The Effects of Vaccination in a Core Group

by

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Abstract

This study examines a mathematical model in which a vaccine without complete effectiveness is applied to a core group. The prevalence of the disease within the core group determines the recruitment rate into the core group. The recruitment function in this model is set up for the case dependent on the proportion of infectious individuals. In particular, we study the possible oscillations of the disease over time caused by the vaccination rate and vaccine efficiency.

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Introduction

Induced immunity against infectious diseases has been a primary concern for centuries, from the inoculation of individuals in the Ming dynasty to Edward Jenner's small pox vaccination. In the late 1700's, during the widespread European epidemic of small pox, Jenner observed milkmaids' immunity to small pox. Further investigation revealed that the microbe species infecting cows with cow pox was closely related to small pox. Vaccination originated when Jenner inoculated susceptible individuals, persons capable of becoming infected, to induce small pox immunity with material taken from a pustular cow pox lesion. In 1760, Daniel Bernoulli introduced the first mathematical model to study the spread of infectious diseases and the benefits of vaccination.

A vaccine provides either permanent or temporary immunity. Successful implementation of a vaccination strategy will ultimately result in disease eradication. However, a vaccine can adopt any of the following properties: permanent immunity and complete effectiveness, temporary immunity and complete effectiveness; permanent immunity and partial effectiveness or temporary immunity and partial effectiveness.

Social relevance of vaccination models is significant. Vaccination models are used by health care officials to combat an infectious disease epidemic. Viruses do not recognize geographical or political borders. As a result, most successful disease eradication programs require cooperation of international, governmental, and local officials. Vaccination models are applied to improve the management of financial and human resources; vaccinating the core group requires less financial effort than vaccinating an entire population.

Our research examines the effects of a vaccination in a core group model. A core group is a subgroup of the population whose members are more prone to becoming infected and transmitting the disease. The concept of core group was first introduced by Hethcote and Yorke (1984), who studied effective treatment methods for gonorrhea in a prostitute core group. The results enabled limited financial resources to be used efficiently to control the epidemic.

Our research will incorporate vaccination into a similar model studied by Velasco-Hernandez, Brauer, and Castillo-Chavez (1996), in order to explore the behavior of a disease as the vaccination rate and the vaccine inefficiency

vary. This study is organized as follows: Section A defines and describes the equations used in this model; Section B describes computation and analysis of the basic reproductive number and the disease-free equilibrium; Section C proves the numerical existence of an endemic equilibrium; Section D includes graphical interpretations of the model; Section E considers death induced by disease; Section F concludes the paper and presents limitations to the model.

Section A: The Model

Let S, I and V represent the population size of susceptible, infected and vaccinated individuals respectively in the core group. The model equation is as follows:

$$\begin{aligned} \frac{dS}{dt} &= \Lambda e^{-(I/N)} - \mu S - \phi S - \beta S\frac{I}{N} \\ \frac{dI}{dt} &= \beta S\frac{I}{N} - \mu I + \sigma\beta V\frac{I}{N} \qquad (*) \\ \frac{dV}{dt} &= \phi S - \mu V - \sigma\beta\frac{I}{N}. \end{aligned}$$

Let the population of the core group be denoted by N, where $N = S + I + V$. The susceptible individuals become infected through contact with infected individuals at a rate β. Susceptible individuals are vaccinated at a rate ϕ individuals per unit time. Although the vaccination is permanent, it is not completely effective. As a result, there exists a group of vaccinated susceptibles. Vaccine inefficiency is measured by σ, where $0 < \sigma < 1$. If $\sigma = 0$ the vaccine is one hundred percent effective and if $\sigma = 1$, the vaccine is completely ineffective. The natural mortality rate is represented by μ. Λ is the recruitment rate of susceptible individuals into the core group from the general population. The fear or information factor affecting the recruitment of the non-core group into the core group is defined as α.

The following compartmental diagram schematically illustrates the model.

$$S \rightarrow I \rightarrow V$$

The rate of change of the population of susceptible individuals is determined by the product of the constant recruitment rate (Λ) and the prevalence-dependent factor ($e^{-aI/N}$) less the sum of the natural mortality of susceptibles, the number of vaccinated susceptibles, and the number of susceptibles

becoming infected. The rate of change of the population of infected individuals consists of the sum of susceptibles becoming infected and the number of vaccinated individuals who became infected less the natural mortality of the infected individuals. The rate of change of the population of vaccinated individuals consists of vaccinated susceptibles less the individuals who die naturally and those who become infected. The inefficiency of a vaccine directly contributes to the sum of infected individuals, because the ineffective vaccine causes the susceptible individual to become infected through contact with infectious individuals.

