

With respect to the likelihood of an outbreak of homelessness, the parameters β_2 and k_H have a positive relationship, whereas γ_H, $k\gamma_D$, and k_D all have a negative relationship. Because β_2 corresponds to the social interactions drawing disaffiliated individuals into homelessness, there is neither a clear nor simple method of lowering this parameter value. For k_H, though, it is necessary to increase the time individuals remain in temporary housing programs. Sociologically, this represents protecting the most vulnerable individuals from both disaffiliation and homelessness. On the other hand, with our parameters of negative relationships, it is necessary to decrease the average time individuals remain homeless or disaffiliated, not a shocking revelation. But with k_D, it is also advisable to reduce the time individuals remain in vocational training programs as well; unlike those in housing assistance programs, individuals in vocational training programs will be better prepared and more likely to re-enter the vulnerable population.

In Section 4.2.3, we considered the success rate of our intervention strategies on the prevalence of homelessness after 10 years. For each variable q and s, the optimal proportion for reducing homelessness was found, which corresponded to the largest slope of the graph and indicated the points where the interventions would bring about the most change in endemic homeless prevalence. Additionally, due to diminishing marginal returns of each programs, after certain thresholds of successful individuals the programs cease to decrease endemic homeless prevalence. In fact, this lower limit of 2 percent signifies that both intervention strategies are ineffective in reducing homelessness beyond this small number. In addition, based on the data from the REDF studies and the Los Angeles Homeless Authority, the cost per person was also calculated for each intervention strategy. In the future, using this data, a cost analysis could be conducted to determine the costs of the optimal proportions for q and s found in the results sections. Adding the cost factor would be useful in broadening the sociological interpretation of our research. Finally, a comparison could be drawn between the daily or yearly costs of each intervention program to also aid in social significance.

To synthesize our analytical and numerical results succinctly, we have that the likelihood of a homeless outbreak will occurring in a city with a minor homeless population depends primarily on social interactions and interventions such as housing assistance and vocational training programs. Moreover, contrary to our hypothesis about the Levins model considerations, the impact of impoverished environments on the prevalence of homelessness are relatively negligible. Under current intervention rates or higher, Social interactions are the leading factor that can influence the likelihood of an outbreak. However, with extremely low or no interventions, environmental influence does seem to play a larger role than social interactions.

With our time series and work with sensitivity analysis, we have that housing assistance programs are the most effective at lowering the likelihood of an outbreak in cities with low homeless population. However, if the goal is to lower endemic levels of homelessness, a focus on vocational training programs is the most appropriate aproach.

5.1 Conclusions

Homelessness is a major socioeconomic issue in the United States, especially in L.A. County, where the homeless population is about 53,000 individuals [8], which is almost 10 percent of the total homeless population [14]. Studies have suggested several factors associated with lifetime homelessness including poor family functioning, regional socioeconomic disparity, isolation from and association with social network communities, mental health issues, and addiction problems. However, what has made the homeless issue difficult is the inability to clearly target a vulnerable population and then implement a set of strategies or interventions to keep them from being homeless. The purpose of this study was to properly determine the efficiency of two types of interventions, temporary housing and vocational training, on the overall homeless rate in L.A. County. To this end, a compartmental model was used to separate the poor population into three stages: vulnerable, disaffiliated, and homeless, all based on the Levins biological model developed in 1969. Overall, our results indicate that temporary housing is a more effective way to reduce the homeless rate than vocational training.

One of the largest challenges we faced during our study was with respect to our mathematical analysis of the most general model. We were able to find limited endemic equilibria for each of the smaller cases and struggled with finding closed form expressions for stability of such equilibria. To add, when we attempted time scale analysis to reduce the dimension of our model, our resultant system equations was increasingly complex. The existence of a homeless-free equilibrium was dependent on parameter relations and also only existed when $p^* = 0$. Attempting to find endemic equilibria or stability information was almost as lengthy as the general model. Another limitation during the entirety of our study was our decision on a research question and a general specificity of our research scope.

