

A Socially Transmitted Disease:
Teacher Qualifications and Dropout Rates

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Abstract

The main goal of this study is to quantify the impact of teacher interactions on student achievement to facilitate recommending policy strategies that minimize high school dropout rates. This study derives a system of differential equations that examine the effects that teachers have on minority high school students' learning experience in California and Arizona. The first mathematical model focuses on the impact that teacher dynamics have on a schools' faculty composition. Teacher dynamics are coupled with a second system that models student responses to teacher preparation and experience in order to investigate the effects of these interactions on high school dropout and completion rates.

1 Background

Almost all people share a common socializing experience: sitting in a classroom with a teacher. The effects of the encounter are far reaching since education shapes a person's future in countless ways. Yet, too many high school students drop out, forgoing the benefits a formal education affords. For Latinos, who are more likely to drop out than any other groups (NCES 1994), it is an especially serious concern. Searching for explanations, studies tend to name students' backgrounds as the primary cause. However, research is beginning to show that there exists a stronger relationship between teacher qualifications and student achievement than between student characteristics and academic success (Darling-Hammond 1998). Not only does this agree with the intuitive account that better teachers make better students, but it also implies that there are perceptible differences in teachers' effectiveness. Consequently, while students are all taught by teachers, the caliber of instruction varies and this study attempts to quantify and analyze the impact teachers' qualifications have on high school dropout rates.

One relatively well-defined indicator of teacher quality is teacher preparedness. This term is difficult to quantify because of the somewhat subjective nature of assessing how qualified teachers are when they leave a certification program and enter into the school system. What skills determine how prepared a teacher is to educate students? Anyone can take a class. The real test presents itself when these teachers are placed in classrooms with thirty students. Will they be able to perform? Will they be able to facilitate their students' learning (Kasprzyk 1999)? Studies show that much of what students learn depends on the expertise of the teacher (Darling-Hammond 1998), however researchers have been unable to fully quantify the effectiveness of teachers' interactions with students.

The recent teacher shortage in California is resulting in the hiring of unqualified teachers. In California, 12.7% of teachers are unqualified, meaning they have no certification or less than full certification (Darling-Hammond 1998). This shortage is more prevalent in high minority areas where there is a higher concentration of unqualified teachers. As a result, these minority students have a low achievement rate due to poor quality teachers (Darling-Hammond 1998). The problem is what should be done to alleviate this situation. Should students be packed into classrooms, 40-45 students at a time, with one qualified teacher? Or should more unqualified teachers be hired and brought into the system to reduce the student-teacher ratio? Both questions bring about interesting dilemmas, but this study focuses on the effects that teachers' qualifications have upon students. Based on the assumption that master teachers have a significantly positive effect on the students in their classrooms, there is a growing body of research that indicates how student achievement is highly associated with teacher education (Darling-Hammond 1998, Ingersoll 1999). It is important that the community know that teacher qualifications play a significant role in student achievement. Then, community members will be able to ask questions about the process of becoming a teacher, the prevalence of unqualified

teachers in schools and, if need be, affect policy in order to change the situation.

In light of the growing concern over the condition of education and the need for teachers, if there is an association between teacher qualifications and student achievement, then it should be explored. Thus, the purpose of this study is to investigate the effects of teacher dynamics on the high school student population and the dropout rate.

2 Introduction

By using two separate systems, one representing teacher population and another representing student population, it is possible to quantify how teachers affect high school completion and dropout rates. In the model derived below, the presence of master teachers in the system plays a vital role given that these teachers positively influence fully qualified teachers. It is the fully qualified and master teachers who have the most positive effect on the student population since their students are more likely to graduate from high school.

In this study, three categories of teachers are defined according to the amount of education and certification they have (Darling-Hammond 2000). Teachers who have a bachelor's degree but do not have a teaching credential are defined as less than qualified teachers. These teachers are working under emergency credentials, waivers or temporary certification. It is assumed that this group of teachers is less effective in the classroom. This implies that the proportion of students they affect are more likely to drop out as opposed to graduating. Yet, there is a proportion of students who do graduate after interacting with less than qualified teachers. Research indicates that these teachers have a higher attrition rates that can lead to instability in the school environment (NCES 1999).

Fully qualified teachers are those who have a teaching credential and a bachelor's degree. In this study, it is assumed that the teacher population that is fully certified is more effective in the classroom than less than qualified teachers. Hence, the dropout rate of the proportion of students most affected by fully qualified teachers is lower than that of the students most affected by less than qualified teachers.

Finally, teachers that complete the necessary credential programs, have long-term experience, and have a master's degree are defined as master teachers. This group of teachers is essential because it is assumed that they have the most positive effect on students in that a majority of students who come in contact with them will graduate. These teachers influence a proportion of fully qualified teachers to become master teachers. The result is that there will be more master teachers, in effect, decreasing the overall dropout rate and increasing the completion rate.

The second system breaks down the student population into four different groups. Students that have yet to be influenced by teachers are defined as unaffected students. These students exit the system by other means such as transferring to another school or taking the GED, General Educational Devel-

opment Test. The rates at which these students leave the system are relatively large compared to the exit rates of the other groups of students because of the way schools and states report completion and dropout data(NCES).

The other three groups of students are influenced by one of the three types of teachers defined above. One group is affected by less than qualified teachers. The last two groups are affected by fully qualified and master teachers, respectively. In the proceeding section, a diagram gives a better understanding of the two systems described above. Obviously, this is a highly simplified system of student-teacher interactions and its' value will depend on the conclusions that can be derived from its' analysis. For some types of questions, more detailed models may be required. However, in most cases, a complex and detailed model (while satisfactory at one level) is very likely to be too complicated to analyze.

In this model of a student population, it is assumed that on average a student is affected by one type of teacher. While in a more complicated model, students would be affected by all types of teachers, analyzing that level of dynamics is too complicated for the resources available to this study. Therefore, this study looks at the average experience a student has rather than the separate interactions between a student and the different types of teachers.

3 Equations

Two systems of differential equations are introduced. When coupled together, they model the effects teachers, with different levels of qualifications, have on high school dropout rates. The equations representing teachers are:

$$\frac{dT_1}{dt} = p\Lambda - \gamma T_1 - \delta_1 T_1, \quad (1)$$

$$\frac{dT_2}{dt} = (1-p)\Lambda - \frac{\beta^* T_2 T_3}{M} + \gamma T_1 - \delta_2 T_2, \quad (2)$$

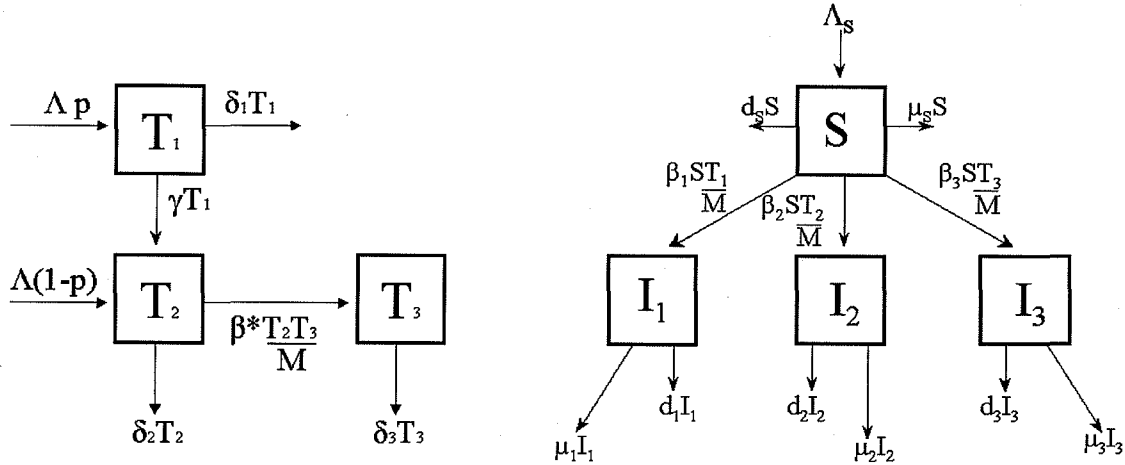
$$\frac{dT_3}{dt} = \frac{\beta^* T_2 T_3}{M} - \delta_3 T_3, \quad (3)$$

where $M = T_1 + T_2 + T_3$ and $\Lambda = \delta_1 T_1 + \delta_2 T_2 + \delta_3 T_3$, meaning Λ is not constant. In this system, it is assumed that the teacher population, M , remains constant. Here, Λ is a variable that represents newly hired teachers per unit of time and depends on the number of teachers leaving the school per unit time.

