Preventing Crack Babies: Different Approaches of Prevention

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Abstract

Crack-cocaine use among pregnant women is of major social and public health concern for many reasons including the birth of "crack babies" and its relations to HIV infection. Many programs exist that focus on the rehabilitation of women and mothers who use cocaine. We use deterministic and stochastic approaches to model the effectiveness of these programs. The focus will be on populations of women (often commercial sex workers) who are encouraged or forced to use drugs by drug dealers, pimps or both. The impact of drug rehabilitation and other treatment programs among particular groups is explored as well as the role of drug enforcement on the dynamics of this system. In particular, the role of pimp's pressure on women to use drugs, the inability of drug users to quit due to addiction, and the relapse among those in rehabilitation programs are explored. The effect of longer jail terms for drug dealers and pimps is discussed in the context of the model and data available.

1 Introduction

Substance abuse during pregnancy is a serious public-health issue, consuming valuable health-care resources and contributing to infant mortality and morbidity. Data for drug abuse indicates that substance abuse by women during pregnancy continues to increase. The spreading abuse of marijuana, cocaine, alcohol, cigarettes and other drugs has intensified concerns about the implications of maternal drug use for unborn children.

Medical reports show that cocaine can be the most harmful illicit drug. It can increase the risk of hemorrhaging and premature delivery, threatening the lives of both mothers and children. Cocaine users give birth to babies with low average birth weight, more than 5.5 pounds less than babies of women who do not use cocaine [6]. It is estimated that the national cost to care for a "cocaine baby" is about 3 billion dollars [5]. The consequences of drug use among pregnant women and their children are multiple. Mothers who smoke cigarettes during pregnancy may increase the risk that their child will have autism [2]. Marijuana users give birth to babies who are three ounces lighter and $\frac{1}{5}$ inch shorter than babies born to women who do not use marijuana [1]. Head size is often smaller in infants exposed to narcotics. While growth erases some of the physical differences, there may be subtle, long-term deficits in mental or neurological functioning in infants exposed to drugs in the womb. Women who use intravenous drugs or share drugs are at a higher risk of contracting deadly diseases like AIDS. Scientists are just beginning to explore how various drugs may effect the development of physical coordination, language and emotional interactions.

A survey found that an estimated $113,000$ white women, $75,000$ African-American women, and $28,000$ Hispanic women in the USA use illicit drugs during pregnancy [3]. At some point during their pregnancy, 20.4%, or $820,000$, pregnant women smoked cigarettes and 18.8%, or $757,000$, drank alcohol [3]. Among those women who used both cigarettes and alcohol, 20.4% also used marijuana and 9.5% took cocaine [3]. African-American women had the highest rates of cocaine use, mainly "crack", during pregnancy [3]. About 4.5% of African American, 0.4% of white women and 0.7% of Hispanic women use crack-cocaine [3]. The researchers estimate that each year, as many as $375,000$ infants may be affected by their mothers' drug use [1].

ResearchF indicates that women can become addicted quickly to certain drugs, even after casual or experimental; use and more than 4 million need treatment [4]. Therefore, by the time a woman enters treatment she may be severely addicted, successful treatment in this case is more difficult. Hence, treatment should include an evaluation of other serious health problems as-

sociated with drug abuse. Many drug-using women do not seek treatment because they are afraid that they will not be able to keep or care for their children. Some of these women often fear reprisal from their spouses, boyfriends or punishment from authorities in the community. Also, many programs have refused to accept pregnant women or have been unable to provide them services that they need, including prenatal care, parenting skills instruction, childcare and transportation. Many women report that their drug-using male sex partners initiate them into drug abuse and then sabotaged their efforts to quit [4].

The war of drugs has been a long drawn out affair and we are losing. The victims of this drug war are all who are addicted, regardless of their age, race or gender. Drug addiction is not an easy process to endure. Children are our most precious assets and it is essential that we protect them.

In this project we want to analyze the drug abuse situation, creating a model that examines the influence of a program that educates pregnant and non-pregnant women about the dangers and consequences of drug abuse. We focus our research on crack-cocaine abuse among women, how we can reduce their rate of drug use, and the impact of encouraging pregnant and non-pregnant women to go to rehabilitation. We carry this project in the context of a system driven by a population of males who use the power and influence of drug addiction to use and control women. Our findings from our modeling effort are that we cannot eliminate drug use among women unless we include both incarceration for drug dealing men and rehabilitation for men and women.

Our paper is organized as follows: section 2 introduces a basic deterministic model where we consider only rehabilitation for men and women; section 3 compares a stochastic version of our model with the deterministic model from section 2; section 4 introduces drug induced mortality to our basic model; section 5 replaces the rehabilitation class from section 4 with a jail class for the male case to explore the effects of incarceration; section 6 combines a jail class with a rehabilitation class for men to see how these two factors can reduce the population of drug using women; section 7 takes our basic model and applies an age structure system to it; in section 8 we discuss the results of our models; our conclusions are drawn in section 9; finally section 10 states what we have left for future work.

2 The Deterministic Model

A basic deterministic model is introduced to study the impact of education on drug abuse in pregnant and non-pregnant women. The model, a two-sex male driven system, consists of nine nonlinear differential equations. This system is driven by a three-dimension system that models the dynamics of drug use among males. The first model (see Figure 1) does not incorporate drug-induced mortality explicitly as it assumes that the average residence time in the system for males is $\frac{1}{\mu_m}$ and for females is $\frac{1}{\mu_f}$. In this assumption the reasons for their departure are included. Hence, it is assumed that all males are equally likely to leave the system by a variety of reasons (murdered, natural death, drug use, etc.). Females are also especially likely to leave the system regardless of drug use habits or pregnancy status. Furthermore, we are assuming that $\mu_m = \mu_f = \mu$. This is of course not true but the conditions will be relaxed later on.

The assumption of equal but gender specific exit rates let us normalized the system, letting $X = \frac{S_m}{N_m}$, $Y = \frac{D_m}{N_m}$, $Z = \frac{R_m}{N_m}$, $P = \frac{S_f}{N_f}$, $Q = \frac{D_f}{N_f}$, $R = \frac{R_f}{N_f}$, $S = \frac{P_s}{N_f}$, $T = \frac{P_d}{N_f}$, and $U = \frac{P_r}{N_f}$. We arrive at the following system of equations:

$$\begin{aligned}
\frac{dX}{dt} &= \mu - \beta_m XY - \mu X \\
\frac{dY}{dt} &= \beta_m XY + \rho_m Z - (\gamma_m + \mu)Y \\
\frac{dZ}{dt} &= \gamma_m Y - (\rho_m + \mu)Z \\
\frac{dP}{dt} &= \mu + \lambda_1 S - \beta_f PY - (\mu + \phi_1)P \\
\frac{dQ}{dt} &= \beta_f PY + \rho_1 R + \lambda_2 T - (\mu + \phi_2 + \gamma_1)Q \\
\frac{dR}{dt} &= \gamma_1 Q + \lambda_3 U - (\rho_1 + \phi_3 + \mu)R \\
\frac{dS}{dt} &= \phi_1 P - \beta_f SY - (\lambda_1 + \mu)S \\
\frac{dT}{dt} &= \beta_f SY + \phi_2 Q + \rho_2 U - (\lambda_2 + \gamma_2 + \mu)T \\
\frac{dU}{dt} &= \gamma_2 T + \phi_3 R - (\rho_2 + \lambda_3 + \mu)U
\end{aligned}$$

where $X + Y + Z = 1$ and $P + Q + R + S + T + U = 1$.

Here, P represents the proportion of susceptible women, Q the proportion of drug using women, R the proportion of women in rehabilitation, S the

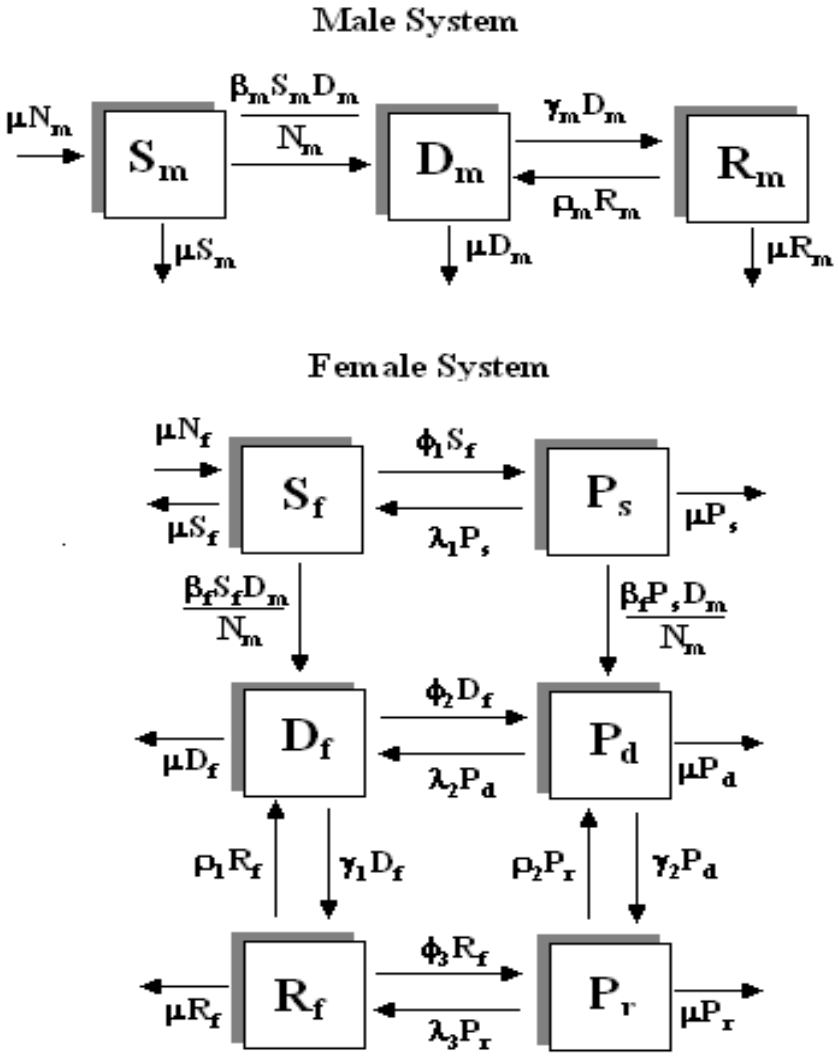

Figure 1: The Deterministic Model

proportion of susceptible pregnant women, T the proportion of drug using pregnant women, U the proportion of pregnant women in rehabilitation, X the proportion of non-drug using men, Y the proportion of drug using men, and Z the proportion of men in rehabilitation.