In order to compute more detailed behavior related to the endemic homelessness equilibrium and stability it is necessary to look more deeply into the different time scales that our model presents in order to do a better analysis from that point of view. Another possible research approach would be analyzing the dynamics of the system under a limited budget scenario, in order to have a more realistic representation of the system. In that scenario we would look for the optimal investment forwarded to each strategy that makes for the most effective combination. For further research we would be like to investigate how the dynamics or parameters might change when we limit our population to just one race, focusing on African Americans. Concentrating on just African Americans may affect both our dynamics and parameters because the targeted population would then be more homogeneous. Additionally, the reason for this approach is that African Americans are an over-represented proportion in the homeless population. Finally, in order to keep track of how effective the interventions are over time at large time scales, we would make the transitions between stages dependent on time, perhaps based on seasonal fluctuations of homelessness, to better identify the most optimal time to strengthen each of them.

Acknowledgements

We would like to thank Dr. Carlos Castillo-Chávez, Founding and Co-Director of the Mathematical and Theoretical Biology Institute (MTBI), for giving us the opportunity to participate in this research program. We would also like to thank Co-Director Dr. Anuj Mubayi as well as Coordinator Ms. Rebecca Perlin and Management Intern Ms. Sabrina Avila for their efforts in planning and executing the day to day activities of MTBI. We also want to give special thanks to our mentors Dr. Viswanathan Arunachalam, Dr. Christopher Kribs, Dr. Carlos M. Hernández-Suárez, Dr. José Flores, Dr. Asma Azizi, and Cesar Montalvo. This research was conducted as part of 2019 MTBI at the Simon A. Levin Mathematical, Computational and Modeling Sciences Center (MCMSC) at Arizona State University (ASU). This project has been partially supported by grants from the National Science Foundation (NSF – Grant MPS-DMS-1263374 and NSF – Grant DMS-1757968), the National Security Agency (NSA – Grant H98230-J8-1-0005), the Office of the President of ASU, and the Office of the Provost of ASU.

A Appendices

A.1 Jacobian Calculations and Numerical Stabilities

For our cases which need numerical values for our eigenvalues, we give them here after substituting our equilibria and parameter values into the corresponding Jacobians. For Case 1, our endemic equilibrium is dependent on the roots of a cubic polynomials, which only yielded one real root and thus one set of eigenvalues. We give the Jacobians and then numerical eigenvalues:

$$J_{M1} = \begin{pmatrix} -\mu-\sigma & \kappa & 0 & 0 \\ \sigma & -\beta_1 p - \beta_2 h - \kappa - \mu & \gamma - \beta_2 d & -\beta_1 d \\ 0 & \beta_1 p + \beta_2 h & \beta_2 d - \gamma - \mu & \beta_1 d \\ 0 & 0 & cp(1-p) & ch(1-2p) - \epsilon \end{pmatrix}$$

Substituting at HFE, the Jacobian matrix is

$$J_{M1}(HFE) = \begin{pmatrix} -\mu-\sigma & \kappa & 0 & 0 \\ \sigma & -\kappa-\mu & \gamma - \frac{\beta_2\sigma}{\kappa+\mu+\sigma} & -\frac{\beta_1\sigma}{\kappa+\mu+\sigma} \\ 0 & 0 & -\gamma-\mu+\frac{\beta_1\sigma}{\kappa+\mu+\sigma} & \frac{\beta_1\sigma}{\kappa+\mu+\sigma} \\ 0 & 0 & 0 & -\epsilon \end{pmatrix}$$

$$EFE = \{-271.234, -0.01272, -0.3333, 10.9029\} \text{ (Unstable)}$$

$$EE = \{-22.4029, -4.7756, -0.2685, -0.0127\} \text{ (Stable)}$$

In Case 2, we have our eigenvalues for the EFE. However, our endemic equilibrium does not exist, as it depends on h^* being the root of a cubic polynomial; our parameter values only gave negative or complex roots.