In order to simplify the model several assumptions were made:

- Infectious individuals die at the same rate as susceptible and vaccinated individuals.
- Disease induced death was not taken into consideration.
- The number of infected individuals outside the core-group is negligible; there is no chance that an infected individual will join the core group.

Section B: Reproductive Number

The analysis of the model begins with the computation of the basic reproductive number R_3. The basic reproductive number expresses the average number of individuals that an infectious person infects when he/she is introduced into a population that is previously uninfected. Since R_0 assumes an initially uninfected population, we begin by computing the disease-free equilibrium (S_0, I_0, V_0). Since R assumes an initially uninfected population, we begin by computing the disease-free equilibrium, $(S_0, I_0, V_0)^{\mathrm{T}}$.

$$F(S,I,V) = \begin{pmatrix} \frac{\Lambda e^{-\beta SI/N}}{\mu+\phi} \\ \frac{\beta SI+\sigma\beta VI/N}{\mu} \\ \frac{\phi-\sigma\beta VI/N}{\mu} \end{pmatrix}.$$

Since $(S_0, I_0, V_0)^{\mathrm{T}}$ is the fixed point of F for which $I_0 = 0$, it follows that

$$(S_0, I_0, V_0) = \begin{pmatrix} \frac{\Lambda}{\mu+\phi} \\ 0 \\ \frac{\phi\Lambda}{\mu(\mu+\phi)} \end{pmatrix} = \begin{pmatrix} \frac{\mu}{\mu+\phi}N \\ 0 \\ \frac{\phi}{\mu+\phi}N \end{pmatrix},$$

where $N = \frac{\Lambda}{\mu}(= S_0 + I_0 + V_0)$.

The dominant eigenvalue of the Jacobian matrix of F, J, evaluated at the disease-free equilibrium reveals the basic reproductive number.

$$J(S_0, I_0, V_0) = \begin{bmatrix} 0 & \frac{-\mu(\beta+\alpha\mu+\alpha\phi)}{(\mu+\phi)^2} & 0 \\ 0 & \frac{\beta(\mu+\sigma\phi)}{\mu(\mu+\phi)} & 0 \\ 0 & \frac{-\beta\phi\sigma}{\mu(\mu+\phi)} & 0 \end{bmatrix}.$$

The resulting dominant eigenvalue, which is also the basic reproductive number, R, is $\beta/\mu \quad (\mu + \sigma\phi)/(\mu + \phi)$. Since vaccination is considered in this model, R is taken as a function of ϕ, the vaccination rate, and σ, the inefficiency of the vaccination such that:

$$R(\phi, \sigma) = \left(\frac{\beta(\mu + \sigma\phi)}{\mu(\mu + \phi)}\right).$$

If $R(\phi, \sigma) < 1$ then the disease-free equilibrium, $(S_0, I_), V_0)^{\mathrm{T}}$, is asymptotically stable. Considering $R(phi, \sigma)$ when there is no vaccination (i.e., $\phi = 0$) it is found:

$$R(0, \sigma) = \beta/\mu = R_0.$$

Consideration of a completely ineffective vaccination, $(\sigma = 1)$, results in:

$$R(\phi, 1) = \beta/\mu = R_0.$$

$R_0 = \beta/\mu$ is the basic reproductive number of the model without vaccination. Notice that if $\beta < \mu$, the disease dies out naturally and no vaccination is necessary.

Considering that

- $1/\mu$ is the mean duration of infectiousness,
- $1/\phi$ is the mean duration of susceptibility,
- $1/(\mu + \phi)$ is the average life of a susceptible, and
- $1/(\mu + \sigma\phi)$ is the average life of a vaccinated,

R can be rewritten as

$$R(\phi,\sigma) = R_0 \cdot \frac{1/(\mu+\phi)}{1/(\mu+\sigma\phi)} = R_0 \cdot \frac{\text{average life as susceptible}}{\text{average life as vaccinated}}.$$

When $\sigma > 1$, the vaccine increases the chance of a susceptible becoming infected; consequently, $R)\phi,\sigma)$ increases. In contrast, when $\sigma < 1$, the vaccine decreases the chance of a susceptible becoming infected, and $R(\phi,\sigma)$ decreases. Infection caused by vaccination is not biologically feasible and is not considered relevant, thus we take $\sigma < 1$.