The dynamics of less than qualified teachers are represented in Equation (1). The term $p\Lambda$ is the rate less than qualified teachers enter the school system per unit time, γT_1 is the rate at which less than qualified teachers become fully qualified and $\delta_1 T_1$ is the rate at which less than qualified teachers leave the system.

The dynamics of fully qualified teachers are represented by Equation (2). The term $(1-p)\Lambda$ is the rate at which fully qualified teachers enter the system per unit time, $\frac{\beta^* T_2 T_3}{M}$ is the rate at which fully qualified teachers are influenced into becoming master teachers, γT_1 is the rate at which less than

Model



Parameter	Description
M	Total teacher population
T_1	Less than qualified teachers
T_2	Fully qualified teachers
T_3	Master Teachers
Λ	Total teacher population turnover rate per unit time
p	Probability that less than qualified teachers are hired
$1-p$	Probability that fully qualified are hired
γ	Rate at which less than qualified teachers become certified
δ_1	Rate at which less than qualified teachers leave the school
δ_2	Rate at which fully qualified teachers leave school
δ_3	Rate at which master teachers leave school
β^*	Rate at which master teachers influenced fully qualified teachers

Parameter	Total number of students ...
N	in school population
S	not affected by teachers
I_1	affected by less than qualified teachers
I_2	affected by fully qualified teachers
I_3	affected by master teachers
Λ_S	entering the school system

Parameter	Rate at which ...
β_1	less than qualified teachers affect students
β_2	fully qualified teachers affect students
β_3	master teachers affect students
μ_S	unaffected students graduate
d_S	unaffected students drop out
μ_1	students affected by less than qualified teachers graduate
d_1	students affected by less than qualified teachers drop out
μ_2	students affected by fully qualified teachers graduate
d_2	students affected by fully qualified teachers drop out
μ_3	students affected by master teachers graduate
d_3	students affected by master teachers drop out

qualified teachers become fully qualified and $\delta_2 T_2$ is the rate at which fully qualified teachers leave the system.

The dynamics of well qualified teachers are represented by Equation (3). The term $\frac{\beta^* T_2 T_3}{M}$ is the rate at which fully qualified teachers become master teachers as a result of interactions between the two groups and $\delta_3 T_3$ is the rate at which master teachers leave the system. In this model, it is assumed that δ_2 is greater than δ_3 , which means that the average career span of T_3 teachers, $\frac{1}{\delta_3}$, is greater than that of T_2 teachers, $\frac{1}{\delta_2}$.

The equations representing students are:

$$\frac{dS}{dt} = \Lambda_S - \frac{\beta_1 S T_1}{M} - \frac{\beta_2 S T_2}{M} - \frac{\beta_3 S T_3}{M} - \alpha_S S, \quad (4)$$

$$\frac{dI_1}{dt} = \frac{\beta_1 S T_1}{M} - \alpha_1 I_1, \quad (5)$$

$$\frac{dI_2}{dt} = \frac{\beta_2 S T_2}{M} - \alpha_2 I_2, \quad (6)$$

$$\frac{dI_3}{dt} = \frac{\beta_3 S T_3}{M} - \alpha_3 I_3, \quad (7)$$

where $N = S + I_1 + I_2 + I_3$, $\alpha_S = \mu_S + d_S$, and $\alpha_i = \mu_i + d_i$, $i = 1, 2, 3$.

The dynamics of unaffected students is represented in Equation (4). The term Λ_S is the rate at which students enter the system, $\frac{\beta_1 S T_1}{M}$ is the rate at which students are affected by less than qualified teachers, $\frac{\beta_2 S T_2}{M}$ is the rate at which students are affected by fully qualified teachers and $\frac{\beta_3 S T_3}{M}$ is the rate at which students are affected by master teachers. $\alpha_S S$ is the rate at which unaffected students leave the system, either by graduating or dropping out.

The dynamics of students who have been affected by less than qualified teachers is represented in Equation (5). The term $\frac{\beta_1 S T_1}{M}$ is the rate at which students are affected by less than qualified teachers and $\alpha_1 I_1$ is the rate at which students affected by less than qualified teachers leave the system, either by graduating or dropping out.

The dynamics of the proportion of students who have been affected by fully qualified teachers is represented in Equation (6). The term $\frac{\beta_2 S T_2}{M}$ is the rate at which students are affected by fully qualified teachers and $\alpha_2 I_2$ is the rate at which students affected by fully qualified teachers leave the system, either by graduating or dropping out.

The dynamics of the proportion of students who have been affected by master teachers is represented in Equation (7). The term $\frac{\beta_3 S T_3}{M}$ is the rate at which students are affected by master teachers and $\alpha_3 I_3$ is the rate at which students affected by master teachers leave the system, either by graduating or dropping out.

In this model of a student population, it is assumed that on average a student is affected by one type of teacher. While in a more complicated model students would be affected by all types of teachers, analyzing that level of dynamics is too complicated for the resources available to this study. Therefore, this study looks at the average experience a student has rather than the separate

interactions between a student and the different types of teachers.

4 Finding Equilibrium Points for the Teacher System

A question of interest is what is the long-term time evolution of the teacher system when $t \rightarrow \infty$. Hence, it is proceeded to find whether steady states are possible with no well qualified teachers in the system, $T_3 = 0$, and with well qualified teachers in the system, $T_3 > 0$. To find the equilibrium points for the teacher model, the system is reduced to two dimensions by substituting $T_3 = M - T_1 - T_2$ into Λ :

$$\begin{aligned}\frac{dT_1}{dt} &= p(\delta_1 T_1 + \delta_2 T_2) + p\delta_3(M - T_1 - T_2) - \gamma T_1 - \delta_1 T_1, \\ \frac{dT_2}{dt} &= (1-p)(\delta_1 T_1 + \delta_2 T_2 + \delta_3(M - T_1 - T_2)) \\ &\quad - \frac{\beta^* T_2 (M - T_1 - T_2)}{M} + \gamma T_1 - \delta_2 T_2.\end{aligned}\quad (8)$$

Both equations are set equal to zero and added together.

$$\delta_1 T_1 + \delta_2 T_2 + \delta_3(M - T_1 - T_2) - \frac{\beta^* T_2 (M - T_1 - T_2)}{M} - \delta_1 T_1 - \delta_2 T_2 = 0$$

Algebraic manipulation and simplification steps yield the following equation:

$$(M - T_1 - T_2)\left(\delta_3 - \frac{\beta^* T_2}{M}\right) = 0 \quad (9)$$

Equation (9) implies there are two solutions.

Case(1): $M - T_1 - T_2 = 0$. So, $M = T_1 + T_2 + T_3$ implies that $T_3 = 0$. Therefore $T_3^* = 0$ corresponds to the T_3 value for one equilibrium point.

Case(2): $\delta_3 - \frac{\beta^* T_2}{M} = 0$. Thus, $T_2 = \frac{\delta_3 M}{\beta^*}$ corresponds to the T_2 value for the second equilibrium point. So $T_2^{**} = \frac{\delta_3 M}{\beta^*}$.

Equilibrium: (T_1^*, T_2^*, T_3^*)

The first equilibrium point from case (1) has $T_3^* = 0$. This equilibrium point represents the demographic steady state, in which there are no master teachers. Hence,

$$\begin{aligned}\frac{dT_1}{dt} &= p(\delta_1 T_1 + \delta_2 T_2) - (\delta_1 + \gamma)T_1, \\ \frac{dT_2}{dt} &= (1-p)(\delta_1 T_1 + \delta_2 T_2) - \delta_2 T_2 + \gamma T_1.\end{aligned}$$

The two equations are set equal to zero and solved for T_1 using $T_2 = M - T_1$.

$$(\delta_1 + \gamma - p\delta_1)T_1 = p\delta_2(M - T_1).$$

Hence,

$$T_1 = \frac{p\delta_2 M}{(1-p)\delta_1 + p\delta_2 + \gamma}.$$

which implies,

$$T_2 = \frac{M((1-p)\delta_1 + \gamma)}{(1-p)\delta_1 + p\delta_2 + \gamma}.$$

Therefore,

$$\begin{aligned} T_1^* &= \frac{p\delta_2 M}{(1-p)\delta_1 + p\delta_2 + \gamma}, \\ T_2^* &= \frac{M(q\delta_1 + \gamma)}{(1-p)\delta_1 + p\delta_2 + \gamma}, \\ T_3^* &= 0. \end{aligned}$$

Since the parameters q , Λ , δ_1 , δ_2 , δ_3 , and γ are all positive, the equilibrium point (T_1^*, T_2^*, T_3^*) always exists.