$$J_{M2} = \begin{pmatrix} -\mu-\sigma & 0 & 0 & 0 & 0 \\ \sigma & -\beta_1 p - \beta_2 h - \mu & f_H q k_H - \beta_2 d & hqk_H & -\beta_1 d \\ 0 & \beta_1 p + \beta_2 h & \beta_2 d - \gamma_H + f_H(1-q)k_H - \mu & h(1-q)k_H & \beta_1 d \\ 0 & 0 & \gamma_H & -k_H - \mu & 0 \\ 0 & 0 & c(1-p)p & 0 & ch(1-2p) - \epsilon \end{pmatrix}$$

Substituting at HFE, we get

$$J_{M2}(HFE) = \begin{pmatrix} -\mu-\sigma & 0 & 0 & 0 & 0 \\ \sigma & -\mu & -\frac{\beta_2\sigma}{\mu+\sigma} & 0 & -\frac{\beta_1\sigma}{\mu+\sigma} \\ 0 & 0 & \gamma_H - \mu + \frac{\beta_2\sigma}{\mu+\sigma} & 0 & \frac{\beta_1\sigma}{\mu+\sigma} \\ 0 & 0 & \gamma_H & k_H - \mu & 0 \\ 0 & 0 & 0 & 0 & -\epsilon \end{pmatrix}$$

$$EFE = \{-4.9445, -1.1587, 0.1506 + 0.3529i, 0.1506 - 0.3529i, -0.2627\} \text{ (Unstable)}$$

In case 3, we give the general Jacobian, the Jaobian at HFE, and stability information for our EFE and EE:

$$J_{M3} = \begin{pmatrix} -\mu - \sigma & 0 & 0 & k_D s & 0 \\ \sigma & -\beta_1 p - \beta_2 h - \mu - \gamma_D & -\beta_2 d & k_D(1-s) & -\beta_1 d \\ 0 & \beta_1 p + \beta_2 h & \beta_2 d - \mu k - \gamma_H & 0 & \beta_1 d \\ 0 & \gamma_D & 0 & -\mu - k_D & 0 \\ 0 & 0 & cp(1-p) & 0 & ch(1-2p) - \epsilon \end{pmatrix}$$

Then, replacing the HFE equilibrium we can find the following Jacobian matrix

$$J_{M3}(HFE) = \begin{pmatrix} -\mu - \sigma & 0 & 0 & k_D s & 0 \\ \sigma & -\mu - \gamma_D & -\frac{\beta_2 \sigma (\mu + k_D)}{c_1} & k_D(1-s) & -\frac{\beta_1 \sigma (\mu + k_d)}{c_1} \\ 0 & 0 & \frac{\beta_2 \sigma (\mu + k_D)}{c_1} - \mu - \gamma_H & 0 & \frac{\beta_1 \sigma (\mu + k_D)}{c_1} \\ 0 & \gamma_D & 0 & -\mu - k_D & 0 \\ 0 & 0 & 0 & 0 & -\epsilon \end{pmatrix}$$

where $c_1 = \mu^2 + (\sigma + \gamma_D + k_D)\mu + (s\gamma_D + \sigma) k_D + \sigma \gamma_D$.

$$EFE = \{-3.6869, -2.4846, 0.0136, -0.0130, -0.012\} \text{ (Unstable)}$$

$$EE = \{-54.1356, -6.1931, -1.2858 + 3.4452i, -1.2858 - 3.4452i, 0.0176\} \text{ (Unstable)}$$