For the male population, μ_m denotes the standard mortality or exit rate for all classes. The rate that men enter the susceptible class is equal to the number of men that have died from all classes, thus keeping the population constant. Once in the susceptible class, men are influenced by other drug-using men to use crack-cocaine via a mass action rate. That is to say, the rate that men cause other men to use drugs is proportional to the density of drug using men in the population. Then, drug using men can go into rehabilitation by a rate γ_m. Once in rehabilitation, the men can either stay in rehabilitation or relapse at a rate ρ_m. Note that men or women cannot go from the rehabilitation class back into the susceptible class. This is because we are assuming that addiction to crack-cocaine is very strong and once someone has become addicted; they always have a stronger chance of falling back into addiction. There is no "recovery" from crack-cocaine addiction, similar to what happens with alcoholism.

For the female system, there is also a standard death rate μ_f and the number of women that enter the susceptible class is equal to the number of women that die, thus keeping the population size constant. We see that pregnant and non-pregnant susceptible women are influenced by men to use crack-cocaine by a mass incidence rate, however women do not cause other women to use crack-cocaine. Often drug using men convince women to use drugs so that the men have better control over the women. This is especially true when the woman is closely associated (married or living together) to a drug using male [4], or in the case of pimps or drug dealers and commercial sex workers. In all classes of non-pregnant women (P, Q, R), we see that they can get pregnant at some rate ϕ_i. These parameters were approximated by taking the proportion of women who are pregnant in each class relative to the total number of women who are either susceptible, drug using, or in rehabilitation [3, 17, 16, 13, 9]. Also, in all classes of pregnant women (S, T, U) they can have miscarriages, give birth earlier and thus return to a state of non-pregnancy at rate λ_i. For the purposes of our models, we only consider a "miscarriage" to occur when the women loses her child after she realizes she is pregnant. Furthermore, we assumed that women cannot get pregnant again until a month after they give birth, have an abortion or miscarriage and that the rate of miscarriage for women who have never used drugs is negligible compared to that of drug using women [6, 8]. When both pregnant and non-pregnant women are in the drug using class, they can go to rehabilitation at a rate γ_i. Once in rehabilitation, they can relapse back into

the drug using class at a rate ρ_i. Note that we did not include an education program for men because we are assuming that women receive more social support than men from a variety of sources such as families, friends and coworkers and that women are more likely to maintain a social network and engage in treatment [19].

This system can be directly applied to the population of drug dealers, pimps, and commercial sex workers. In this scenario, susceptible men include pimps and drug dealers who are not addicted to crack-cocaine. Male drug-dealers and pimps can cause other males and females to become addicted to crack-cocaine, but female sex workers generally do not cause other women or men to abuse crack-cocaine. Pimps and drug dealers will often sabotage a female sex worker's attempt to get out of the drug culture so that he can remain in control, and thus the male population is the driving force on this system of crack-cocaine abuse.

Table 1: Parameter List

Parameters	Description
μ	natural mortality rate
ϕ_1	rate at which women that are not using drugs become pregnant
ϕ_2	rate at which women that are using drugs become pregnant
ϕ_3	rate at which women that are in education (rehabilitation) become pregnant
λ_1	rate at which women are susceptible to become pregnant again
λ_2	rate of which women drug users are susceptible to become pregnant again
λ_3	rate of which women in education (rehabilitation) are susceptible to become pregnant again
β_f	contact rate of women with drug using men
γ_1	rehabilitation rate of non-pregnant women
γ_2	rehabilitation rate of pregnant women
ρ_1	relapse rate of non-pregnant women
ρ_2	relapse rate of pregnant women
ρ_m	relapse rate of men
d	drug induced mortality for men
w	drug induced mortality for women
β_m	contact rate of men with drug using men
γ_m	rehabilitation rate of men

2.1 Definition of the basic reproductive number $\mathcal{R}_0$

To evaluate our system of equations we analyze the basic reproductive number, $\mathcal{R}_0$ of drug-abuse, interpreted in epidemiological models as the average number of secondary cases caused by a drug using male. In our system, $\mathcal{R}_0$ represents the average number of women, pregnant or non-pregnant, coerced to use drugs by men at the beginning of the drug epidemic.

To calculate $\mathcal{R}_0$, we consider the drug-abuse free equilibrium. The $\mathcal{R}_0$ of our system is derived from the male equations, because they are the only group generating secondary cases of drug addiction. In section 4, we show that $\mathcal{R}_0$ is given by:

$$\mathcal{R}_0 = \frac{\beta_m(\rho_m + \mu)}{\mu(\gamma_m + \mu + \rho_m)}$$

and illustrate its role in the stability of the drug free and endemic drug abuse equilibria.

2.2 Calculation of drug-abuse free equilibrium

One possible end state for this model is the drug-abuse free equilibrium. The drug-abuse free equilibrium of the male system is given by $(X^*, Y^*, Z^*) = (1, 0, 0)$, in which the entire population is susceptible but there is no drug abuse and no one in the rehabilitation class. The male system is give by

$$\begin{aligned}
\frac{dX}{dt} &= \mu - \beta_m XY - \mu X && (1)\\
\frac{dY}{dt} &= \beta_m XY + \rho_m Z - (\gamma_m + \mu)Y && (2)\\
\frac{dZ}{dt} &= \gamma_m Y - (\rho_m + \mu)Z && (3)
\end{aligned}$$

where $X + Y + Z = 1$.

We have the following theorem:

Theorem 2.1. *Let $\vec{x}_\infty(DF) = (1, 0, 0)$ be the disease free equilibrium of (1) - (3) then is locally asymptotically stable if and only if $\mathcal{R}_0 < 1$.*

Proof. The Jacobian given from the linearization at this equilibrium is:

$$J(1,0,0) = \begin{bmatrix} -\mu & -\beta & 0 \\ 0 & \beta_m - \mu - \gamma_m & \rho_m \\ 0 & \gamma_m & -(\rho_m + \mu) \end{bmatrix}.$$

Since $-\mu$ is an eigenvalue, we only need to consider the trace and the determinant of

$$A = \begin{bmatrix} \beta_m - \mu - \gamma_m & \rho_m \\ \gamma_m & -(\rho_m + \mu) \end{bmatrix}.$$

The $det(A) > 0 \Leftrightarrow \mathcal{R}_0 < 1$ and this implies that the $trace(A) < 0$

□

The drug abuse free equilibrium is stable for the males. This allows us to treat Y as a constant and thus linearize the female equations. Doing so, we can calculate the drug abuse free equilibrium for the whole system as:

$$(S_m^*, D_m^*, R_m^*, S_f^*, D_f^*, R_f^*, P_s^*, P_d^*, P_r^*) = (1, 0, 0, \frac{\lambda_1 + \mu}{\lambda_1 + \mu + \phi_1}, 0, 0, \frac{\phi_1}{\lambda_1 + \mu + \phi_1}, 0, 0)$$

Having $\mathcal{R}_0 < 1$ gives us the condition for local stability of the whole system.

2.3 Endemic Equilibrium and Stability Analysis

The nonzero solutions of the normalized system are: $X^* = \frac{1}{\mathcal{R}_0}$, $Y^* = \frac{\mu}{\beta_m}(\mathcal{R}_0 - 1)$, and $Z^* = \frac{\gamma_m \mu}{\beta_m(\rho_m + \mu)}(\mathcal{R}_0 - 1)$.

From the Jacobian matrix at $\vec{x}_\infty(EE)$ we obtained:

$$J(\vec{x}_\infty) = \begin{bmatrix} -\mu(\mathcal{R}_0 - 1) - \mu & -\frac{\beta_m}{\mathcal{R}_0} & 0 \\ \mu(\mathcal{R}_0 - 1) & \frac{\beta_m}{\mathcal{R}_0} - \mu - \gamma_m & \rho_m \\ 0 & \gamma_m & -(\rho_m + \mu) \end{bmatrix}$$

Solving for the eigenvalues of J:

$$(J - \lambda I) = \begin{bmatrix} -\mu(\mathcal{R}_0 - 1) - \mu - \lambda & -\frac{\beta_m}{\mathcal{R}_0} & 0 \\ \mu(\mathcal{R}_0 - 1) & \frac{\beta_m}{\mathcal{R}_0} - \mu - \gamma_m - \lambda & \rho_m \\ 0 & \gamma_m & -(\rho_m + \mu) - \lambda \end{bmatrix}$$

$$= -(\mu+\lambda)\begin{bmatrix} 1 & 1 & 1 \\ 0 & \beta_m-\mu-\gamma_m-\lambda+\mu(\mathcal{R}_0-1) & \rho_m+\mu(\mathcal{R}_0-1) \\ 0 & \gamma_m & -(\rho_m+\mu+\lambda) \end{bmatrix}$$

Since $-\mu$ is an eigenvalue, we only need to consider the trace and the determinant of:

$$A_E = \begin{bmatrix} \beta_m-\mu-\gamma_m-\lambda+\mu(\mathcal{R}_0-1) & \rho_m+\mu(\mathcal{R}_0-1) \\ \gamma_m & -(\rho_m+\mu+\lambda) \end{bmatrix}$$

The $det(A_E) > 0$ and $trace(A_E) < 0$. This implies that the endemic solution is locally asymptotically stable.