Equilibrium: $(T_1^{**}, T_2^{**}, T_3^{**})$

The second equilibrium comes from case (2), where $T_2^{**} = \frac{\delta_3 M}{\beta^*}$ is substituted into Equation (8):

$$p \left(\delta_1 T_1 + \delta_2 \frac{\delta_3 M}{\beta^*} + \delta_3 \left(M - T_1 - \frac{\delta_3 M}{\beta^*} \right) \right) - \gamma T_1 - \delta_1 T_1 = 0$$

Expanding and solving for T_1 gives:

$$T_1^{**} = \frac{p\delta_3 M}{\beta^*} \left(\frac{\beta^* + \delta_2 - \delta_3}{(1-p)\delta_1 + p\delta_3 + \gamma} \right).$$

Finally, since $T_3^{**} = M - T_1^{**} - T_2^{**}$, then

$$T_3^{**} = M - \frac{\delta_3 M}{\beta^*} - \frac{p\delta_3 M}{\beta^*} \left(\frac{\beta^* + \delta_2 - \delta_3}{(1-p)\delta_1 + p\delta_3 + \gamma} \right).$$

Therefore, $(T_1^{**}, T_2^{**}, T_3^{**})$ represents the second equilibrium point. As with the first equilibrium point, $(T_1^{**}, T_2^{**}, T_3^{**})$ must also have all positive values. In this case, T_2^{**} is automatically positive since δ_3 and β^* are always positive and

$$T_2^{**} = \frac{\delta_3 M}{\beta^*}.$$

For T_1^{**} to exist, the only condition stems from the terms in the numerator. So,

$$T_1^{**} = \frac{p\delta_3 M}{\beta^*} \left(\frac{\beta^* + \delta_2 - \delta_3}{(1-p)\delta_1 + p\delta_3 + \gamma} \right) > 0.$$

T_1^{**} is positive as δ_2 is always greater than δ_3 .

For T_3^{**} to exist, the value cannot equal zero and must be positive. Thus,

$$\begin{aligned} T_3^{**} &= M - \frac{\delta_3 M}{\beta^*} - \frac{p\delta_3 M}{\beta^*} \left(\frac{\beta^* + \delta_2 - \delta_3}{(1-p)\delta_1 + p\delta_3 + \gamma} \right) > 0, \\ \Rightarrow 1 &> \frac{\delta_3}{\beta^*} + \frac{\delta_3}{\beta^*} \left(\frac{p\delta_2 - p\delta_3 + p\beta^*}{(1-p)\delta_1 + p\delta_3 + \gamma} \right), \\ \Rightarrow \beta^* \gamma + \beta^* \delta_1 - p\delta_1 + \beta^* p\delta_3 &> \gamma\delta_3 + \delta_1\delta_3 q + \delta_2\delta_3 p + \beta^* p\delta_3, \\ \Rightarrow \frac{\beta^*}{\delta_3} \frac{(1-p)\delta_1 + \gamma}{(1-p)\delta_1 + p\delta_2 + \gamma} &> 1. \end{aligned}$$

For simplicity, let $q = (1-p)$ where $q \geq 0$ for all p . Then,

$$R_0 \equiv \frac{\beta^*}{\delta_3} \frac{\delta_1 + \gamma}{q\delta_1 + p\delta_2 + \gamma}. \quad (10)$$

So, T_3^{**} exists when Equation (10) is true. Therefore, the condition for the existence of the equilibrium point where master teachers are present is the same condition as for the existence of T_3^{**} . When R_0 is less than 1, master teachers will leave the system before influencing other teachers to take their place. When R_0 is greater than one, the number of master teachers will grow. In this study, the teacher interaction ratio, denoted by R_0 , is essential because it defines a threshold of change when fully qualified teachers are influenced enough on the average to become master teachers. The teacher interaction ratio gives the threshold value for the teacher system.

Then $\frac{q\delta_1 + \gamma}{q\delta_1 + p\delta_2 + \gamma}$ is the probability that a T_3 teacher will come into contact with a T_2 teacher, $\frac{1}{\delta_3}$ represents the average career span of a T_3 teacher and β^* is the probability that a T_2 teacher will convert to a T_3 teacher. Hence, R_0 is the ratio of fully qualified teachers that are influenced to become master teachers. Hence, $R_0 < 1$ implies that $T_3 \rightarrow 0$, while $R_0 > 1$ implies that T_3 will initially grow.

5 Stability of Equilibria

Since the teacher system has been reduced to a two dimensional system, then the stability depends on the values of the trace and the determinant of the Jacobian, matrix A , evaluated at the respective equilibrium points. The Jacobian is:

$$A = \begin{pmatrix} p\delta_1 - p\delta_3 - \gamma - \delta_1 & p\delta_2 - p\delta_3 \\ (1-p)(\delta_1 - \delta_3) + \gamma + \frac{\beta^* T_2}{M} & (p-1)\delta_3 - p\delta_2 - \beta^* + \frac{\beta^* T_1}{M} + \frac{2\beta^* T_2}{M} \end{pmatrix}.$$

In order for the equilibria to be locally asymptotically stable, the Routh-Hurwitz conditions $trace(A) < 0$ and $det(A) > 0$ must be satisfied.

Stability of (T_1^*, T_2^*, T_3^*)

For the first equilibrium point, the Jacobian, denoted as A^* , is obtained by evaluating A at T_1^* and T_2^* .

$$A^* = \begin{pmatrix} p(\delta_1 - \delta_3) - \delta_1 - \gamma & p(\delta_2 - \delta_3) \\ (1-p)(\delta_1 - \delta_3) + \frac{\beta^*(q\delta_1 + \gamma)}{(q\delta_1 + p\delta_2 + \gamma)} + \gamma & (1-p)(\delta_2 - \delta_3) + \frac{\beta^*(q\delta_1 + \gamma)}{(q\delta_1 + \delta_2 + \gamma)} - \delta_2 \end{pmatrix}.$$

Condition 1:

$$trace(A^*) = p(\delta_1 - \delta_3) - \delta_1 - \gamma + (1-p)(\delta_2 - \delta_3) + \frac{\beta^*(q\delta_1 + \gamma)}{(q\delta_1 + \delta_2 + \gamma)} - \delta_2.$$

After some manipulation, the condition for the trace to be negative is found to be:

$$p\delta_1 - \delta_1 - p\delta_2 - \delta_3 - \gamma + \frac{\beta^*(q\delta_1 + \gamma)}{q\delta_1 + p\delta_2 + \gamma} < 0.$$

Factoring and rearranging yields the condition:

$$-\delta_3(1 - R_0) - q\delta_1 - p\delta_2 - \gamma < 0.$$

Note that $R_0 < 1$ is sufficient to make the trace negative and the first condition is satisfied.

Condition 2:

$$det(A^*) = -p\delta_1\delta_3 + p\beta^*\delta_1 + \delta_1\delta_3 - \beta^*\delta_1 + p\delta_2\delta_3 - \beta^*\gamma + \delta_3\gamma.$$

By rearranging the expression for $det(A^*)$, it is clear when it is positive. For the determinant to be positive, the following condition must be met:

$$\frac{\beta^*}{\delta_3} \frac{q\delta_1 + \gamma}{q\delta_1 + p\delta_2 + \gamma} < 1.$$

Thus, condition 2 is satisfied. Therefore, whenever $R_0 < 1$, (T_1^*, T_2^*, T_3^*) is locally asymptotically stable.