For our general model:

$$J = \begin{pmatrix} -\sigma - \mu & 0 & 0 & 0 & k_D\, s & 0 \\ \sigma & -\beta_1\, p - \beta_2\, h - \gamma_D - \mu & -\beta_2\, d & k_H\, q & k_D\, (1-s) & -\beta_1\, d \\ 0 & \beta_1\, p + \beta_2\, h & \beta_2\, d - \gamma_H - \mu & k_H\, (1-q) & 0 & \beta_1\, d \\ 0 & 0 & \gamma_H & -k_H - \mu & 0 & 0 \\ 0 & \gamma_D & 0 & 0 & -k_D - \mu & 0 \\ 0 & 0 & cp\,(1-p) & 0 & 0 & ch\,(1-2p) - \epsilon \end{pmatrix}$$

$$HFE = \{-5.0834, -1.9260, -1.1921, -0.3010, -0.0127, -0.012\}\ \text{(Unstable)}$$

$$EFE = \{-5.0978, 1.4837, -1.1626 + 0.2106i, -1.1626 - 0.2106i, -0.0251, -0.0127\}\ \text{(Unstable)}$$

$$EE = \{-5.2835, -5.2342, -1.6190 + 0.5156i, -1.61890 - 0.5156i, -0.2459, -0.0127\}\ \text{(Stable)}$$

after substituting our disease-free equilibrium and parameters. So our HFE in the general model is stable and for our EFE is unstable.

A.2 $\mathcal{R}_{HD}$ Calculations

Before beginning our work, we take our equilibrium as $t \to \infty$ so that our population is assumed constant. Then, we can normalize our system and work with proportions of our original state variables with respect to the entire population. These variables will be lowercase in contrast of our original uppercase state variables. We consider our calculation for the $\mathcal{R}_0$ of the whole system, which includes v, d, h, f_H, f_D, and p. The other three $\mathcal{R}_0$ values were calculated similarly and are therefore omitted. We start with the disease-free equilibrium, or in our case the homeless-free equilibrium, by considering the six equations for the whole system and setting h, the homeless variable, equal to 0.

Our homeless-free equilibrium is found by setting $h^* = 0$. Thus, from here, the sixth equation reduces to:

$$0 = 0 - \epsilon p \longrightarrow p^* = 0$$

And by equation 3, we have

$$0 = (1-q)k_H f_H \longrightarrow f_H^* = 0$$

From equation 1,

$$0 = \mu - \sigma v + k_D s f_D - \mu v \longrightarrow v^* = \frac{\mu + k_D s f_D}{\sigma + \mu}$$

From equation 5,

$$0 = \gamma_D d - k_D f_D - \mu f_D \longrightarrow d^* = \frac{f_D(k_D + \mu)}{\gamma_D} \tag{1}$$

by substitution, we get:

$$\begin{aligned}
v^* &= \frac{\mu(\gamma_D + \mu) + k_D(s\gamma_D + \mu)}{(\gamma_D + \mu)(\mu + \sigma) + k_D(s\gamma_D + \mu + \sigma)} \\
d^* &= \frac{(k_D + \mu)\sigma}{(\gamma_D + \mu)(\mu + \sigma) + k_D(s\gamma_D + \mu + \sigma)} \\
f_D^* &= \frac{(\gamma_D)\sigma}{(\sigma_D + \mu)(\mu + \sigma) + k_D(s\gamma_D + \mu + \sigma})
\end{aligned}$$

Now, the Homeless Free Equilibrium is $(v^*, d^*, h^*, f_H^*, f_D^*, p^*)$

$$X = [h, f_H, p]^{\mathrm{T}}$$

$$Y = [v, d, f_D]^{\mathrm{T}}$$

$$\mathcal{F} = \begin{pmatrix} \beta_1 pd + \beta_2 hd \\ 0 \\ chp(1-p) \end{pmatrix}, \quad \mathcal{V} = \begin{pmatrix} -(1-q)k_H f_H + \gamma_h + \mu h \\ -\gamma_H h + k_H q f_H + (1-q)k_H f_H + \mu f_H \\ \epsilon p \end{pmatrix}$$

Taking the Jacobians of these vectors gives:

$$F = \begin{pmatrix} \beta_2 d & 0 & \beta_1 d \\ 0 & 0 & 0 \\ cp(1-p) & 0 & ch(1-2p) \end{pmatrix} \quad V = \begin{pmatrix} \gamma_H + \mu & -(1-q)k_H & 0 \\ -\gamma_H & k_H + \mu & 0 \\ 0 & 0 & \epsilon \end{pmatrix}$$