2.4 Deterministic Simulations

We analyzed numerical simulations using the same initial conditions and parameters to see the behavior of drug use and to get a more complete understanding of our model. In the simulations we vary parameters such as the contact rate of men and women and the rehabilitation rate of pregnant and non-pregnant women to see what effects these parameters would have on the proportion of pregnant women that used drugs. In doing so, we could determine how effective our education program would have to be in order to attain a certain amount of success defined by a level of decrease in the number of pregnant women that abused crack-cocaine. In this way we model an education program by altering the parameters that cause pregnant women to abuse crack-cocaine.

We analyze the effects of the interaction of women with drug-using men, see Figure (2). In Figure (b) the value of β_m is decreased from 0.0714 to 0.0414 and in (a) the value of β_m is increased to 0.0914. When comparing the (a) and (b), as we increase the value of β_m we get a correspondingly larger number of drug using women. This shows a direct correlation between the number of drug using women and the rate that women interact with drug using men.

We also analyze the situation of drug using women going into rehabilitation, see Figure (3). In (b) the value of γ_1 and γ_2 is decreased to 0.001108 and 0.004967. In (a) the value of γ_1 and γ_2 is increased from 0.007108, 0.014967 to 0.009108 and 0.054967 respectively. We see that as the value of γ_i is increased, there will be more people in rehabilitation. When the value of γ_i increases, the effectiveness of the education program for women increases and vice versa.

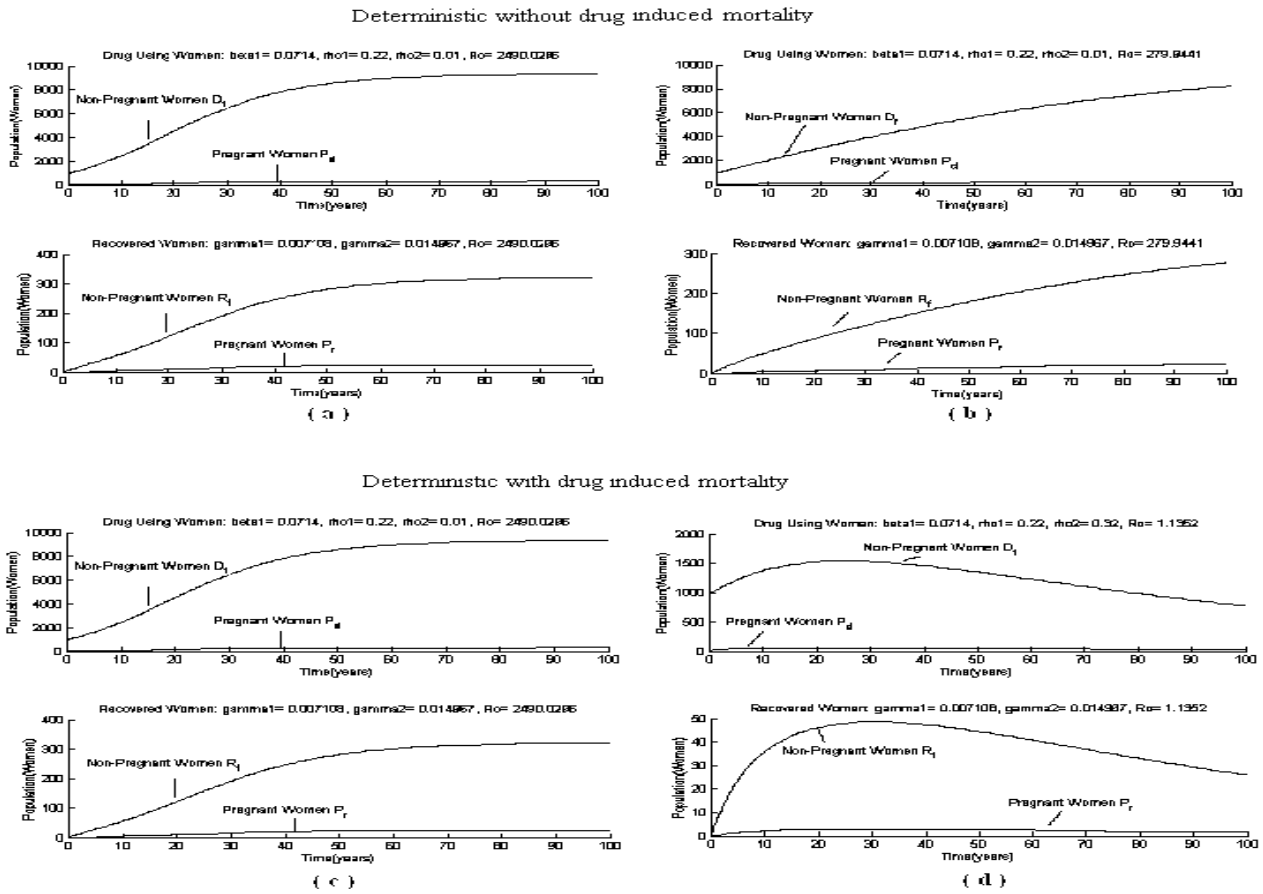

Figure 2: (a): Increasing β_m, (b): Decreasing β_m, (c): Increasing β_m, (d): Decreasing β_m.

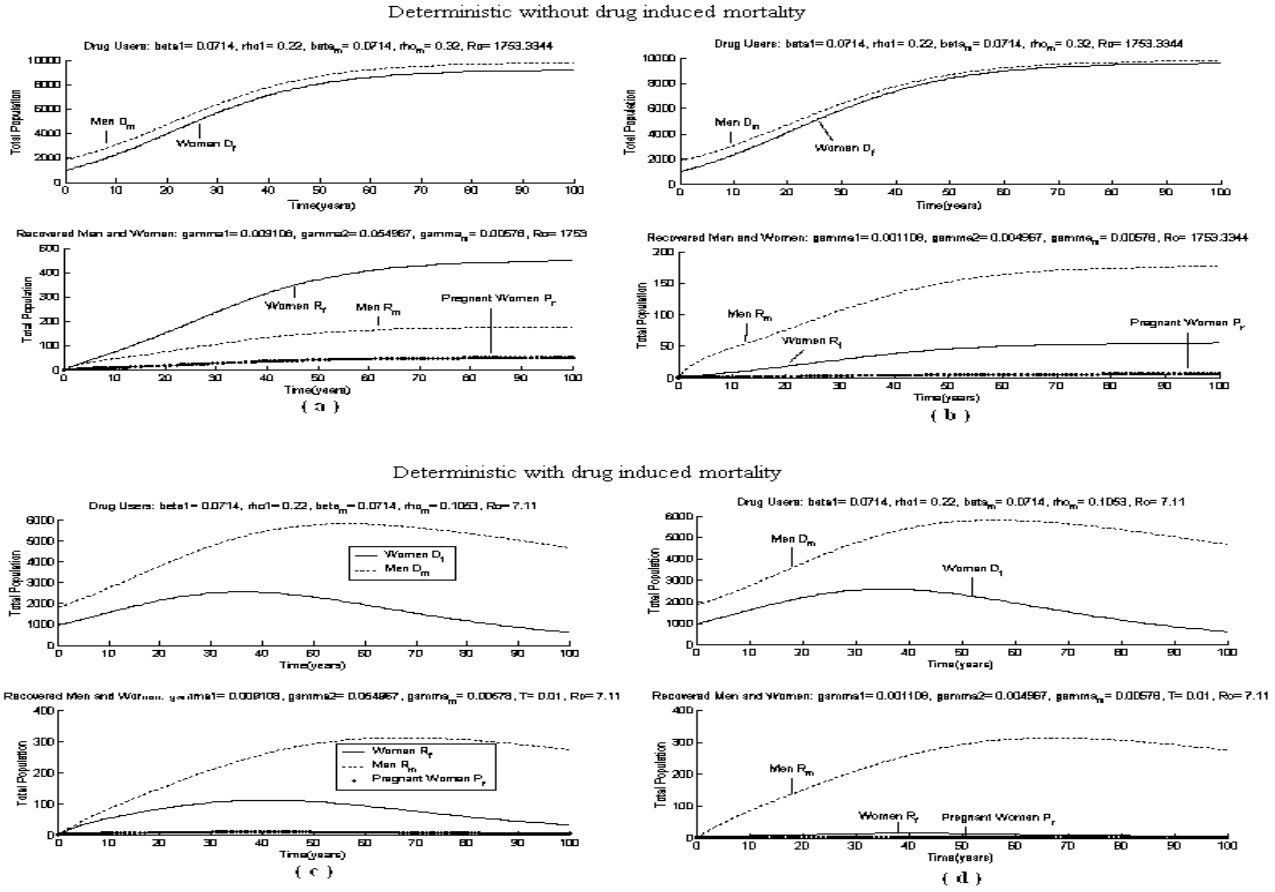

Figure 3: (a): Increasing γ_i, (b): Decreasing γ_i, (c): Increasing γ_i, (d): Decreasing γ_i.

3 Stochastic vs. Deterministic

There are many factors that contribute to drug use and pregnancy that we could not put into our model and still get meaningful results. The rate that women get pregnant is inherently probabilistic, even with the use of birth control. The rate that drug-using men convince women to use drugs is also very random and can depend on such factors as family upbringing and friends who may have died through drug use. It is difficult to consider these types of situations in a deterministic model. These circumstances motivated the creation of a stochastic version of our model.

Unfortunately, one of the drawbacks of a stochastic model is that they tend to be hard to analyze analytically. These computationally intensive study is time consuming and not totally satisfactory. We consider the stochastic analog of our deterministic model and discuss its mean behavior to that of our deterministic version.