Section C: Endemic Equilbrium

In the previous section, the disease-free equilibrium was explicitly calculated. Finding the exact expression of a single endemic point is difficult so that only its existence will be discussed.

The endemic points $(S,I,V)^{\mathrm{T}}$ of the system are the fixed points of F for which $I > 0$. Setting $\Lambda = I/N$, the equations $F\left((S,I,T)^{\mathrm{T}}\right) = 0$ can be written as:

$$\begin{aligned} S &= \frac{\Lambda e^{-\alpha\Gamma}}{\mu+\phi} - \frac{\beta S\Gamma}{\mu+\phi} \\ I &= \frac{\beta}{\mu}(S+\sigma V)\Gamma \qquad (*) \\ V &= \frac{\phi}{\mu}S - \frac{\sigma\beta V\Gamma}{\mu}. \end{aligned}$$

The next steps involve some manipulating of the equations in order to reach a single equation representing the entire system.

$$I + V = \frac{S(\beta\Gamma+\phi)}{\mu} = N - S$$

so

$$N = \left[1 + \frac{beta\Gamma - \phi}{\mu}\right] S$$

thus

$$S/N = \frac{\mu}{\mu+\phi+\beta\Gamma}.$$

From (*),

$$\begin{aligned} V/N &= \frac{\phi}{\mu}\frac{S}{N} - \sigma\Gamma\frac{\beta}{\mu}\frac{V}{N} \\ V/N &= \frac{S}{N}\left(\frac{\phi}{\mu+\sigma\beta\Gamma}\right). \end{aligned}$$

Since $N = S + I + V$, it follows that

$$\begin{aligned} \Lambda &= \frac{I}{N} = 1 - \left(\frac{S}{N} + \frac{V}{N}\right) \\ &= 1 - \frac{S}{N}\left(1 + \frac{\phi}{\mu+\sigma\beta\Gamma}\right) \\ &= \left(\frac{\mu}{\mu+\phi+\beta\Gamma}\right)\left(1 + \frac{\phi}{\mu+\sigma\beta\Gamma}\right) \\ &= \frac{(\phi\sigma\beta + \beta\mu + \beta^2\sigma\Gamma)\Gamma}{(\mu+\phi+\beta\Gamma)(\mu+\sigma+\beta\Gamma)}. \end{aligned}$$

For $\Gamma \neq 0$ (in particular we are interested in $\Gamma > 0$) this equation reduces to:

$$\Gamma^2 + a\Gamma + b = 0,$$

where

$$a = \frac{\sigma(\mu+\phi) + \mu - \beta\sigma}{\beta\sigma}$$

and

$$b = \frac{\mu(\mu+\phi) - \beta(\mu+\sigma\phi)}{\beta^2\sigma}.$$

When solving roots of Γ, the quadratic equation results in:

$$\Gamma = \frac{-a \pm \sqrt{(a2 - 4b)}}{2}.$$

Hence, to have $\Gamma > 0$, $b < 0$ is necessary:

$$\begin{aligned} b &< 0 \\ \frac{\mu(\mu+\phi)}{\beta^2\sigma} &< \frac{\beta(\mu+\sigma\phi)}{\beta^2\sigma} \\ 1 &< \frac{\beta}{\mu}\frac{\mu+\sigma\phi)}{(\mu+\phi)}. \end{aligned}$$

However, $R(\phi, \sigma) = \frac{\beta}{\mu} \frac{(\mu+\sigma\phi)}{(\mu+\phi)}$; therefore, it can be concluded that an endemic point exists if:

$$R(\phi, \sigma) > 1.$$

By introducing a vaccine into a core group, it is expected that the disease and the core population will coexist (note that this implies that stability of the endemic equilibria is expected). This coexistence will take place in either of two forms: one possibility is that the core population will reach a stable equilibrium, meaning that the population will reach a fixed proportion of susceptible, infected and vaccinated individuals; the second possibility is that the population of susceptibles, infected and vaccinated individuals will fluctuate. There will be a bound on how large and how small the population can reach. This fluctuation is what is the reoccurrence of epidemics. The introduction of the vaccine should attempt to stabilize the population, or if that is not possible, limit the fluctuation to a small range. It is crucial to note that the number of infectious, susceptible and vaccinated individuals is never zero.