Now, substituting our HFE into F and V:

$$F_{HFE} = \begin{pmatrix} \frac{\beta_2(k_D + \mu\sigma)}{(\gamma_D + \mu)(\mu + \sigma) + k_D(s\gamma + \mu + \sigma)} & 0 & \frac{\beta_1(k_D + \mu\sigma)}{(\gamma_D + \mu)(\mu + \sigma) + k_D(s\gamma_D + \mu + \sigma)} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$V_{HFE} = \begin{pmatrix} \gamma_H + \mu & (q-1)k_H & 0 \\ \gamma_H & k_H + \mu & 0 \\ 0 & 0 & \epsilon \end{pmatrix}$$

Now we multiply to get FV^{-1} and find our largest eigenvalue to achieve our $\mathcal{R}_0$:

$$FV^{-1} = \begin{pmatrix} \frac{\beta_2\sigma(k_H+\mu)(k_D+\mu)}{J(\mu(\gamma_H+k_H+\mu)+\gamma_H k_H q)} & \frac{\beta_2 k_H(k_D+\mu)\sigma(1-q)}{J(\mu(\gamma_H+k_H+\mu)+\gamma_H k_H q)} & \frac{\beta_1(k_D+\mu)\sigma}{J\epsilon} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

A.3 Time Scale Analysis

In order to supplement our analysis with respect to our inability of solving large systems for equilibria and stability, we utilize a methodology in Brauer [4] in the study of vector-transmitted diseases. To simplify large, complex systems, one can consider the various time scales present in the overall system. In our case, our intervention strategies and disaffiliated class last on anywhere from three months [24] to two years at the longest [20]. These would be considered fast, short term time scales in the general picture of homelessness. On the other hand, the influence or growth of poverty may be seen on the time scale of decades [6]; this would function as our slow, long term time scale. Existing in between these two time scales is our main focus for mathematical analysis: the transition between vulnerable and homeless classes, this whole cycle lasting up to approximately ten years [12] and is our medium speed time scale.

With our fast and slow time scale segments of the population and environmental factors, we wish to collapse them so that they might be substituted within our vulnerable and homeless classes, generating a smaller two-dimensional system, albeit with more complex equations. For sompler equations, we assume that our intervention programs are always successful, i.e. $q = s = 1$. Before beginning analysis, though, we need to determine the parameters for our ϵ value to justify our *quasi-steady-state*, as in Brauer [4].

Between our fast and medium classes, we have that $k_D \gg \beta_2$ so that

$$\epsilon := \frac{\beta_2}{k_D} \ll 1$$

is an appropriate value to make our quasi-steady-state. Calling $\vec{X}$ our long term time scale (consisting of p)

and $\vec{Y}$ our short term time scale (consisting of D, F_H, and F_D, we have that

$$\vec{X}' = F(\vec{X}) \approx 0$$
$$\epsilon\vec{Y}' = G(\vec{Y}).$$

For our small ϵ, in the overall view of our system, $\vec{X}$ would be considered a constant and $\vec{Y}$ would be at a quasi-steady-state. For our normalized system, we solve our three fast time scales equations in terms of v and h:

$$\begin{aligned}
p &= \rho \\
d &= \frac{\sigma^* v}{\beta_1^*\rho + \beta_2^* h + \mu^* + \gamma_D^*} + \frac{k_H^*\gamma_H^* h}{(k_H^* + \mu^*)(\beta_1^*\rho + \beta_2^* h + \mu^* + \gamma_D^*)} \\
f_H &= \frac{\gamma_H^*}{k_H^* + \mu^*} h \\
f_D &= \frac{\gamma_D^*}{\beta_2 + \mu^*}\left(\frac{\sigma^* v}{\beta_1^*\rho + \beta_2^* h + \mu^* + \gamma_D^*} + \frac{k_H^*\gamma_H^* h}{(k_H^* + \mu^*)(\beta_1^*\rho + \beta_2^* h + \mu^* + \gamma_D^*)}\right)
\end{aligned}$$