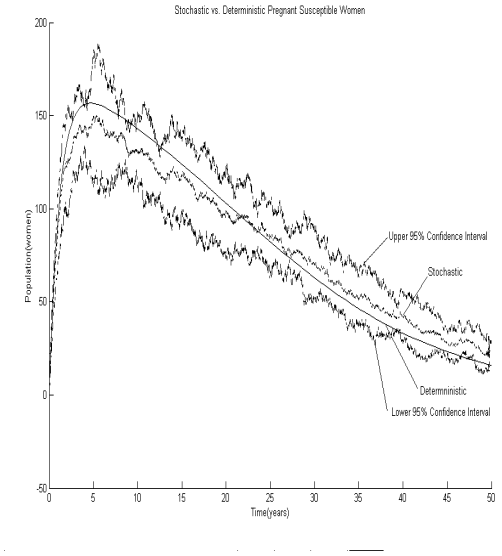

Figure 4: Comparison Between the Stochastic and Deterministic Models Pregnant Susceptible Women

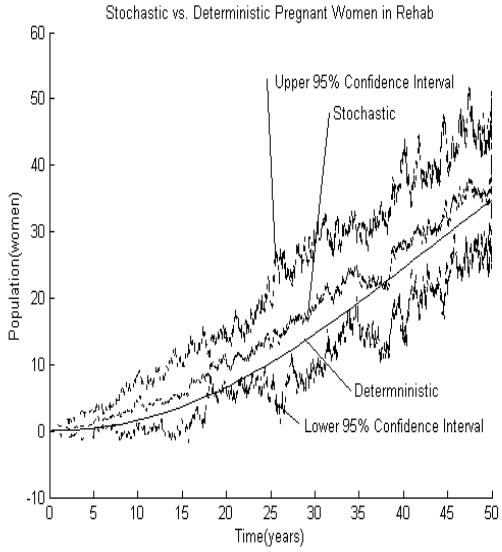

Figure 5: Comparison Between the Stochastic and Deterministic Models: Pregnant Women in Rehabilitation

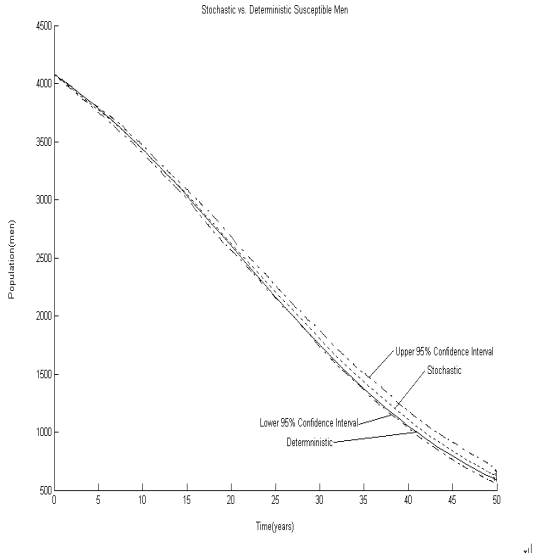

Figure 6: Comparison Between the Stochastic and Deterministic Models Susceptible Men

3.1 Confidence Interval, Variance and Mean of Deterministic and Stochastic models

We make a statistical comparison between our deterministic and its analog the stochastic model. Hence, we compute the mean, variance, and 95% confidence interval for repeated stochastic simulations for selected parameters. We used $Minitab$ Statistical Software package and $Microsoft$ Excel. After calculating the mean values of our stochastic model and creating a 95% confidence interval, we superimposed our deterministic and stochastic models on the same graph using $Matlab$.

Looking at Figure (4) we can see that there is relatively good agreement between both models. Although there is a lot of noise even after averaging several stochastic simulations, it is clear that the trend predicted by the deterministic model is supported by the stochastic model: a sharp increase in the number of susceptible pregnant women for the first five years, and then a gradual decline. Many more stochastic simulations can be run and averaged to reduce the amount of noise in the Figure (4), but that is not necessary to observe the qualitative features of this stochastic model.

In Figure (5) all of the parameters are the same except for ρ_1 which is equal to 0.022 and ρ_2 which is 0.001. Here we examine what happens when we decrease the rate that women in rehabilitation will relapse back into drug use. In particular, Figure (5) shows the frequency of pregnant women that relapse. We notice that this number slowly increases as a function of time and that the deterministic model is a monotonically increasing function on our time scale. Although we would not expect the stochastic version to be monotonically increasing due to its inherently random nature, we see that it is generally increasing with time. We also note that the deterministic model resides almost exclusively within the 95% confidence interval. Again there seems to be a large amount of noise, but the qualitative characteristics of the stochastic model agree well with that of the deterministic model, and further stochastic simulations may not provide additional insight sufficient enough to justify the cost in time of the simulations.

Next we consider a case where we again have altered the ρ_1 parameter to equal 0.022 and ρ_2 to equal 0.001. In Figure (6) represents the case of susceptible males. We observe that the numbers of men decrease slowly with time as more and more men fall into drug use and that the relationship is almost linear on the time scale under consideration. Again, the plot of the deterministic model resides almost exclusively within the 95% confidence interval. The standard deviation in this case ranges from .3 to 42.77. Although a standard deviation of 42.77 may seem large, it represents only 1.6% of the mean population. One interesting feature of this plot is the lack of

the amount of noise we saw in the previous two plots. This may be due to the scale of the population of susceptible males. The size of the population of pregnant susceptible women was in the hundreds. For pregnant women in rehabilitation, the scale was in the tens. In this case, the scale is in the thousands. If we compare the three plots, it seems the degree of noise decreases as a function of the order of magnitude of the size of the population of the variable under consideration. In all three cases, the total population of males and females are the same, but the actual proportion of individuals in each particular class changes with time. We can thus conclude that the amount of noise seems to decrease as the number of individuals in the particular class we are considering increase. This is true because as we increase the number of people in the class we are investigating, the differences in the stochastic simulations become relatively small and less noticeable.

4 Deterministic Model with Drug Induced Mortality

In this section we want to analyze the original model after we include drug induced mortality. The new rescaled version of the model is:

$$\begin{aligned}
\frac{dX}{dt} &= \mu - \beta_m XY - \mu X + XYd && (4)\\
\frac{dY}{dt} &= \beta_m XY + \rho_m Z - (\gamma_m + \mu + d)Y + Y^2 d && (5)\\
\frac{dZ}{dt} &= \gamma_m Y - (\rho_m + \mu)Z + YZd && (6)\\
\frac{dP}{dt} &= \mu + \lambda_1 S - \beta_f PY - (\mu + \phi_1)P + wP(Q+T) && (7)\\
\frac{dQ}{dt} &= \beta_f PY + \rho_1 R + \lambda_2 T - (\mu + \phi_2 + \gamma_1 + w)Q + wQ(Q+T) && (8)\\
\frac{dR}{dt} &= \gamma_1 Q + \lambda_3 U - (\rho_1 + \phi_3 + \mu)R + wR(Q+T) && (9)\\
\frac{dS}{dt} &= \phi_1 P - \beta_f SY - (\lambda_1 + \mu)S + wS(Q+T) && (10)\\
\frac{dT}{dt} &= \beta_f SY + \phi_2 Q + \rho_2 U - (\lambda_2 + \gamma_2 + \mu + w)T + wT(Q+T) && (11)\\
\frac{dU}{dt} &= \gamma_2 T + \phi_3 R - (\rho_2 + \lambda_3 + \mu)U + wU(Q+T) && (12)
\end{aligned}$$

Where $X + Y + Z = 1$ and $P + Q + R + S + T + U = 1$.

We have the following theorem for this model:

Theorem 4.1. *Let $\vec{x}_\infty(DF) = (1, 0, 0)$ be a disease free equilibrium of (4) - (6) then is locally asymptotically stable if and only if $\mathcal{R}_0 < 1$ where*

$$\mathcal{R}_0 = \frac{\beta_m(\rho_m + \mu)}{\mu(\gamma_m + \rho_m + \mu) + d(\rho_m + \mu)}$$

.

Proof. To find the stability of the disease free where, $J(X^*, Y^*, Z^*) = (1, 0, 0)$ we used the Jacobian matrix of the system.

$$J(1,0,0) = \begin{bmatrix} -\mu & d - \beta_m & 0 \\ 0 & d - \beta_m - \mu - d - \gamma_m & \rho_m \\ 0 & \gamma_m & -(\mu + \rho_m) \end{bmatrix}$$

from that we know that $-\mu$ is an eigenvalue and it is negative since $\mu > 0$. For there we can look at the 2×2 matrix,

$$C = \begin{bmatrix} \beta_m - \mu - d - \gamma_m & \rho_m \\ \gamma_m & -(\mu + \rho_m) \end{bmatrix}$$

$$det(C) > 0 \Rightarrow \beta_m(\rho + \mu) < \mu(\gamma + \rho + \mu) + d(\rho + \mu) \Rightarrow trace(C) < 0$$

which is equivalent to $\mathcal{R}_0 < 1$ where,

$$\mathcal{R}_0 = \frac{\beta_m(\rho_m + \mu)}{\mu(\gamma_m + \rho_m + \mu) + d(\rho_m + \mu)}$$

□

Now we can look some numerical solutions using the same initial conditions and parameters to see the behavior of drug use with drug induced mortality. In the simulations we analyzed the same cases of the deterministic model without drug induce mortality for comparison. We expected the same behavior in the graphs of the deterministic model without drug induce, but with fewer drug users because now are dying from drug use and natural mortality.

We analyzed the effect of the interaction of women with drug-using men, see Figure (2). In (d) the value of β_m is decreased to 0.0114 and in (c) the value of β_m is increased to 0.1014. If we compare it with (a) and (b), we see

it has the same qualitative behavior, but with less women drug users.

We analyzed also the situation of drug-using women going to rehabilitation, see Figure (3). In (d) the value of γ_1 and γ_2 is decreased to 0.001108 and 0.004967 respectively and in (c) the value of β_m is increased to 0.1014. If we compare it with (a) and (b), we also see the same behavior just at a different scale.