Section D: Graphical Analysis

Case 1:

Figure 1 depicts a vaccination rate of 0.70 with a vaccine ineffectiveness of 0.05. The vaccinated population increases rapidly, implying that the recruitment rate and the susceptible population increase while the infected population decreases. A portion of the susceptible and the vaccinated populations becomes infected because the vaccine is not completely effective.

In this graph, the vaccinated population is larger than the susceptible population which in turn is larger than the infected population. Therefore, the infection within the core group is not eradicated, but contained by the vaccine.

Figure 2 graphically represents a vaccination rate of 0.70 and a vaccine inefficiency of 0.30. After the initial increase of the vaccinated population and the susceptible population, the infected population begins to increase. The spread of the infection slows the recruitment into the core group such that less people are susceptible and less people are vaccinated. At the minima of the vaccinated population, the infected population is the largest population.

This is due to the large vaccine inefficiency in combination with a large vaccination rate.

Figure 3 shows the effect of a high vaccination rate, 0.70, with a high vaccine inefficiency, 0.30, and a relatively low fear factor, 3. Initially, the populations of the vaccinated and the susceptible individuals increase. The population of infected individuals increases due to the high inefficiency of the vaccine. We note the recruitment rate will approach a constant value since α, the fear factor, is relatively small. The vaccinated population remains greater than the infected population and the susceptible population.

Case 2:

Figure 4 above displays a vaccination rate ϕ of 0.137, a vaccine inefficiency σ of 0.05, and a relatively high fear factor α of 6. As recruitment rate decreases, all populations decrease to constant values. The infected population is the largest of the three populations, because the vaccination rate is relatively low. The limit on the spread of infection is not affected by the vaccination of the susceptibles in the core group, and reflects an indirect quarantine.

Figure 5 above graphically represents a fear factor of 6, a vaccination rate of 0.137, and a vaccine inefficiency of 0.30. In this graph the populations of the infected, susceptible, and vaccinated approach constant values. The spread of infection is also limited in this graph. The recruitment rate decreases because the proportion of infected individuals is high.

Figure 6 above depicts a fear factor of 4, a vaccination rate of 0.137 and an inefficiency of 0.30. All populations approach constant values. However, the infected population is higher than the graph when the fear factor was 6. This behavior is attributed to the fact that the recruitment into the core group is less affected by the total infected population (I/N) ratio when the fear factor is relatively small. The recruitment into the core group approaches a higher constant.

Section E: Death Induced by Disease

One modification of the model considers death induced by infectious disease, δI. This would be the case in a disease such as HIV/AIDS (if there

were a vaccine for it). It is no longer assumed that infected individuals die at the same rate as susceptible or vaccinated individuals. Death from infectious disease surpasses natural mortality. It is reasonable to assume that $\delta > \mu$. Only one equation changes in (*), that is

$$\frac{dI}{dt} = \beta S\frac{I}{N} + \sigma\beta V\frac{I}{N} - (\mu + \delta)I$$

and

$$I = \frac{\beta S\frac{I}{N} + \sigma\beta V\frac{I}{N}}{\mu + \delta}.$$

The disease-free equilibrium is not affected by the introduction of δ. The resulting equilibrium remains the same:

$$(S_0, I_0, V_0) = \begin{pmatrix} \frac{\mu}{\mu+\phi}N \\ 0 \\ \frac{\mu}{\mu+\phi}N \end{pmatrix}.$$

The Jacobian matrix evaluated at (S_0, I_0, V_0) results in:

$$J(S_0, I_0, V_0) = \begin{bmatrix} 0 & \frac{-\mu(\beta_\alpha\mu+\alpha\phi)}{(\mu+\phi)^2} & 0 \\ 0 & \frac{\beta(\mu+\sigma\phi)}{(\delta+\mu)(\mu+\phi)} & 0 \\ 0 & \frac{-\beta\phi\sigma}{\mu^2+\mu\phi} & 0 \end{bmatrix}.$$

The only eigenvalue of $J[(S_0, I_0, V_0)^{\mathrm{T}}]$ is:

$$\frac{(\mu + \phi)}{(\delta + \mu)(\mu + \phi)}.$$

The only modification in $R(\phi, \sigma)\delta$ is that, instead of $R(0, \sigma) = R(\phi, 1) = \beta/\mu$, we get $R(\phi, \sigma)\delta = R_0(\phi, \sigma)\delta = \beta/(\delta + \mu)$. Note that $1/(\delta + \mu)$ is the average time an infectious individual lives before he/she dies from the disease. We find that with the introduction of death induced by disease, we no longer encounter oscillations for reasonable values of δ. As δ approaches zero, $-\delta I$ becomes insignificant in dI/dt. In this case, we get our original model and conditions for oscillations to occur.