Note that because we multiplied by our small ϵ value we have hidden ϵ values in many of our parameters; for example, $\mu^* = \epsilon\mu > 0$, $\sigma^* = \epsilon\sigma > 0$, etc. Plugging these extreme time scale values back into our medium term time scale equations, we get our new two-dimensional system:

$$\dot{v} = \mu - (\sigma + \mu)v + k_D\left(\frac{\gamma_D^*}{\beta_2 + \mu^*}\left(\frac{\sigma^* v}{\beta_1^*\rho + \beta_2^* h + \mu^* + \gamma_D^*} + \frac{k_H^*\gamma_H^* h}{(k_H^* + \mu^*)(\beta_1^*\rho + \beta_2^* h + \mu^* + \gamma_D^*)}\right)\right) \quad (2)$$

$$\dot{h} = (\beta_1\rho + \beta_2 h)\left(\frac{\sigma^* v}{\beta_1^*\rho + \beta_2^* h + \mu^* + \gamma_D^*} + \frac{k_H^*\gamma_H^* h}{(k_H^* + \mu^*)(\beta_1^*\rho + \beta_2^* h + \mu^* + \gamma_D^*)}\right) - (\gamma_H + \mu)h \quad (3)$$

Because this reduced system has quasi-steady-states, it is not appropriate to find a homeless-free equilibrium. With t near 0, our reduced system is not a good approximation. However, we give a solution for can give a solution for the endemic solution. We rename the above equations with

A.4 Stochastic Event Probabilities

Here we give the probabilities of each of our 16 events happening, in terms of the rates from our deterministic model.

$$\text{Prob}\left(\Delta X(t) = (a,b,c,d,e,f)|X(t)\right) = \begin{cases} \Lambda\Delta t + o(\Delta t) & a = 1 \\ \mu V(t)\Delta t + o(\Delta t) & a = -1 \\ \sigma V(t)\Delta t + o(\Delta t) & a = -1, b = 1 \\ k_D s F_D \Delta t + o(\Delta t) & a = 1, e = -1 \\ (\beta_1 D(t)p(t) + \beta_2 D(t)V(t)/N(t))\Delta t + o(\Delta t) & b = -1, c = 1 \\ \mu D(t)\Delta t + o(\Delta t) & b = -1 \\ \gamma_D H(t)\Delta t + o(\Delta t) & b = -1, e = 1 \\ k_D(1-s)F_D(t) + o(\Delta t) & b = 1, e = -1 \\ k_H q F_H \Delta t + o(\Delta t) & b = 1.d = -1 \\ \mu H(t)\Delta t + o(\Delta t) & c = -1 \\ \gamma_H H(t)\Delta t + o(\Delta t) & c = -1, d = 1 \\ k_H(1-q)F_H(t)\Delta t + o(\Delta t) & c = 1, d = -1 \\ \mu F_H(t)\Delta t + o(\Delta t) & d = -1 \\ \mu F_D(t)\Delta t + o(\Delta t) & e = -1 \\ \frac{H(t)}{N(t)} p(t)(N_p - p(t))\Delta t + o(\Delta t) & f = 1 \\ \epsilon p(t)\Delta t + o(\Delta t) & f = -1 \end{cases}$$

References

[1] LA:RISE – the Los Angeles Regional Initiative for Social Enterprise, REDF. `https://redfworkshop.org/larise/what`. Accessed: 2019-07-11.

[2] Los Angeles Home Values. `https://www.zillow.com/los-angeles-ca/home-values/`. Accessed: 2019-07-25.

[3] Temporary assistance for needy families. `https://www.benefits.gov/benefit/627`. Accessed: 2019-

07-11.