5 Deterministic Model with Jail Term, No Rehabilitation for Men, and Drug Induced Mortality

We now add a different stage in the male population and include drug induced mortality for men and women where both are different due to the fact that we are assuming that men have a higher risk of getting killed or dying through drug use. For the women we assume that this rate is equal for both pregnant and non-pregnant women. This new approach to the model tells us approximately how long an individual (male) who uses drugs has to remain in jail in order to prevent women from using drugs. There is also the idea that rehabilitation in men is not as effective and the relapse rate is much higher than the time they stay in jail for drug possession, in our case crack cocaine. We will now use a deterministic approach in order to analyze our problem.
The model equations are:

$$\frac{dX}{dt} = \mu - \beta_m XY - \mu X + Xd(Y+Z) \quad (13)$$
$$\frac{dY}{dt} = \beta_m XY + \rho_m Z - (\gamma_m + \mu + d)Y + Yd(Y+Z) \quad (14)$$
$$\frac{dZ}{dt} = \gamma_m Y - (\rho_m + \mu + d)Z + Zd(Y+Z) \quad (15)$$
$$\frac{dP}{dt} = \mu + \lambda_1 S - \beta_f PY - (\mu + \phi_1)P + wP(Q+T) \quad (16)$$
$$\frac{dQ}{dt} = \beta_f PY + \rho_1 R + \lambda_2 T - (\mu + \phi_2 + \gamma_1 + w)Q + wQ(Q+T) \quad (17)$$
$$\frac{dR}{dt} = \gamma_1 Q + \lambda_3 U - (\rho_1 + \phi_3 + \mu)R + wR(Q+T) \quad (18)$$
$$\frac{dS}{dt} = \phi_1 P - \beta_f SY - (\lambda_1 + \mu)S + wS(Q+T) \quad (19)$$
$$\frac{dT}{dt} = \beta_f SY + \phi_2 Q + \rho_2 U - (\lambda_2 + \gamma_2 + \mu + w)T + wT(Q+T) \quad (20)$$
$$\frac{dU}{dt} = \gamma_2 T + \phi_3 R - (\rho_2 + \lambda_3 + \mu)U + wU(Q+T) \quad (21)$$

Where $X + Y + Z = 1$ and $P + Q + R + S + T + U = 1$.

In order to do the stability analysis of the system we will only focus on the male equations, which are the driving force of the system. The women equations do not play a role in the stability analysis. The disease free equilibrium is $(X^*, Y^*, Z^*) = (1, 0, 0)$, and to find its stability we looked at the Jacobian matrix. In this case our population is not constant, hence we cannot reduce it to a two dimensional system. Nevertheless we looked at the 3×3 Jacobian matrix where $-\mu$ is an eigenvalue where $\mu > 0$, and we have the following theorem:

Theorem 5.1. *Let $\vec{x}_\infty(DF) = (1, 0, 0)$ be a disease free equilibrium of (13) - (15) then it is locally asymptotically stable if and only if $\mathcal{R}_0^J < 1$ where*

$$\mathcal{R}_0^J = \frac{\beta_m(\rho_m + \mu + d)}{(\mu + d)(\gamma_m + \rho_m + \mu + d)}.$$

Proof.

$$J(1,0,0) = \begin{bmatrix} -\mu & d - \beta_m & d \\ 0 & \beta_m - \gamma_m - \mu - d & \rho_m \\ 0 & \gamma_m & -(\rho_m + \mu + d) \end{bmatrix}$$

and the matrix can be reduced to a 2×2 matrix,

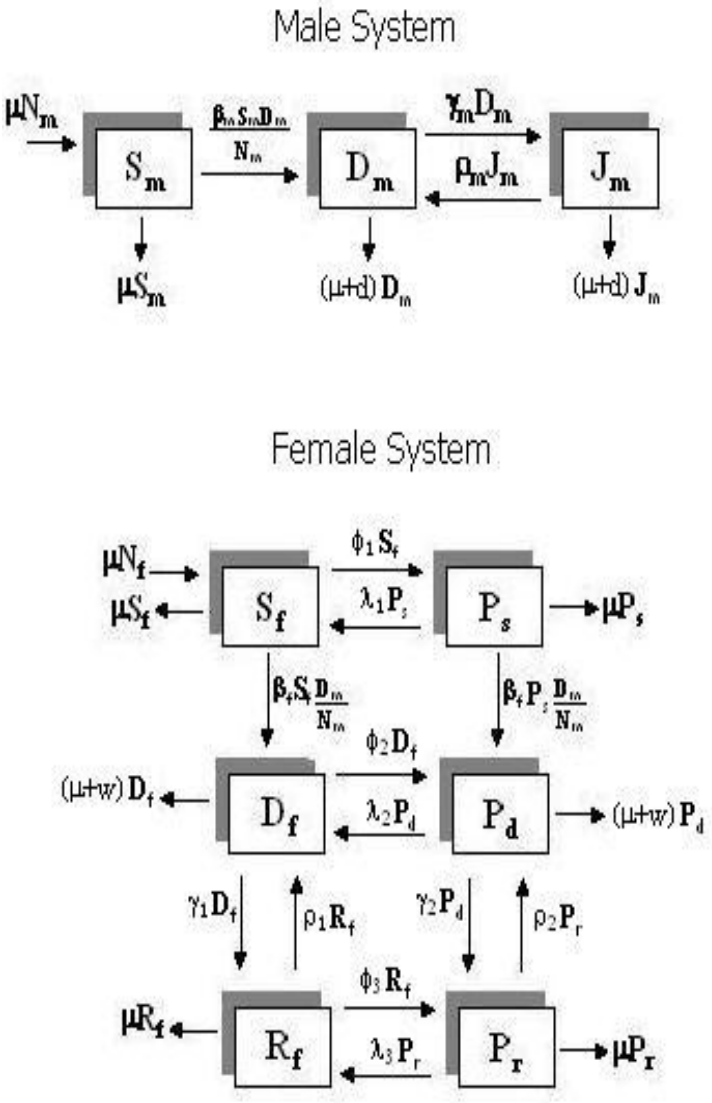

Figure 7: Deterministic model with jail term and no rehabilitation in men population

$$B = \begin{bmatrix} \beta_m - \gamma_m - \mu - d & \rho_m \\ \gamma_m & -(\rho_m + \mu + d) \end{bmatrix}$$

$det(B) > 0 \Rightarrow \beta_m(\rho_m + \mu + d) < (\mu + d)(\gamma_m + \rho_m + \mu + d) \Rightarrow trace(B) < 0$, which is equivalent to $\mathcal{R}_0^J < 1$ where,

$$\mathcal{R}_0^J = \frac{\beta_m(\rho_m + \mu + d)}{(\mu + d)(\gamma_m + \rho_m + \mu + d)}.$$

□

In the deterministic simulations we fixed some parameters; $\beta_f = 0.0714$, $\lambda_1 = 0.8$, $\lambda_2 = 0.7$, $\lambda_3 = 0.75$, $\gamma_1 = 0.007108$, $\gamma_2 = 0.014967$, $\phi_1 = 0.028489$, $\phi_2 = 0.023313$, $\phi_3 = 0.049089$, $\rho_1 = 0.22$, $\rho_2 = 0.01$, $\mu = 0.00004$, $d = 0.01$, and $w = 0.05$. We will vary β_m, γ_m, and ρ_m. Our initial conditions are: $D_f(0) = 977$, $P_d(0) = 20$, and $D_m(0) = 1840$. Our starting population is $N_m = 10000$, and $N_f = 10000$.

In Figure (8,9) we see as the number of men in jail increases the number of men using drugs also increases, which tells us that even if we send men to jail for a significant amount of time they will be replaced by upcoming drug users. We can also notice the number of women using drugs increasing but as soon as there are not enough men using drugs the number of women in rehabilitation (education) increases significantly. What is somehow surprising is that the number of pregnant women using drugs does not increase. Based on our model we can say that men do not have as large an impact on pregnant women as they do on non-pregnant women.

In Figure (10,11) when we decrease β_m and we see that there is only a slight outbreak of women using drugs, pregnant or non-pregnant, and there is a noticeable increase in women in rehabilitation. In this case $\mathcal{R}_0^J < 1$ so eventually there will be no one using crack cocaine which is not realistic but based on our model tells us how critical the situation is and if we were to try to get rid off crack cocaine dramatic measures have to be taken some of which we will discuss in our conclusions.

In Figure (12,13) we doubled the sentence for individuals sent to jail for crack cocaine possession which is 10 years [18]. By varying ρ_m from 10 to 20 years we see a decrease in the number of men using drugs but increases slowly with time. In contrast, the number of men sent to jail increases exponentially. In the case of women there is only a slight increase and decreases slowly which indicates that even when $\mathcal{R}_0^J > 1$ it is still possible to reduce the number of drug using women by having prosecuted men stay in jail longer.

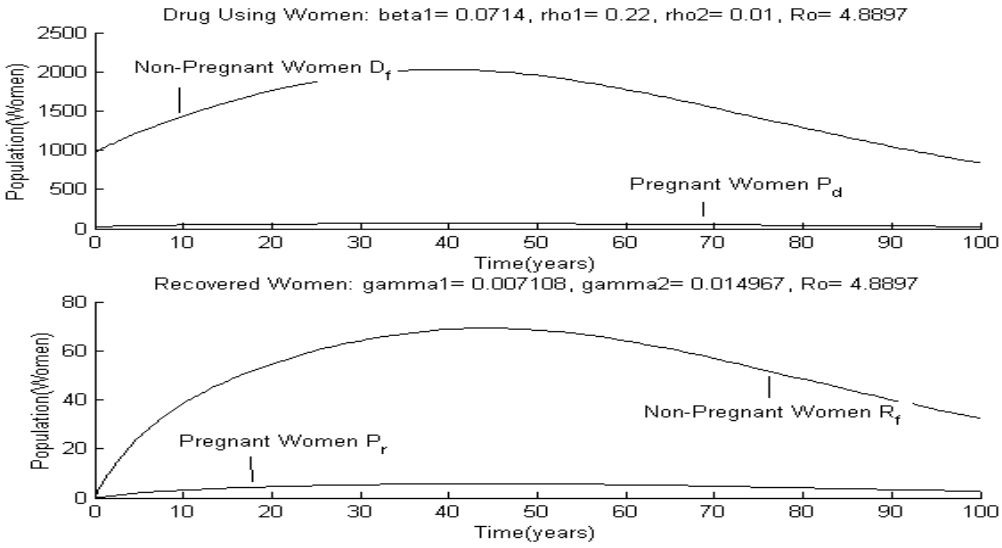

Figure 8:

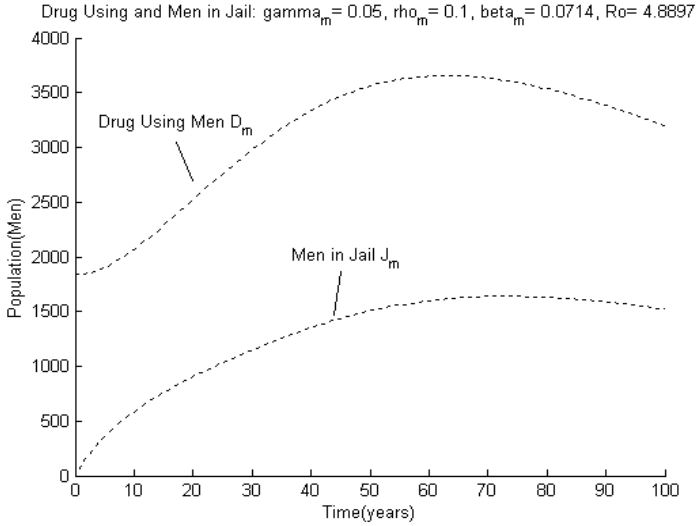

Figure 9:

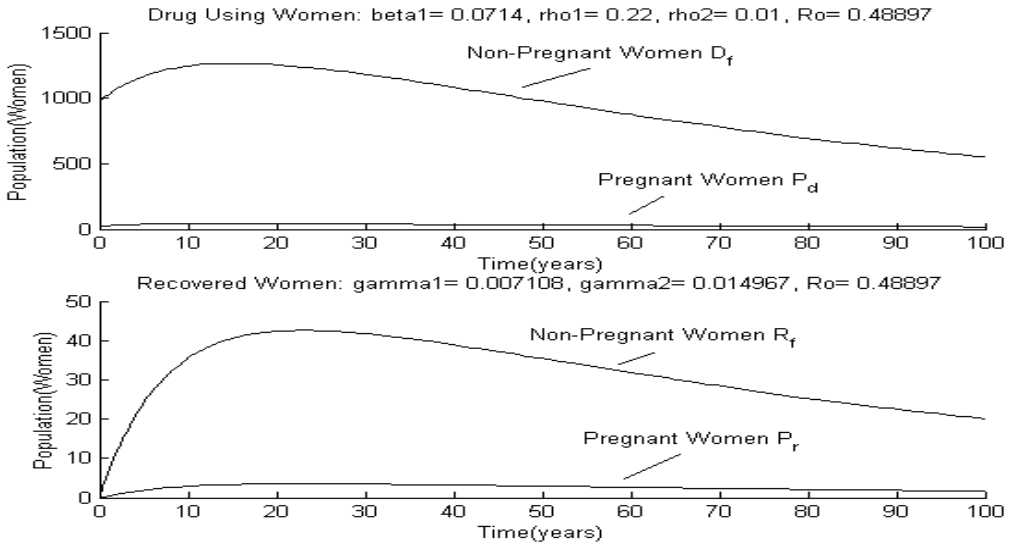

Figure 10:

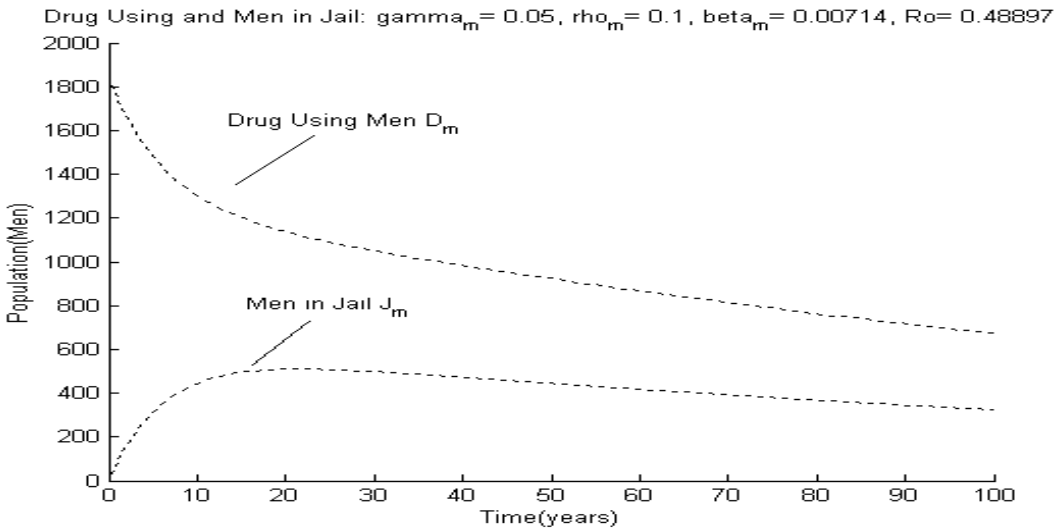

Figure 11:

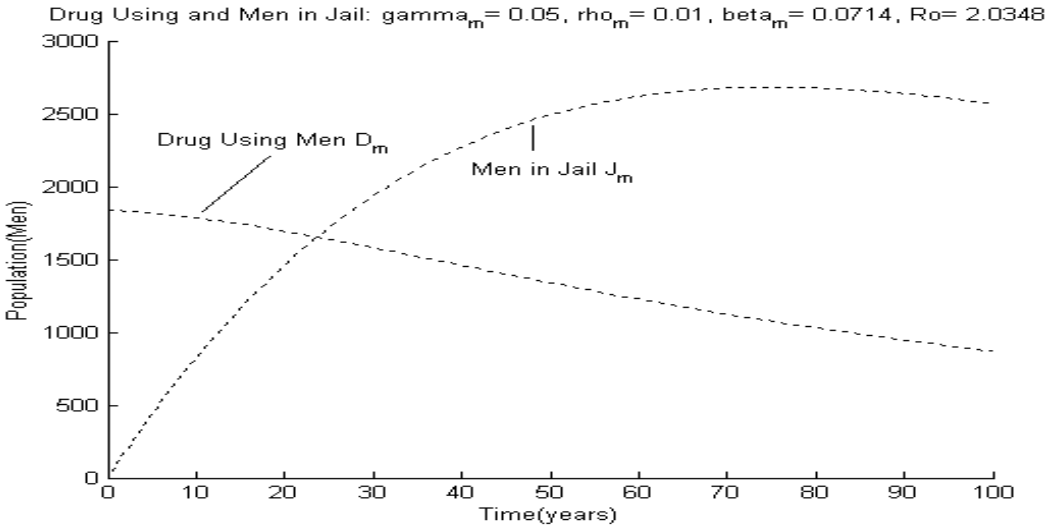

Figure 12:

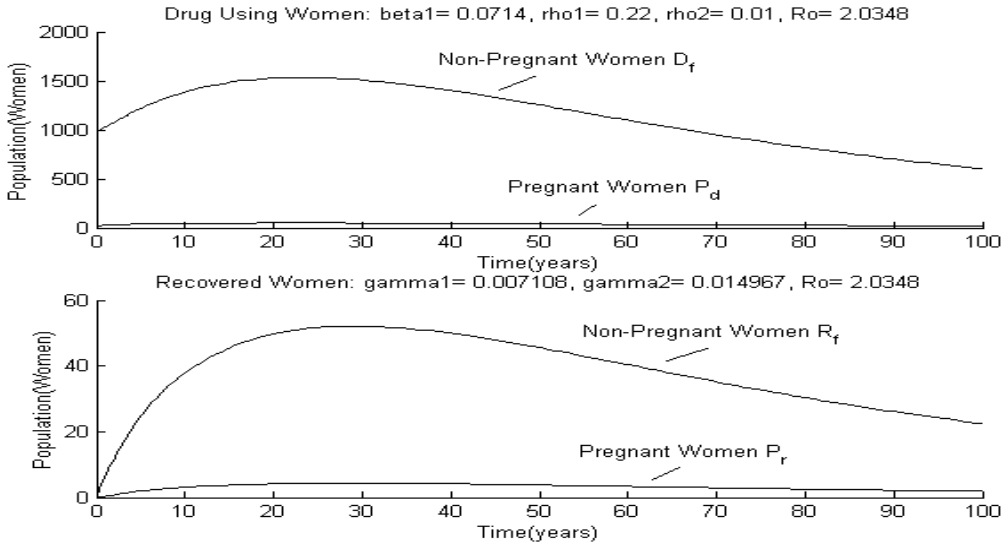

Figure 13:

6 Jail Term and Rehabilitation in Men

As seen in previous sections, based on our model and reality it is practically impossible to make drug abuse disappear. Our model that included jail term in men and no rehabilitation told us that jail is not enough to lower the endemicity of drug use. Hence, this tells us that if we were to lower the number of men using crack cocaine there are a number of factors that have to be taken into consideration, and as we show in this section keeping men "crack-free" is one way to prevent drug abuse in pregnant and non-pregnant women. In reality, it is difficult to keep men or in fact anyone off crack cocaine. We show in this section that by keeping a small proportion of the male population off of drug use, we can lower the number of women who abuse crack-cocaine. The model is normalized and the new variable is $W = \frac{R_m}{N_m}$.