Section F: Conclusion

The recruitment rate, Λ, is influenced by the fear/information factor, α, and by the proportion of infected individuals in the core group in the following manner: $\Lambda e^{-(\alpha I/N)}$. Previous studies have shown that the fear/information factor strongly influences oscillatory behavior of a system. Fluctuating values of the recruitment rate cause these reoccurring oscillations. However, the prevalence dependent factor, $e^{-(\alpha I/N)}$, has two other dynamic variables I and N. A high percentage of infectious individuals, I/N, will cause the recruitment into the core group to decrease. In contrast, a lower value of I/N will cause a higher recruitment. These values are directly affected by the rate of vaccination and the inefficiency of the vaccine. If the fear factor, α, is relatively large, then the system oscillates at higher amplitudes with increased frequencies. A relatively smaller fear factor causes smaller oscillatory amplitudes with smaller frequency. If individuals belonging to the non-core group are educated about the actual number of infected individuals rather than a percentage of infected individuals (i.e., eliminating N from the prevalence-dependent factor) then the recruitment rate will decrease and oscillation in the model will cease to exist.

This model can be utilized to control and eradicate a disease in a core group; however, it does not take into account the disease spread in the general population. This model is limited by the specific application to diseases for which a vaccine has been developed.

This model has some serious implications when attempting to control outbreaks of a disease. It is essential to consider the information people receive about the core population and the vaccination procedures within it. It can be useful to public health officials in focusing social resources to a core population.

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Section D: **Graphical Analysis**

Case 1:

Figure 1

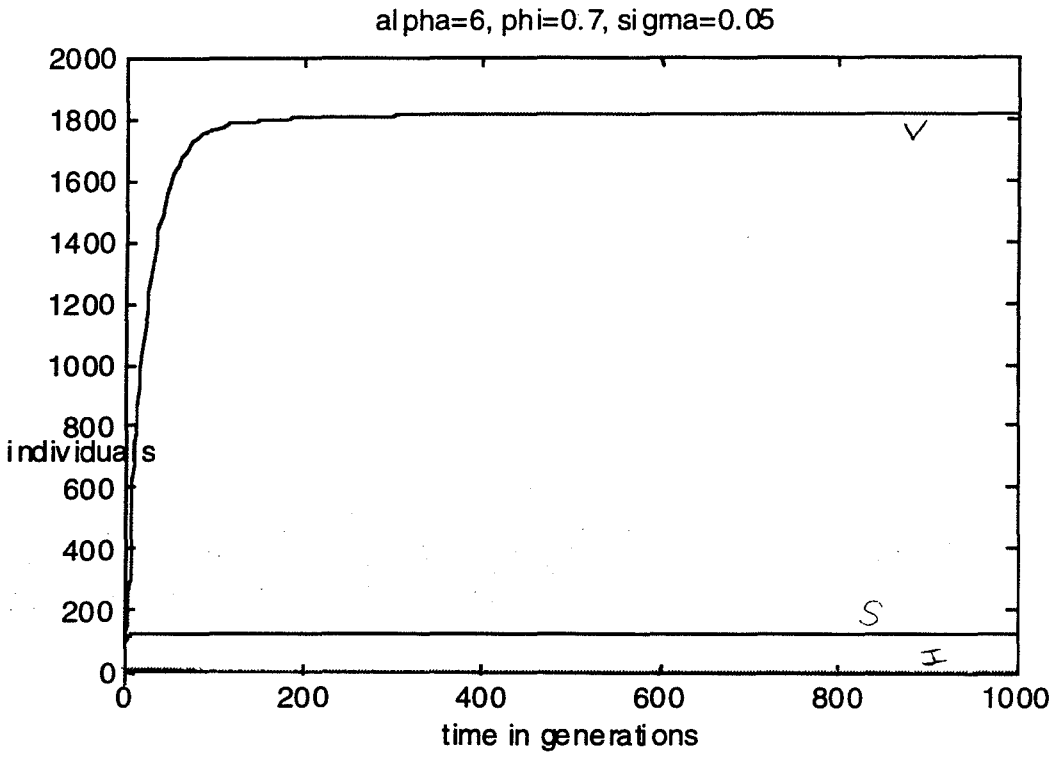

Figure 1 depicts a vaccination rate of 0.70 with a vaccine ineffectiveness of 0.05. The vaccinated population increases rapidly, implying that the recruitment rate and the susceptible population increase while the infected population decreases. A portion of the susceptible and the vaccinated populations becomes infected because the vaccine is not completely effective.