[4] Fred Brauer. A singular perturbation approach to epidemics of vector-transmitted diseases. *Infectious Disease Modelling*, 4:115–123, 2019.

[5] U.S. Census Bureau. Income and poverty in the united states. `https://www.census.gov/library/publications/2018/demo/p60-263.html`. Accessed: 2019-07-10.

[6] U.S. Census Bureau. Los angeles county quickfacts. `https://www.census.gov/quickfacts/fact/table/\losangelescountycalifornia/PST045218`. Accessed: 2019-07-10.

[7] Elijah Chilland. Low-income requirements for Los Angeles updated for 2017.

[8] Arden Dier. In Los Angeles, a Disaster of Epic Proportions. 2019.

[9] Marta Elliott and Lauren J. Krivo. Structural determinants of homelessness in the united states. *Social Problems*, 38(1):113–131, February 1991.

[10] The National Coalition for the Homeless. Why are people homeless? `https://www.nationalhomeless.org/factsheets/why.html`. Accessed: 2019-07-10.

[11] Vincent A Fusaro, Helen G Levy, and H Luke Shaefer. Racial and ethnic disparities in the lifetime prevalence of homelessness in the United States. *Demography*, 55(6):2119–2128, 2018.

[12] Charles Grigsby, Donald Baumann, Steven E Gregorich, and Cynthia Roberts-Gray. Disaffiliation to entrenchment: A model for understanding homelessness. *Journal of Social Issues*, 46(4):141–156, 1990.

[13] Lisa Guerin. Amount and duration of unemployment compensation in California. `https://www.nolo.com/legal-encyclopedia/collecting-unemployment-benefits-california-32504-2.html`. Accessed: 2019-07-10.

[14] Meghan Henry, Anna Mahathey, Tyler Morrill, Anna Robinson, Azim Shivji, and Rian Watt. The 2018 annual homeless assessment report (ahar) to congress, 2018.

[15] Sven Erik Jørgensen and Brian D. Fath. Introduction. In *Developments in Environmental Modelling*, pages 1–18. Elsevier, 2011.

[16] AA Lacey, DF Parker, K Parrott, D Simpson, Warren Smith, and JAD Wattis. Modelling of homeless populations.

[17] Richard Levins. Some demographic and genetic consequences of environmental heterogeneity for biological control. *American Entomologist*, 15(3):237–240, 1969.

[18] Takefumi Nakazawa. Introducing stage-specific spatial distribution into the Levins metapopulation model. *Scientific reports*, 5:7871, 2015.

[19] U.S. Department of Health & Human Services. 2019 poverty guidelines. `https://aspe.hhs.gov/2019-poverty-guidelines`. Accessed: 2019-07-09.

[20] U.S. Department of Housing and Urban Development. Continuum of care program eligibility requirements. `https://www.hudexchange.info/programs/coc/coc-program-eligibility-requirements/`. Accessed: 2019-07-10.

[21] U.S. Department of Housing and Urban Development. Homeless prevention and rapid re-housing program year 2 summary. 2013.

[22] California Housing Partnership. Los angeles county renters in crisis: a call for action. 2017.

[23] National Vital Statistics Reports. United states life tables, 2017. `https://www.cdc.gov/nchs/data/nvsr/nvsr68/nvsr68_07-508.pdf`. Accessed: 2019-07-10.

[24] Dana Rotz, Nan Maxwell, Adam Dunn, et al. Economic self-sufficiency and life stability one year after starting a social enterprise job. 2015.

[25] Andrea Saltelli. Sensitivity analysis for importance assessment. *Risk Analysis*, 22(3):579–590, June 2002.

[26] Mun C. Tsang. The cost of vocational training. `https://pdfs.semanticscholar.org/d1ed/\bd8880c887f7b7c1e63104cd2da31824a05d.pdf`.

[27] Quentin T Wodon. Providing shelters or low income housing? an optimal control model. *Journal of Housing Economics*, 8(2):90–115, 1999.