Stability for $(T_1^{**}, T_2^{**}, T_3^{**})$

For the second equilibrium point, the Jacobian, denoted by A^{**} , is obtained by evaluating A at T_1^{**} and T_2^{**} .

$$A^{**} = \begin{pmatrix} (p-1)\delta_1 - p\delta_3 - \gamma & p(\delta_2 - \delta_3) \\ (1-p)\delta_1 + p\delta_3 + \gamma & (1+p)\delta_3 - p\delta_2 - \beta^* + p\delta_3 \frac{\delta_2 + \beta^* - \delta_3}{\gamma + \delta_1(1-p) + p\delta_3} \end{pmatrix}.$$

Condition 1:

$$\text{trace}(A^{**}) = (p-1)\delta_1 - \gamma + \delta_1 - \beta^* + p\delta_3 \frac{\delta_2 + \beta^* - \delta_3}{\gamma + \delta_1(1-p) + p\delta_3}.$$

After some manipulation, the condition for a negative trace is found to be:

$$(p\delta_1\delta_3 + \delta_1\beta^* + \gamma\beta^*) - (p\delta_1\beta^* + \delta_1\delta_3 + \gamma\delta_3 + p\delta_3\delta_2) < 0.$$

Factoring and rearranging yields the condition:

$$\frac{\beta^*}{\delta_3} \frac{q\delta_1 + \gamma}{q\delta_1 + p\delta_2 + \gamma} > 1.$$

Condition 2:

$$\det(A^{**}) = ((p-1)\delta_1 - p\delta_3 - \gamma) \left(\delta_3 - \beta^* + p\delta_3 \frac{\beta^* + \delta_2 - \delta_3}{(1-p)\delta_1 + p\delta_3 + \gamma} \right).$$

By simplifying, a condition is produced for $\det(A^{**})$. The determinant is positive if the following condition is met:

$$(\delta_1\delta_3p + \delta_1\beta^* + \gamma\beta^*) - (\delta_1\beta^*p + \delta_1\delta_3 + \gamma\delta_3 + \delta_3\delta_2p) < 0.$$

This can be rewritten as

$$\frac{\beta^*}{\delta_3} \frac{q\delta_1 + \gamma}{q\delta_1 + p\delta_2 + \gamma} > 1.$$

Therefore, if $R_0 > 1$, $(T_1^{**}, T_2^{**}, T_3^{**})$ exists and it is stable.

6 Teacher Equilibria in terms of R_0

The teacher system is analyzed further by rewriting the equilibria in terms of R_0 . This illustrates how the teacher system is affected by the teacher interaction ratio. Since T_3 teachers are not present in the first equilibrium, then it is only necessary to examine T^{**} as it varies with different values of R_0 . For the second equilibrium, let

$$K = \frac{q\delta_1 + \gamma}{q\delta_1 + p\delta_2 + \gamma},$$

$$Q = \frac{p(\beta^* + \delta_2 - \delta_3)}{q\delta_1 + p\delta_3 + \gamma},$$

then

$$T_1^{**} = \frac{KMQ}{R_0},$$

$$T_2^{**} = \frac{KM}{R_0},$$

$$T_3^{**} = M - \frac{KM(1+Q)}{R_0}.$$

As $R_0 \rightarrow \infty$, T_1^{**} and T_2^{**} diminish as T_3^{**} approaches M .

7 Analysis of Student System

From the previous section the dynamics of the teacher system are known, i.e. $R_0 < 1$, $(T_1, T_2, T_3) \rightarrow (T_1^*, T_2^*, T_3^*)$ and $R_0 > 1$, $(T_1, T_2, T_3) \rightarrow (T_1^{**}, T_2^{**}, T_3^{**})$. According to the theory of limiting equations developed by Carlos Castillo-Chávez and Horst Thime in 1995, the equilibrium values of the teacher system can be substituted in the student system. The resulting system is an ODE linear system. For this system, there exists only one equilibrium point that is always stable, and its value depends upon the proportion of teachers at any given time. (See appendix for derivation of equilibrium point.) Coupled together, the equilibria from the teacher system generate corresponding equilibria for the student system. Therefore, to analyze the impact of teachers on the student system is enough to look at the impact of teacher equilibria on student equilibria.

Whereas, the parameter values for the teacher system are all available so that determining the teacher distribution is only a matter of computing the equilibrium values for the given parameters, not all of the parameters for the student system are known or can be estimated realistically. In particular, the rates at which different teachers affect students, β_1 , β_2 , and β_3 , are beyond the scope of the data that is available. In other words, there is no quantitative measures of the role of various types of teachers on students' achievement and retention. Consequently, the best information the model provides is in the form of the relative impact of master teachers compared to other teachers on student dropout and graduation rates.

8 Results

This model illustrates how high school students are affected by teachers and, specifically, by the distribution of less than qualified, fully qualified and master teachers. The student system is analyzed considering the two master teacher population distributions. However, data for the rates at which different teachers affect students is not available. Even though it is possible to estimate the dropout rate for a given school, it is not possible, at present, to quantify how teachers impact students in terms of β_1 , β_2 , and β_3 . One strategy available is to increase the proportion of master teachers in the system. This is based on the assumption that master teachers have a significantly different and positive effect on the students in those classrooms. There is a growing body of research that indicates how student achievement is associated so highly with teacher education that the effects of teacher qualifications can no longer be ignored (Darling-Hammond 1998, Ingersoll). Thus, the best information the model provides at this point is how the parameters affect the proportion of master teachers in a school. Therefore, it is worthwhile to further analyze the nature of the parameters in the teacher model.

To determine how parameters affect T_3^{**} , the partial derivative can be used since it represents how the equilibrium point responds to change. β^* and p are of most interest because hiring practices can be influenced through poli-

cies, and master teachers' abilities to mentor other teachers can be fostered by administrative decisions and community advocacy. The partial derivatives with respect to β^* and p are

$$\frac{\partial T_3^{**}}{\partial \beta^*} = \frac{\delta_3 M}{(\beta^*)^2} \left(\frac{q\delta_1 + p\delta_2 + \gamma}{q\delta_1 + p\delta_3 + \gamma} \right),$$

$$\frac{\partial T_3^{**}}{\partial p} = -\frac{\delta_3 M}{\beta^*} \left(\frac{(\beta^* + \delta_2 - \delta_3)(\delta_1 + \gamma)}{(q\delta_1 + p\delta_3 + \gamma)^2} \right).$$

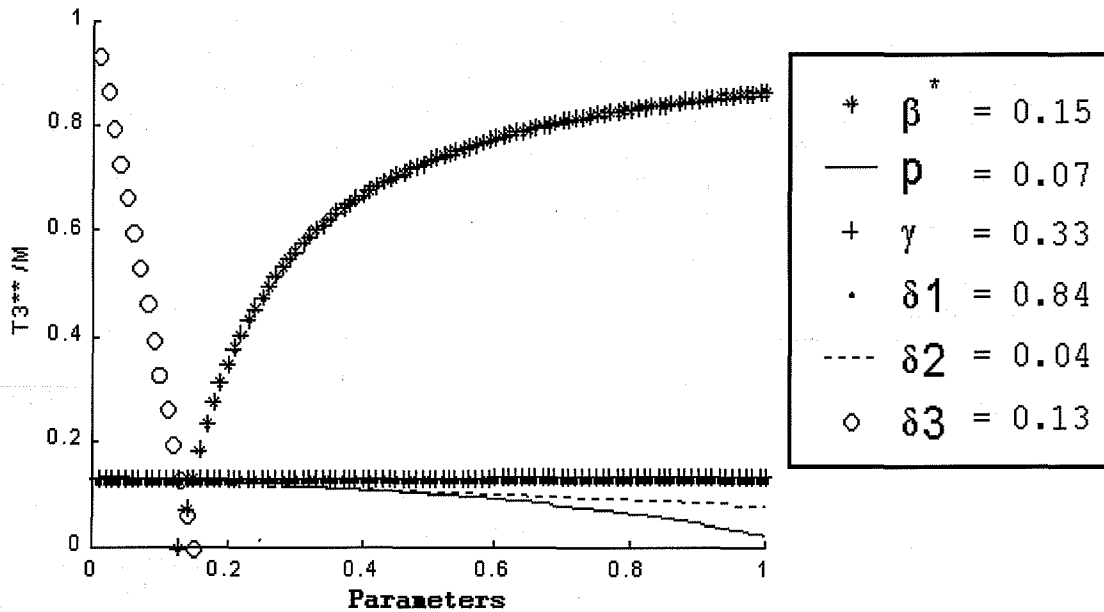
Using data from national and state agencies, the parameters values are estimated to be $\delta_1 = 0.84$, $\delta_2 = 0.04$, $\delta_3 = 0.13$, $\gamma = 0.33$, $p = 0.07$ and $\beta^* = 0.15$ for an average California high school. This gives the value $R_0 = 1.15$. Similarly, the parameter values for an average Arizona high school are found to be $\delta_1 = 0.5$, $\delta_2 = 0.05$, $\delta_3 = 0.06$, $\gamma = 0.5$, $p = .4$ and $\beta^* = 0.10$. This gives the value $R_0 = 1.62$. These sets of parameters are substituted into the partial derivatives of T_3^{**} with respect to changes of β^* and p . For Arizona, the rate of change for β^* is 265, while the rate of change for p is -79.53. For California, the rate of change for β^* is 514.52, while the rate of change for p is -32.32. For California, β^* is large and positive, which means a small increase in β^* results in a relatively large increase in the proportion of master teachers. Likewise, for p , a small change causes a decrease in the proportion of master teachers. Similarly, Arizona has large positive values for β^* and negative values for p . By plotting the proportion of master teachers versus the estimated parameters for Arizona and California, these calculations can be verified.