The model equations are:

$$\begin{aligned}
\frac{dX}{dt} &= \mu - \beta_m XY - \mu X + X(dY + gZ) && (22)\\
\frac{dY}{dt} &= \beta_m XY + \rho_m Z - (\gamma_m + \mu + d)Y + Y(dY + gZ) && (23)\\
\frac{dZ}{dt} &= \gamma_m Y - (\rho_m + \mu + g)Z + Z(dY + gZ) && (24)\\
\frac{dW}{dt} &= \epsilon Z - \mu W + W(dY + gZ) && (25)\\
\frac{dP}{dt} &= \mu + \lambda_1 S - \beta_f PY - (\mu + \phi_1)P + wP(Q + T) && (26)\\
\frac{dQ}{dt} &= \beta_f PY + \rho_1 R + \lambda_2 T - (\mu + \phi_2 + \gamma_1)Q + wQ(Q + T) && (27)\\
\frac{dR}{dt} &= \gamma_1 Q + \lambda_3 U - (\rho_1 + \phi_3 + \mu)R + wR(Q + T) && (28)\\
\frac{dS}{dt} &= \phi_1 P - \beta_f SY - (\lambda_1 + \mu)S + wS(Q + T) && (29)\\
\frac{dT}{dt} &= \beta_f SY + \phi_2 Q + \rho_2 U - (\lambda_2 + \gamma_2 + \mu)T + wT(Q + T) && (30)\\
\frac{dU}{dt} &= \gamma_2 T + \phi_3 R - (\rho_2 + \lambda_3 + \mu)U + wU(Q + T) && (31)
\end{aligned}$$

where $X + Y + Z + W = 1$ and $P + Q + R + S + T + U = 1$.

For the stability analysis we looked at the Jacobian matrix at the disease free equilibrium and we have the following theorem:

Theorem 6.1. *Let $\vec{x}_\infty(DF) = (1, 0, 0)$ be a disease free equilibrium of (22)*

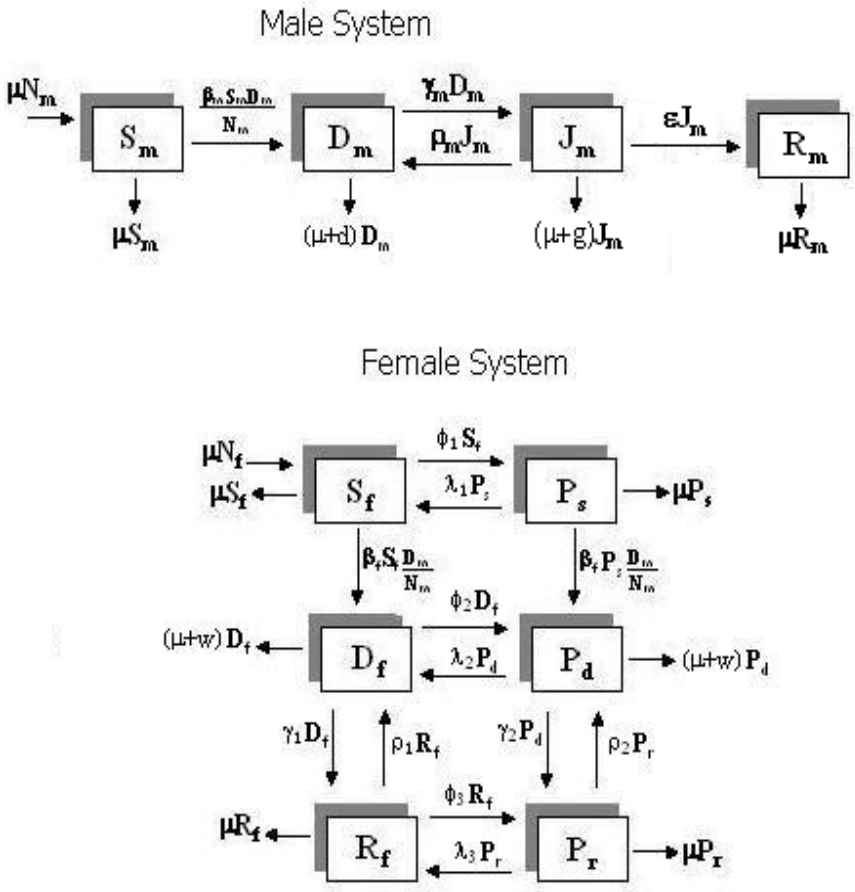

Figure 14: Deterministic model including jail term and rehabilitation in the male population

- (24) then is locally asymptotically stable if and only if $\mathcal{R}_0^{jr} < 1$ *where*

$$\mathcal{R}_0^{jr} = \frac{\beta_m(\rho_m + \mu + g)}{\mu(\rho_m + \mu + g) + d(\mu + \rho_m) + g(\gamma_m + d)}$$

.

Proof.

$$J(1,0,0,0) = \begin{bmatrix} -\mu & d - \beta_m & g & 0 \\ 0 & \beta_m - \gamma_m - \mu - d & \rho_m & 0 \\ 0 & \gamma_m & -(\rho_m + \mu + g) & 0 \\ 0 & 0 & \epsilon & -\mu \end{bmatrix}$$

$-\mu$ and $-\mu$ are both eigenvalues where $\mu > 0$, therefore we can look at the 2×2 matrix to find conditions for stability,

$$D = \begin{bmatrix} -\beta_m - \gamma_m - \mu - d & \rho_m \\ \gamma_m & -(\rho_m + \mu + g) \end{bmatrix}$$

$det(D) > 0 \Rightarrow \beta_m(\mu+\rho_m+g) < \mu(\rho_m+\mu+g+\gamma_m)+d(\rho_m+\mu)+g(\gamma_m+d) \Rightarrow trace(D) < 0$

which is equivalent to $\mathcal{R}_0^{jr} < 1$, and

$$\mathcal{R}_0^{jr} = \frac{\beta_m(\rho_m + \mu + g)}{\mu(\rho_m + \mu + g) + d(\mu + \rho_m) + g(\gamma_m + d)}$$

□

Looking at the Figures (15,16) we see that there is a peak in the number of women and men who abuse crack-cocaine. This peak occurs after the first 40 years and slowly drops off, although to fully appreciate the dynamics of this system we need to analyze it in the long run where our system is not valid. Looking at the male population, we notice that the number of men in rehabilitation will eventually overtake the number of men in jail, but only after 100 years, and overtake the number of drug using men after 150 years.

If we were to double the rate that men go into rehabilitation as in Figures (18,19), then we notice that there is negligible change to the female system. In the male system, there are no qualitative changes, but the number of drug using men and men in jail do decrease. The number of men in rehabilitation also increases substantially, as expected. The number of years it takes for