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Figure 2

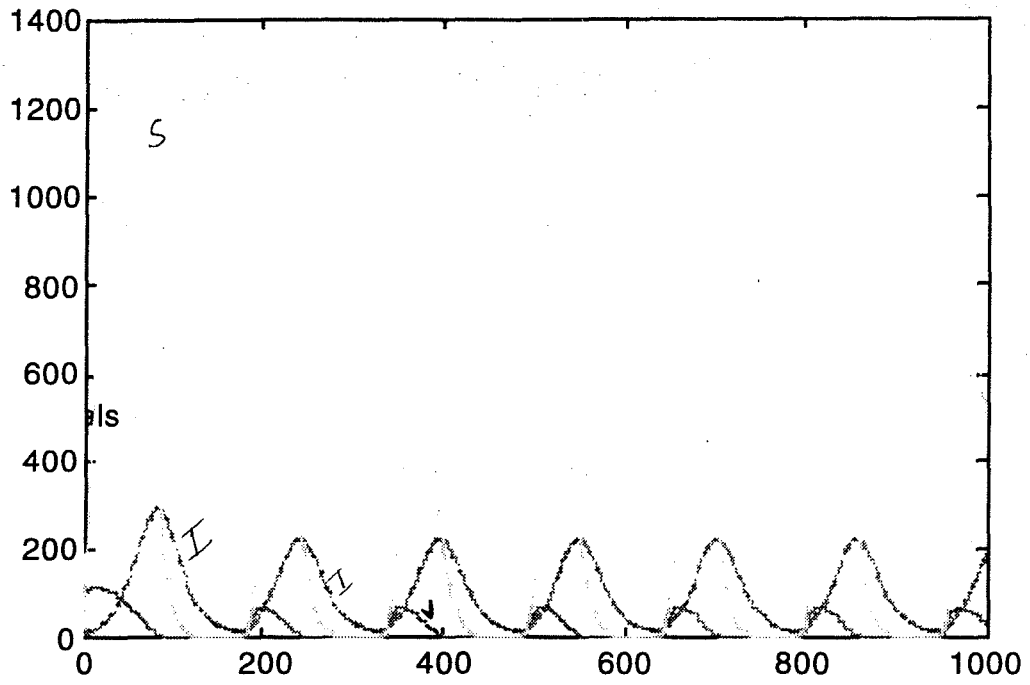
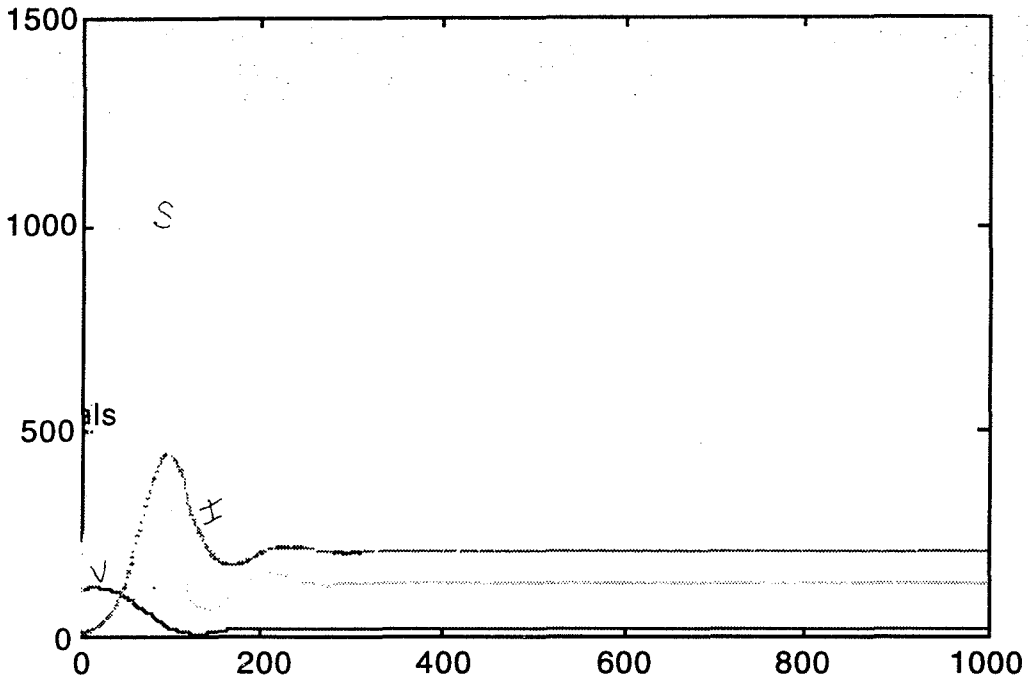