It is assumed that master teachers have the best effect on students, because of their qualifications and experience. Hence, the goal is to find out how to increase the proportion of master teachers in a given school. Since this study focuses on California and Arizona, two questions are considered. What parameters change the proportion of master teachers the greatest and what parameter values make master teachers ineffective at recruiting new master teachers?

9 Discussion

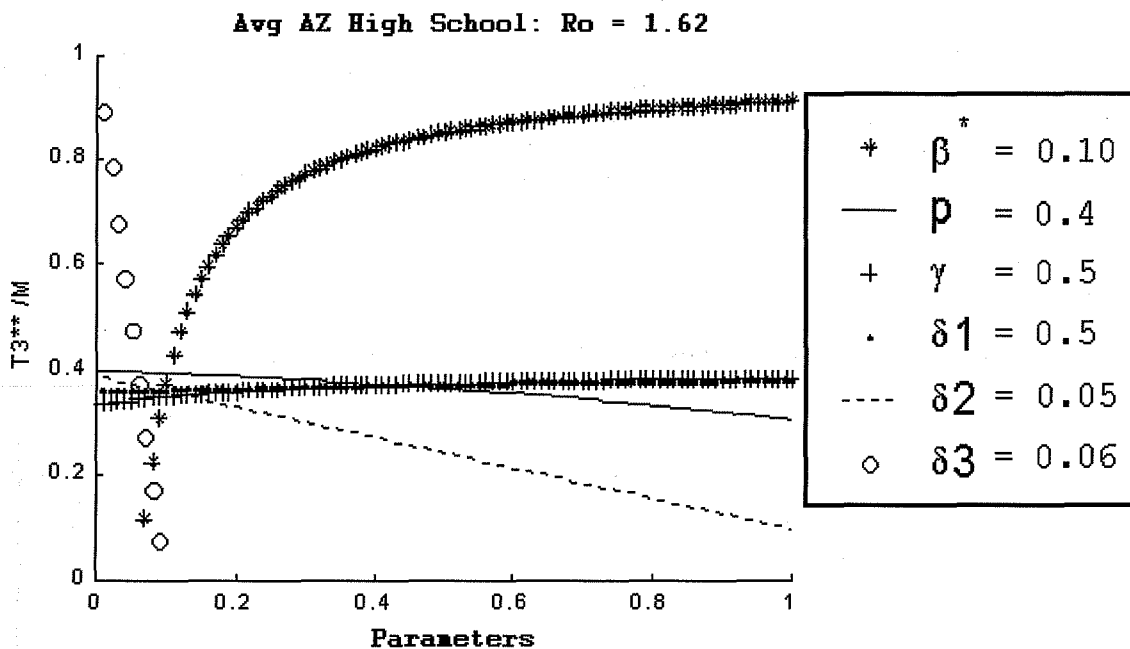
The situation for master teachers to be successful in average California and Arizona high schools is in jeopardy because of many social and economic reasons. For students and teachers in schools with high minority populations, the conditions are even more unstable. Minority and low-income students are more likely to attend schools with a high proportion of less than qualified teachers (Darling-Hammond 1998). Furthermore, Latino students have suffered disproportionate dropout rates, and in areas such as Los Angeles County, they have been instructed by as much as 60% of all the state's less than qualified teachers (CSU 1996). In Arizona, Latinos are even more likely to drop out, especially

Avg CA High School Ro = 1.15



Using the same parameters values for an average California high school, this graph shows how sensitive $\frac{T_3^{**}}{M}$ is to changes in the parameter values. When the value of β^* is nearer the value of δ_3 , increasing β^* for some small amount causes almost exponential growth in the proportion of master teachers. Although the graph of T_3^{**} vs δ_3 has a negative slope, the graph of T_3^{**} vs δ_3 has a steeper negative slope. This implies that if master teachers retire slightly earlier than the given δ_3 , the effect on the proportion of master teachers would be substantially large and negative.

Arizona shows similar conditions.

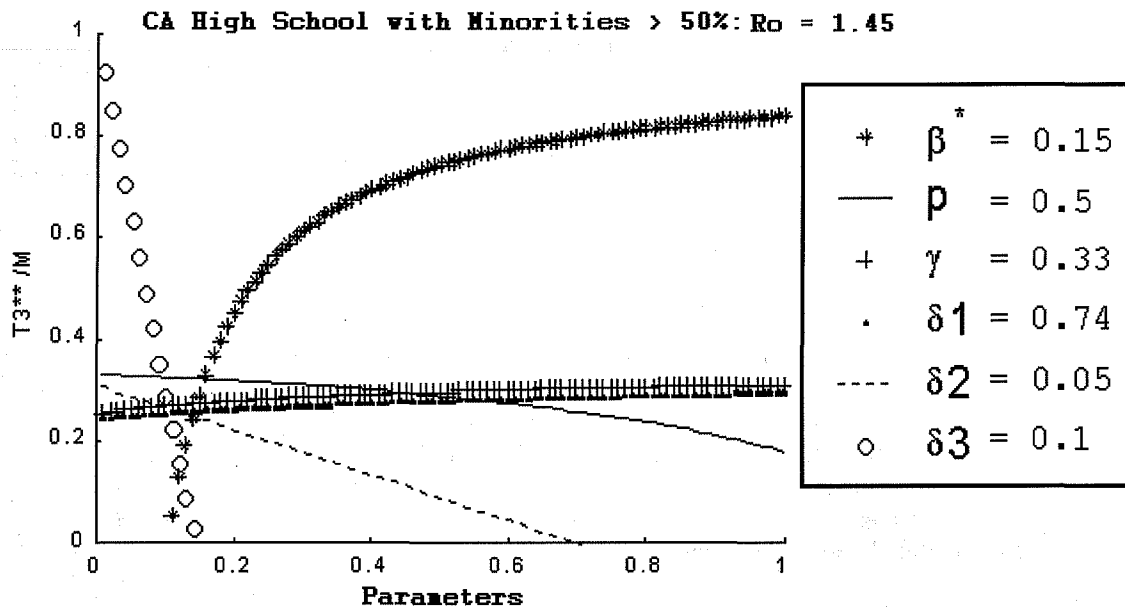


For an average Arizona high school, δ_3 , and β^* are sensitive parameters, but in this case δ_2 has a large negative slope. This means that the master teacher population in an average Arizona high school is more likely than an average California school to feel the impact of fully qualified teachers retiring early. Otherwise, neither California nor Arizona master teacher population demonstrate a strong sensitivity to fluctuations with respect to δ_1 and γ in the less than qualified teacher population.

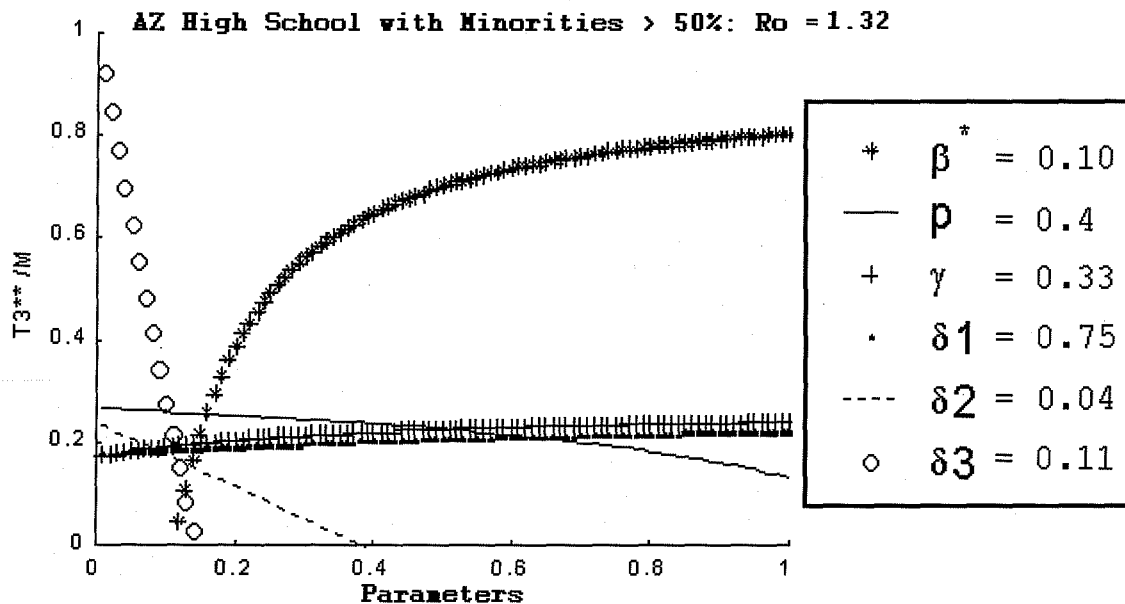
in urban areas (Ingersoll 1999). These patterns translate into parameter values that differ from the average values for the states.