Figure 2 graphically represents a vaccination efficiency rate of 0.70 with and a vaccine inefficiency of 0.30. After the initial increase of the vaccinated population and the susceptible population, the infected population begins to increase. The spread of the infection slows the recruitment into the core group such that less people are susceptible and less people are vaccinated. When the vaccinated population reaches a minimum, the infected population becomes largest population.

Figure 3



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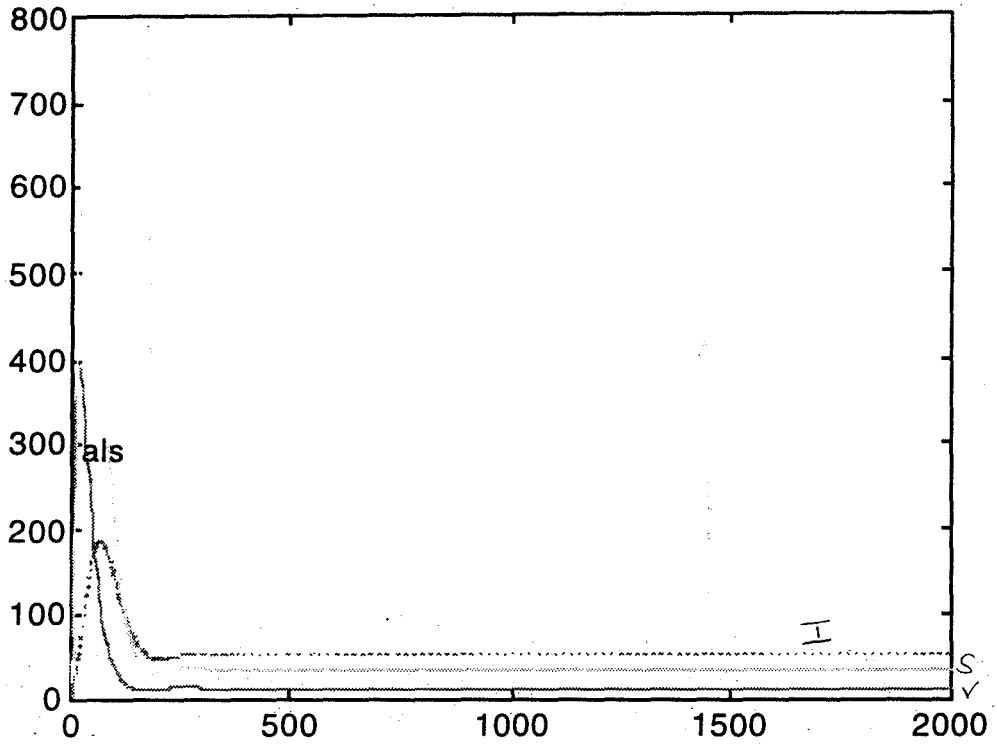