The parameter values are estimated to be $\delta_1 = 0.74$, $\delta_2 = 0.05$, $\delta_3 = 0.1$, $\gamma = 0.33$, $p = 0.5$, and $\beta^* = 0.15$ for an average California high school with minorities > 50%. This gives a value of $R_0 = 1.45$.

For California minority students, similar to the previous graphs, β^* is a significant parameter in that slight increases can change the proportion of master teachers for the better. However, because of social conditions embodied by the high number of newly hired less than qualified teachers, δ_2 is a crucial parameter. As the fully certified teachers leave the system, they are replaced by high numbers of less than qualified teachers. Hence, β^* may be less than 0.10 because a lower value could be more realistic. "Student characteristics such as poverty, ... and minority status are negatively correlated with students outcomes... These student characteristics are [similarly] correlated with the qualifications of teachers; that is the less socially advantaged the students, ... the more likely [teachers] are to have entered teaching without certification" (Darling-Hammond 2000).



Arizona minority students face similar outcomes. The parameter values are estimated to be $\delta_1 = 0.75$, $\delta_2 = 0.04$, $\delta_3 = 0.11$, $\gamma = 0.33$, $p = .4$ and $\beta^* = 0.10$ for an average Arizona high school with minorities > 50%. This gives a value of $R_0 = 1.32$.



In Arizona, δ_2 has an even steeper slope. Minority students experience very similar conditions to those in California. The results show that student's achievement does not necessarily depend on their demographics. In fact, this suggests that there is a link between teachers' and her/his qualifications and the success or failure of students. While there is clearly a combination of social factors implicit in this discussion, what should not be taken for granted is that teachers do not affect students or that students are the only cause for their failures. It is a complex set of issues and circumstances, and as these graphs illustrate, some factors are more related than others. Most importantly, some factors can be targeted because of the stronger effects on the teacher populations and the subsequent student experiences.

10 Conclusion

The initial purpose of this study was to find conditions that would reduce existing school dropout rates. After researching available resources, it was difficult to determine the rates at which less than qualified, fully qualified and master teachers affect students. The available data provides only enough information to analyze the conditions that will enable master teachers to thrive. The results show how necessary it is to have enough qualified teachers in the classroom. Increasing the effectiveness of master teachers in various school settings demonstrates the need of supporting a critical mass of master teachers in each school.

This study finds that less than fully qualified teachers tend to leave average CA and AZ minority high schools before becoming master teachers more than for average CA and AZ high schools. This reduces the pool of teachers for master teachers to work with and recruit from which impacts their effectiveness as master teachers. It also causes master teachers to invest in developing fully qualified teachers who then leave without contributing at the minority school. Furthermore, this study concludes that lowering student teacher ratios does not increase the effectiveness of master teachers if it is based on the hiring of less than fully qualified teachers. Less than fully qualified teachers tend to leave minority schools at a greater rate than the time it takes for them to become fully qualified and then reach the level of experience and education to be an effective master teacher themselves. Finally, this research indicates that the best way to increase the proportion of master teachers is by providing opportunities for them to develop fully qualified teachers.

While student demographics play a role in student achievement, this study shows that teacher qualifications should not be ignored. Beyond the need to hire fully qualified teachers and support master teachers, policy makers and educational advocates need to screen and monitor teacher recruitment programs. Some programs offer incentives and make exceptions for individuals who are less than qualified to be teachers in order to find enough teachers for minority schools. If there are no guarantees that the teachers will stay and become effective as master teachers, then minority communities find themselves paying extra for less than qualified teachers and then watching their investments leave to other schools when they need the experienced teachers the most. Therefore, education advocates and minority community members should request detailed teacher preparedness reports as part of school reform efforts in order to take into account how teacher qualifications effect student achievement.

11 Future Studies

Throughout this project, it has become clear how complicated the present situation is in the high school educational system. Some factors include out-of-field teaching where a teacher has a degree in one subject area but actually teaches

in another area. For example, a teacher with a degree in mathematics is teaching science. There is also evidence that cultural affinity plays a role in student achievement. One possible aspect to analyze would be the effects that Latino master teachers have on Latino students. It is possible to study the effects of different types of teachers on students, with respect to gender and race and the effects of less than qualified teachers on master teacher attrition rates and vice versa. Finally, it would be beneficial to study the teacher recruitment programs which are already in place across the nation. Above all, it is most important to ensure that the community is aware of what is happening in today's educational system so as to create a beneficial learning environment for students and teachers alike.

Appendix

A Derivation of stable equilibrium point for student system

Recall the equations from the student model, i.e, equations (4)-(7). Using these equations, the system is solved in terms of S, I_1, I_2, I_3 . The results show only the following solution:

$$S = \frac{\Lambda M}{\beta_1 T_1 + \beta_2 T_2 + \beta_3 T_3 + \alpha_S M}, \quad (11)$$

$$I_1 = \frac{\Lambda \beta_1 T_1}{\alpha_1 (\beta_1 T_1 + \beta_2 T_2 + \beta_3 T_3 + \alpha_S M)}, \quad (12)$$

$$I_2 = \frac{\Lambda \beta_2 T_2}{\alpha_2 (\beta_1 T_1 + \beta_2 T_2 + \beta_3 T_3 + \alpha_S M)}, \quad (13)$$

$$I_3 = \frac{\Lambda \beta_3 T_3}{\alpha_3 (\beta_1 T_1 + \beta_2 T_2 + \beta_3 T_3 + \alpha_S M)}. \quad (14)$$

The Jacobian for the student system of equations is

$$A_S = \begin{pmatrix} -\left(\frac{\beta_1 T_1}{M} + \frac{\beta_2 T_2}{M} + \frac{\beta_3 T_3}{M} + \alpha_S\right) & 0 & 0 & 0 \\ \frac{\beta_1 T_1}{M} & -\alpha_1 & 0 & 0 \\ \frac{\beta_2 T_2}{M} & 0 & -\alpha_2 & 0 \\ \frac{\beta_3 T_3}{M} & 0 & 0 & -\alpha_3 \end{pmatrix}.$$

Condition 1:

$$\begin{aligned} \text{trace}(A_S) &= -\left(\frac{\beta_1 T_1}{M} + \frac{\beta_2 T_2}{M} + \frac{\beta_3 T_3}{M} + \alpha_s + \alpha_1 + \alpha_2 + \alpha_3\right) \\ &\Rightarrow -\left(\frac{\beta_1 T_1}{M} + \frac{\beta_2 T_2}{M} + \frac{\beta_3 T_3}{M} + \alpha_s + \alpha_1 + \alpha_2 + \alpha_3\right) < 0. \end{aligned}$$

Thus, the trace is always negative and condition (1) is satisfied.

Condition 2:

$$\begin{aligned} \det(A_S) &= -\left(\frac{\beta_1 T_1}{M} + \frac{\beta_2 T_2}{M} + \frac{\beta_3 T_3}{M} + \alpha_s\right)(-\alpha_1)(-\alpha_2)(-\alpha_3) \\ &\Rightarrow \alpha_1 \alpha_2 \alpha_3 \left(\frac{\beta_1 T_1}{M} + \frac{\beta_2 T_2}{M} + \frac{\beta_3 T_3}{M} + \alpha_s\right) > 0. \end{aligned}$$

Thus, the determinant is always positive and condition (2) is satisfied. Since the determinant is positive and the trace is negative for any point, T^* and T^{**} will be stable precisely one at a time. Therefore, a stable equilibrium point for the teacher system generates a corresponding stable equilibrium point for the student system.