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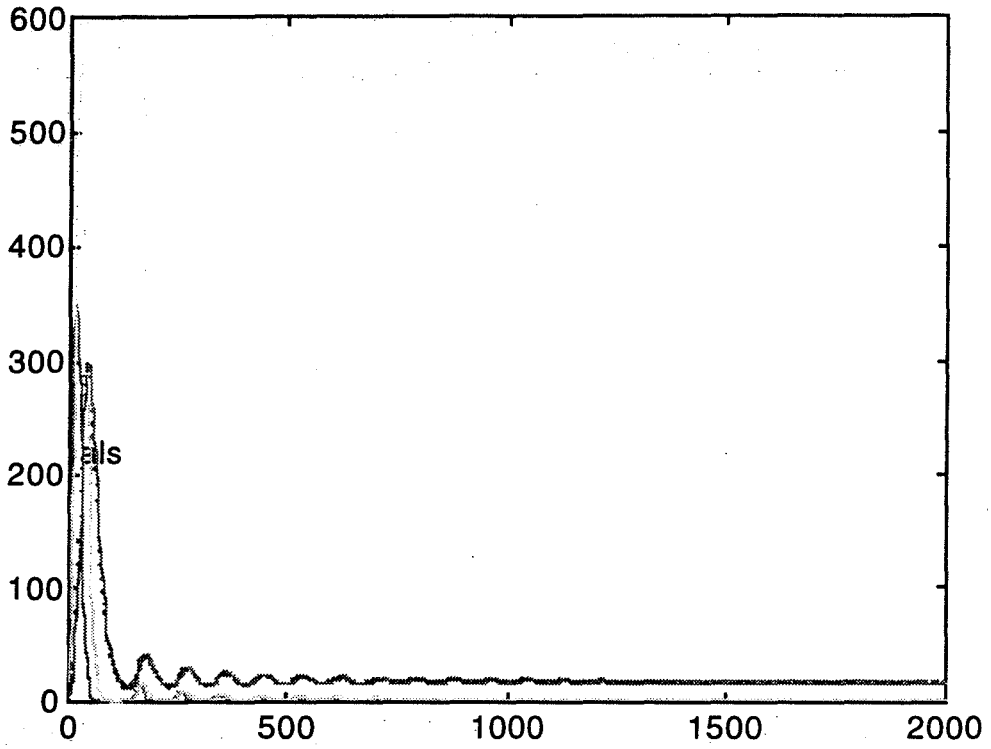

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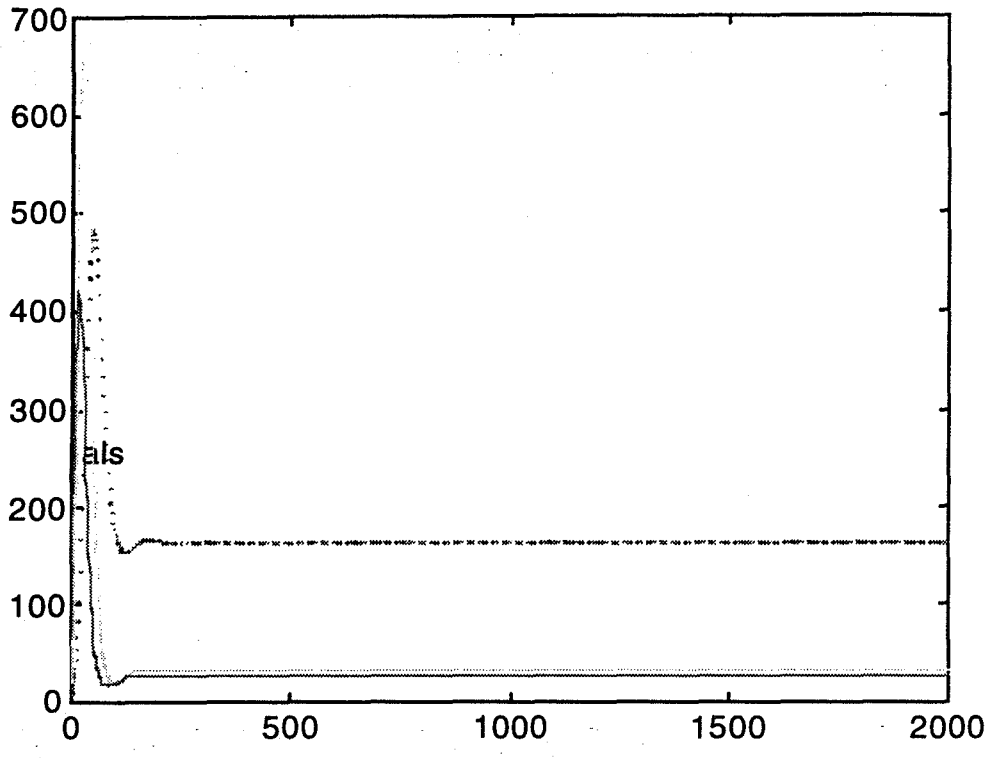

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