B Writing equilibria in terms of R_0 for the student model

Using T^* , let

$$z^* = R_0 p \beta_1 \delta_2 \delta_3 + R_0 \beta_2 \delta_3 (q \delta_1 + \gamma) + \alpha_s \beta^* (q \delta_1 + \gamma)$$

Then,

$$\begin{aligned} S^* &= \frac{\beta^* \Lambda (q \delta_1 + \gamma)}{z^*} \\ I_1^* &= \frac{R_0 p \beta_1 \delta_2 \delta_3 \Lambda}{\alpha_1 z^*} \\ I_2^* &= \frac{R_0 \beta_2 \delta_3 \Lambda (q \delta_1 + \gamma)}{\alpha_2 z^*} \\ I_3^* &= 0 \end{aligned} \tag{15}$$

Using T^{**} , let

$$z^{**} = \beta_1 \left(\frac{KQ}{R_0}\right) + \beta_2 \left(\frac{K}{R_0}\right) + \beta_3 \left(1 - \frac{K}{R_0} - \frac{KQ}{R_0}\right).$$

Then,

$$\begin{aligned}
 S^{**} &= \frac{\Lambda}{z^{**}} \\
 I_1^{**} &= \frac{\beta_1 \Lambda K Q}{R_0 \alpha_1 z^{**}} \\
 I_2^{**} &= \frac{\beta_2 \Lambda K}{R_0 \alpha_2 z^{**}} \\
 I_3^{**} &= \frac{\beta_3 \Lambda}{\alpha_3 z^{**}} \left(1 - \frac{K}{R_0} - \frac{KQ}{R_0} \right).
 \end{aligned}
 \tag{16}$$

C Partial Derivatives of Teacher Equilibrium

To check how sensitive the equilibrium point, T_3^{**} , is to small changes in the parameters, the point is differentiated with respect to each parameter in R_0 . This will give an expression in terms of the parameters that can be solved. Note that the partial derivative of T_3^{**} with respect to M is not computed because it is constant. Below are the computed partial derivatives with respect to the parameters found in R_0 :

$$\frac{\partial T_3^{**}}{\partial \beta^*} = \frac{\delta_3 M}{(\beta^*)^2} \left(\frac{(1-p)\delta_1 + p\delta_2 + \gamma}{(1-p)\delta_1 + p\delta_3 + \gamma} \right)$$

$$\frac{\partial T_3^{**}}{\partial p} = -\frac{\delta_3 M}{\beta^*} \left(\frac{(\beta^* + \delta_2 - \delta_3)(\delta_1 + \gamma)}{(p\delta_1 - \delta_1 - p\delta_3 - \gamma)^2} \right)$$

$$\frac{\partial T_3^{**}}{\partial \gamma} = \frac{\delta_3 M p}{\beta^*} \left(\frac{\beta^* + \delta_2 - \delta_3}{(p\delta_1 - \delta_1 - p\delta_3 - \gamma)^2} \right)$$

$$\frac{\partial T_3^{**}}{\partial \delta_1} = \frac{\delta_3 M p}{\beta^*} \left(\frac{(\beta^* + \delta_2 - \delta_3)(1-p)}{(p\delta_1 - \delta_1 - p\delta_3 - \gamma)^2} \right)$$

$$\frac{\partial T_3^{**}}{\partial \delta_2} = \frac{\delta_3 M p}{\beta^*} \left(\frac{1}{p\delta_1 - \delta_1 - p\delta_3 - \gamma} \right)$$

$$\frac{\partial T_3^{**}}{\partial \delta_3} = \frac{M}{\beta^*} \left(\frac{p(-\beta^* - \delta_2 + 2\delta_3)}{(1-p)\delta_1 + p\delta_3 + \gamma} + \frac{p^2 \delta_3 (\beta^* + \delta_2 - \delta_3)}{((1-p)\delta_1 + p\delta_3 + \gamma)^2} - 1 \right).$$

D MATLAB code

The following MATLAB code is used to generate the graphs used in this study.

```
%T3** = Z

%X = p
%Y = Beta*
%This is the code we use to plot T3/M vs Beta* and p
%Z = 1 - (delta3./X) - (Y*delta3./X).*(delta2+X-delta3)./(delta1-
%Y*delta1+gamma+Y*delta3);
%axis([0 1 0 1 0 1])

figure
subplot(2,2,1)
hold on

delta1=0.5;
delta2=0.04;
delta3=0.06;
gamma=0.75;
p=0.15;
beta=0.15;

t = .01:.01:1;
y(1)=beta;
y = 1-(delta3./t)-(p*delta3./t).*(delta2+t-delta3)./(delta1-
p*delta1+gamma+p*delta3);
plot(t,y,'*')
y(2)=p;
y = 1-(delta3/beta)-(t*delta3./beta).*(delta2+beta-delta3)./
(delta1-t*delta1+gamma+t*delta3);
plot(t,y,'-')
y(3)=gamma;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(delta1-p*delta1+t*p*delta3);
plot(t,y,'+')
y(4)=delta1;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(t-p*t+gamma+p*delta3);
plot(t,y,'.')
y(5)=delta2;
y = 1-(delta3/beta)-(p*delta3./beta).*(t+beta-delta3)./(delta1-
p*delta1+gamma+p*delta3);
plot(t,y,':')
```

```

y(6)=delta3;
y = 1-(t./beta)-(p*t./beta).*(delta2+beta-t)./(delta1-p*delta1+
gamma+p*t);
plot(t,y,'o')

axis([0 1 0 1])
title('Avg National High School: Ro > 1')
xlabel('Parameters: \beta^* = 0.15, p = 0.15, \gamma = 0.75,
\delta1 = 0.5, \delta2 = 0.04, \delta3 = 0.06');
ylabel('T3** /M');
%legend('\beta^*', 'p', '\gamma', '\delta1', '\delta2', '\delta3');
zoom
hold off

subplot(2,2,2)
hold on

delta1=0.84;
delta2=0.04;
delta3=0.13;
gamma=0.33;
p=0.07;
beta=0.15;

t = .01:.01:1;
y(1)=beta;
y = 1-(delta3./t)-(p*delta3./t).*(delta2+t-delta3)./(delta1-
p*delta1+gamma+p*delta3);
plot(t,y,'*')
y(2)=p;
y = 1-(delta3/beta)-(t*delta3./beta).*(delta2+beta-delta3)./
(delta1-t*delta1+gamma+t*delta3);
plot(t,y,'-')
y(3)=gamma;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(delta1-p*delta1+t+p*delta3);
plot(t,y,'+')
y(4)=delta1;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(t-p*t+gamma+p*delta3);
plot(t,y,'.')
y(5)=delta2;
y = 1-(delta3/beta)-(p*delta3./beta).*(t+beta-delta3)./(delta1-
p*delta1+gamma+p*delta3);
plot(t,y,':')
y(6)=delta3;

```

```

y = 1-(t./beta)-(p*t./beta).*(delta2+beta-t)./(delta1-p*delta1+
gamma+p*t);
plot(t,y,'o')

axis([0 1 0 1])
title('Avg CA High School: Ro > 1')
xlabel('Parameters: \beta^* = 0.15, p = 0.07, \gamma = 0.33,
\delta1 = 0.84, \delta2 = 0.04, \delta3 = 0.13');
ylabel('T3** /M');
%legend('\beta^*', 'p', '\gamma', '\delta1', '\delta2', '\delta3');zoom
hold off

subplot(2,2,3)
hold on

delta1=0.74;
delta2=0.05;
delta3=0.1; %10 years
gamma=0.33;
p=0.5;
beta=0.15;

t = .01:.01:1;
y(1)=beta;
y = 1-(delta3./t)-(p*delta3./t).*(delta2+t-delta3)./(delta1-
p*delta1+gamma+p*delta3);
plot(t,y,'*')
y(2)=p;
y = 1-(delta3/beta)-(t*delta3./beta).*(delta2+beta-delta3)./
(delta1-t*delta1+gamma+t*delta3);
plot(t,y,'-')
y(3)=gamma;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(delta1-p*delta1+t*p*delta3);
plot(t,y,'+')
y(4)=delta1;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(t-p*t+gamma+p*delta3);
plot(t,y,'.')
y(5)=delta2;
y = 1-(delta3/beta)-(p*delta3./beta).*(t+beta-delta3)./
(delta1-p*delta1+gamma+p*delta3);
plot(t,y,':')
y(6)=delta3;
y = 1-(t./beta)-(p*t./beta).*(delta2+beta-t)./(delta1-p*delta1+

```



```

gamma+p*t);
plot(t,y,'o')

axis([0 1 0 1])
title('CA High School with Minorities > 50%: Ro > 1')
xlabel('Parameters: \beta^* = 0.15, p = 0.5, \gamma = 0.33,
\delta1 = 0.74, \delta2 = 0.05, \delta3 = 0.1');
ylabel('T3** /M');
%legend('\beta^*', 'p', '\gamma', '\delta1', '\delta2', '\delta3');
zoom
hold off

subplot(2,2,4)
hold on

delta1=0.74;
delta2=0.05;
delta3=0.1; %10 years
gamma=0.33;
p=0.5;
beta=0.09;

t = .01:.01:1;
y(1)=beta;
y = 1-(delta3./t)-(p*delta3./t).*(delta2+t-delta3)./(delta1-
p*delta1+gamma+p*delta3);
plot(t,y,'*')
y(2)=p;
y = 1-(delta3/beta)-(t*delta3./beta).*(delta2+beta-delta3)./
(delta1-t*delta1+gamma+t*delta3);
plot(t,y,'-')
y(3)=gamma;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(delta1-p*delta1+t+p*delta3);
plot(t,y,'+')
y(4)=delta1;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(t-p*t+gamma+p*delta3);
plot(t,y,'.')
y(5)=delta2;
y = 1-(delta3/beta)-(p*delta3./beta).*(t+beta-delta3)./(delta1-
p*delta1+gamma+p*delta3);
plot(t,y,':')
y(6)=delta3;
y = 1-(t./beta)-(p*t./beta).*(delta2+beta-t)./(delta1-p*delta1+
gamma+p*t);

```

```

plot(t,y,'o')

axis([0 1 0 1])
title('CA High School with Minorities > 50%: Ro < 1')
xlabel('Parameters: \beta^* = 0.09, p = 0.5, \gamma = 0.33,
\delta1 = 0.74, \delta2 = 0.05, \delta3 = 0.1');
ylabel('T3** /M');
legend('\beta^*', 'p', '\gamma', '\delta1', '\delta2', '\delta3');
zoom
hold off

figure
subplot(2,2,1)
hold on

delta1=0.5;
delta2=0.04;
delta3=0.06;
gamma=0.75;
p=0.15;
beta=0.15;

t = .01:.01:1;
y(1)=beta;
y = 1-(delta3./t)-(p*delta3./t).*(delta2+t-delta3)./(delta1-
p*delta1+gamma+p*delta3);
plot(t,y,'*')
y(2)=p;
y = 1-(delta3/beta)-(t*delta3./beta).*(delta2+beta-delta3)./
(delta1-t*delta1+gamma+t*delta3);
plot(t,y,'-')
y(3)=gamma;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(delta1-p*delta1+t+p*delta3);
plot(t,y,'+')
y(4)=delta1;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(t-p*t+gamma+p*delta3);
plot(t,y,'.')
y(5)=delta2;
y = 1-(delta3/beta)-(p*delta3./beta).*(t+beta-delta3)./(delta1-
p*delta1+gamma+p*delta3);
plot(t,y,':')
y(6)=delta3;
y = 1-(t./beta)-(p*t./beta).*(delta2+beta-t)./(delta1-p*delta1+

```

```

gamma+p*t);
plot(t,y,'o')

axis([0 1 0 1])
title('Avg National High School: Ro > 1')
xlabel('Parameters: \beta^* = 0.15, p = 0.15, \gamma = 0.75,
\delta1 = 0.5, \delta2 = 0.04, \delta3 = 0.06');
ylabel('T3** /M');
%legend('\beta^*', 'p', '\gamma', '\delta1', '\delta2', '\delta3');
zoom
hold off
subplot(2,2,2)
hold on

delta1=0.5;
delta2=0.05;
delta3=0.06;
gamma=0.5;
p=0.4;
beta=0.10;

t = .01:.01:1;
y(1)=beta;
y = 1-(delta3./t)-(p*delta3./t).*(delta2+t-delta3)./(delta1-
p*delta1+gamma+p*delta3);
plot(t,y,'*')
y(2)=p;
y = 1-(delta3/beta)-(t*delta3./beta).*(delta2+beta-delta3)./
(delta1-t*delta1+gamma+t*delta3);
plot(t,y,'-')
y(3)=gamma;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(delta1-p*delta1+t+p*delta3);
plot(t,y,'+')
y(4)=delta1;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(t-p*t+gamma+p*delta3);
plot(t,y,'.')
y(5)=delta2;
y = 1-(delta3/beta)-(p*delta3./beta).*(t+beta-delta3)./(delta1-
p*delta1+gamma+p*delta3);
plot(t,y,':')
y(6)=delta3;
y = 1-(t./beta)-(p*t./beta).*(delta2+beta-t)./(delta1-p*delta1+
gamma+p*t);

```

```

plot(t,y,'o')

axis([0 1 0 1])
title('Avg AZ High School: Ro > 1')
xlabel('Parameters: \beta^* = 0.10, p = 0.4, \gamma = 0.5,
\delta1 = 0.5, \delta2 = 0.05, \delta3 = 0.06');
ylabel('T3** /M');
%legend('\beta^*', 'p', '\gamma', '\delta1', '\delta2', '\delta3'); zoom
hold off

subplot(2,2,3)
hold on

delta1=0.75;
delta2=0.04;
delta3=0.11;
gamma=0.33;
p=0.6;
beta=0.15;

t = .01:.01:1;
y(1)=beta;
y = 1-(delta3./t)-(p*delta3./t).*(delta2+t-delta3)./(delta1-
p*delta1+gamma+p*delta3);
plot(t,y,'*')
y(2)=p;
y = 1-(delta3/beta)-(t*delta3./beta).*(delta2+beta-delta3)./
(delta1-t*delta1+gamma+t*delta3);
plot(t,y,'-')
y(3)=gamma;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(delta1-p*delta1+t+p*delta3);
plot(t,y,'+')
y(4)=delta1;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(t-p*t+gamma+p*delta3);
plot(t,y,'.')
y(5)=delta2;
y = 1-(delta3/beta)-(p*delta3./beta).*(t+beta-delta3)./(delta1-
p*delta1+gamma+p*delta3);
plot(t,y,':')
y(6)=delta3;
y = 1-(t./beta)-(p*t./beta).*(delta2+beta-t)./(delta1-p*delta1+
gamma+p*t);
plot(t,y,'o')

```

```

axis([0 1 0 1])
title('AZ High School with Minorities > 50% : Ro > 1')
xlabel('Parameters: \beta^* = 0.15, p = 0.6, \gamma = 0.33,
\delta1 = 0.75, \delta2 = 0.04, \delta3 = 0.11');
ylabel('T3** /M');
legend('\beta^*', 'p', '\gamma', '\delta1', '\delta2', '\delta3'); zoom
hold off

```

```

subplot(2,2,4)
hold on

```

```

delta1=0.75;
delta2=0.04;
delta3=0.11;
gamma=0.33;
p=0.6;
beta=0.09;

```

```

t = .01:.01:1;
y(1)=beta;
y = 1-(delta3./t)-(p*delta3./t).*(delta2+t-delta3)./
(delta1-p*delta1+gamma+p*delta3);
plot(t,y,'*')
y(2)=p;
y = 1-(delta3/beta)-(t*delta3./beta).*(delta2+beta-delta3)./
(delta1-t*delta1+gamma+t*delta3);
plot(t,y,'-')
y(3)=gamma;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(delta1-p*delta1+t+p*delta3);
plot(t,y,'+')
y(4)=delta1;
y = 1-(delta3/beta)-(p*delta3./beta).*(delta2+beta-delta3)./
(t-p*t+gamma+p*delta3);
plot(t,y,'.')
y(5)=delta2;
y = 1-(delta3/beta)-(p*delta3./beta).*(t+beta-delta3)./
(delta1-p*delta1+gamma+p*delta3);
plot(t,y,':')
y(6)=delta3;
y = 1-(t./beta)-(p*t./beta).*(delta2+beta-t)./
(delta1-p*delta1+gamma+p*t);
plot(t,y,'o')

```

```

axis([0 1 0 1])
title('AZ High School with Minorities > 50%: Ro < 1')

```

```
xlabel('Parameters: \beta^* = 0.09, p = 0.6, \gamma = 0.33,  
\delta1 = .75, \delta2 = 0.04, \delta3 = 0.11');  
ylabel('T3** /M');  
%legend('\beta^*', 'p', '\gamma', '\delta1', '\delta2', '\delta3'); zoom  
hold off  
%
```

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