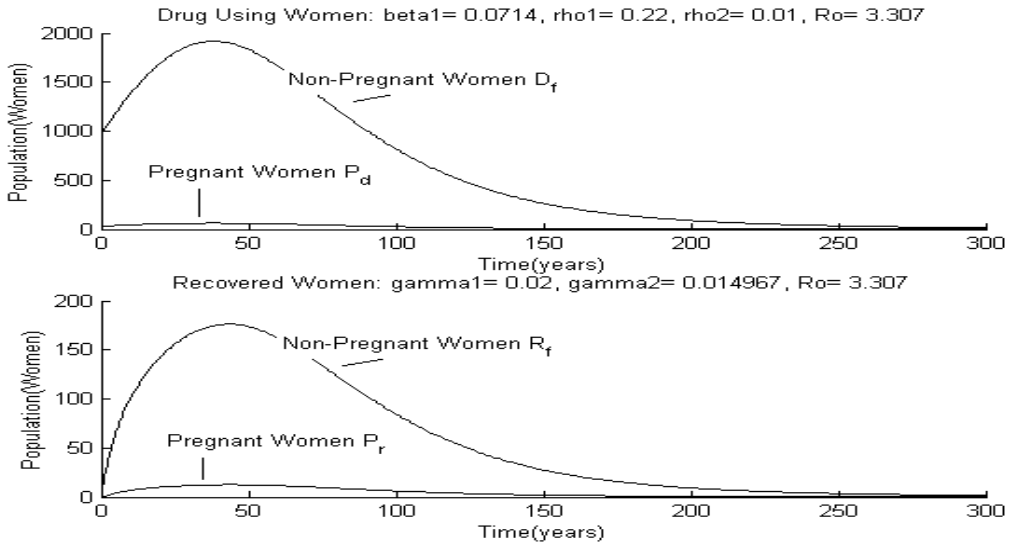

Figure 15: Female Population

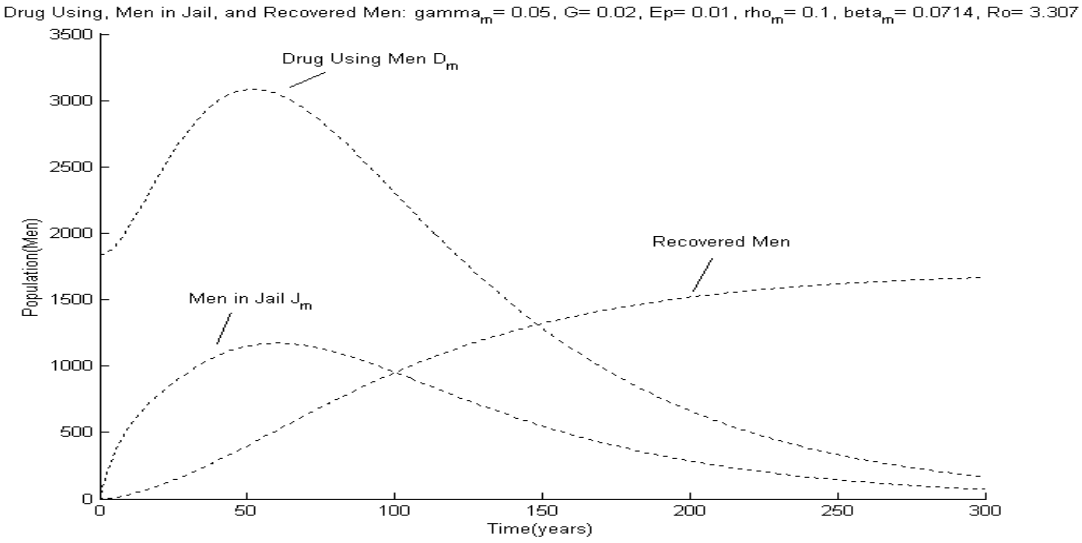

Figure 16: Male Population

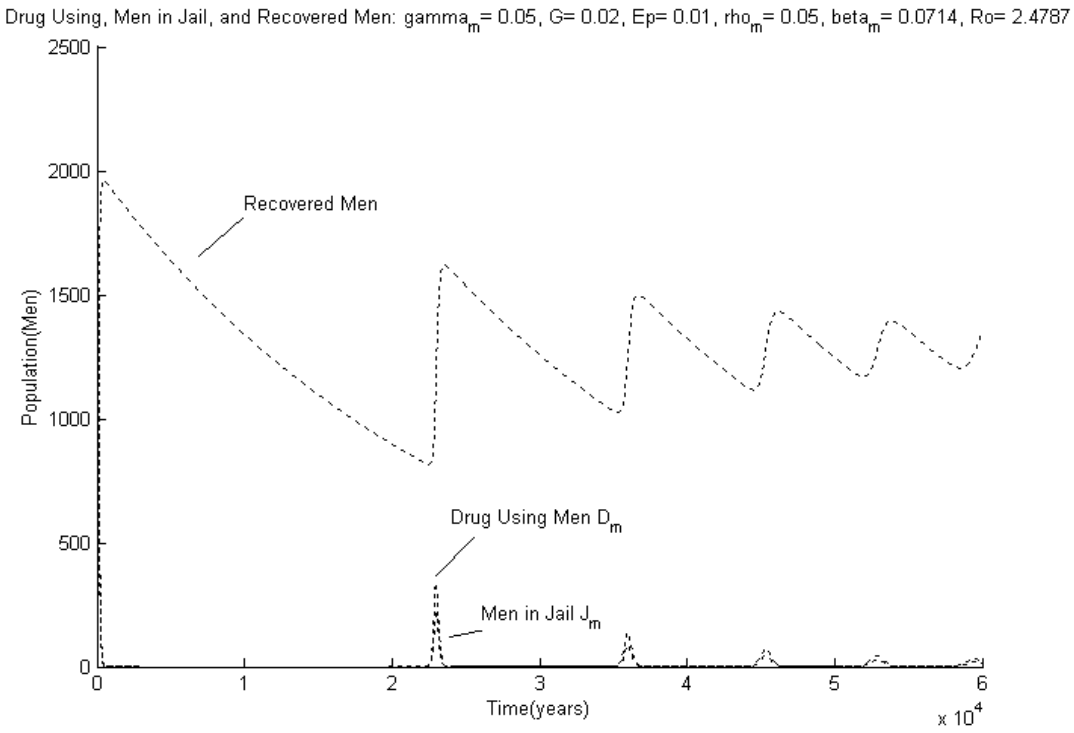

Figure 17: Male Population

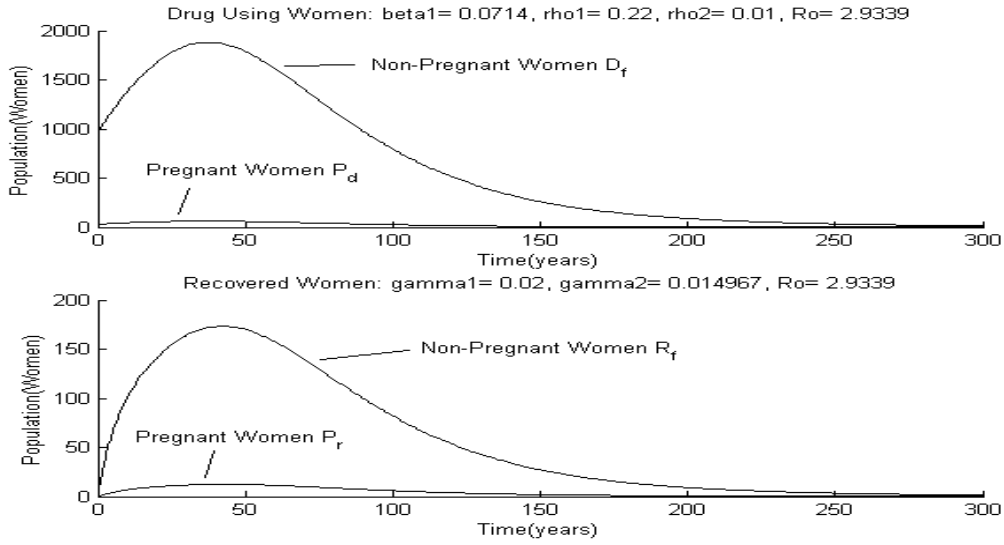

Figure 18: Female Population

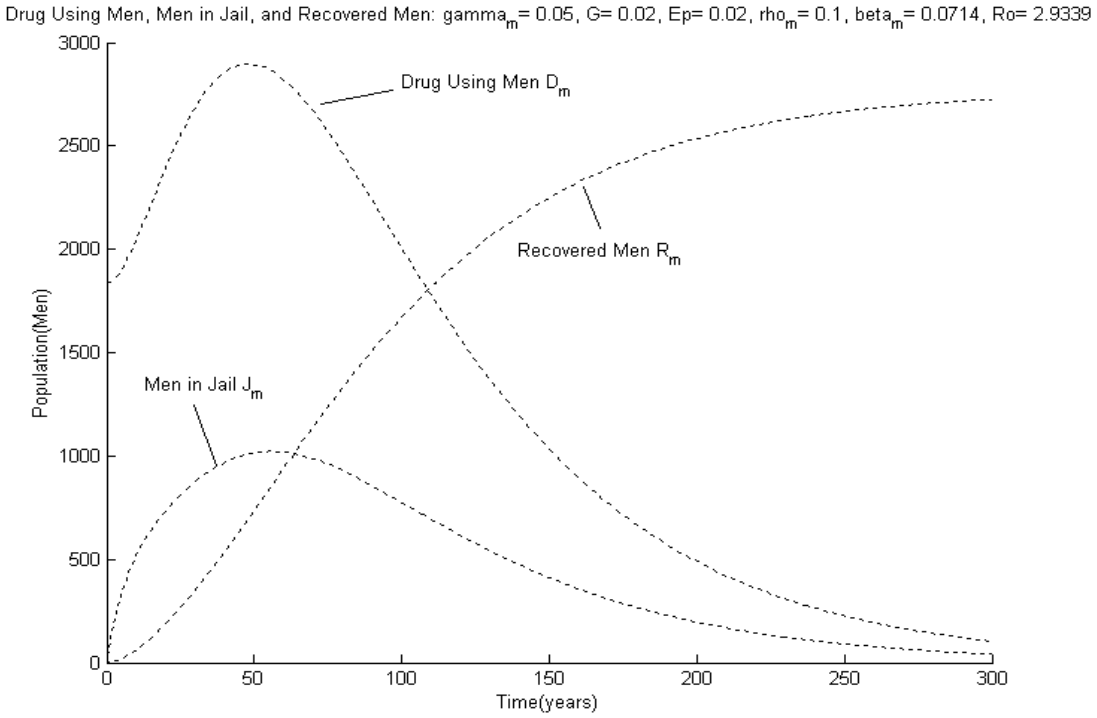

Figure 19: Male Population

the men in rehabilitation to overtake the number of men in jail decreases to approximately 60 years, and 100 to overtake the number of men abusing crack-cocaine.

Looking at Figures (16,19), we notice a curious phenomenon. Although $\mathcal{R}_0$ is greater than one, the number of drug using men seems to drop off to very low levels. Out of purely mathematical curiosity, if we increase the time scale of our analysis, then we notice very interesting behavior in Figure (17). There is a periodic damped oscillation of the number of recovered men, and periodic spikes in the number of drug using men and men in jail that decreases in amplitude as a function of time. The number of recovered men decreases slowly with time, then increases sharply for a short time, then repeats. Then number of men in jail or using drugs exists at low levels, then experiences sharp peaks right after the number of recovered men reach a relative minimum and jump up to a new relative maximum. The dynamics of this motion need further investigation and are left for future work.

7 Drug Abuse in Pregnant and Non-Pregnant Women with Age Structure

In this section we add age structure into the model and analyze the 9-dimensional system to find the $\mathcal{R}_0^{as}$, steady states and stability of the system which in our case is only dependant on the male equations which reduces the problem to a 3-dimensional system of equations. The model equations with age structure are given below,

$$\begin{aligned}
\Big(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\Big)s_m(t,a) &= \mu(a)n_m(t,a)-\beta(a)c(a)B^*s_m(t,a)-\mu(a)s_m(t,a)\\
\Big(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\Big)d_m(t,a) &= \beta_m(a)c(a)B^*s_m(t,a)+\rho_m(a)d_m(t,a)-\gamma_m(a)d_m(t,a)\\
\Big(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\Big)r_m(t,a) &= \gamma_m(a)d_m(t,a)-\rho_m(a)d_m(t,a)-\mu(a)r_m(t,a)
\end{aligned}$$

$$
\begin{aligned}
\Big(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\Big)s_f(t,a) &= \mu(a)n_f(t,a)+\lambda_1(a)p_s(t,a)-\beta_f(a)c(a)B(t)s_f(t,a)\\
&\quad -(\phi_1(a)+\mu(a))s_m(t,a)\\
\Big(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\Big)d_f(t,a) &= \beta_f(a)c(a)B(t)s_f(t,a)+\lambda_2(a)p_d(t,a)+\rho_1(a)r_f(t,a)\\
&\quad -(\mu(a)+\gamma_1(a)+\phi_2(a))d_f(t,a)\\
\Big(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\Big)r_f(t,a) &= \gamma_1(a)d_f(t,a)+\lambda_3(a)p_r(t,a)-(\mu(a)+\rho_1(a)+\phi_3(a))r_f(t,a)\\
\Big(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\Big)p_s(t,a) &= \phi_1(a)s_f(t,a)-\beta_f(a)c(a)B(t)p_s(t,a)-(\lambda_1(a)+\mu(a))p_s(t,a)\\
\Big(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\Big)p_d(t,a) &= \beta_2(a)c(a)B(t)p_s(t,a)+\phi_2(a)d_f(t,a)+\rho_2(a)p_r(t,a)\\
&\quad -(\lambda_2(a)+\gamma_2(a)+\mu(a))p_d(t,a)\\
\Big(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\Big)p_r(t,a) &= \gamma_2(a)p_d(t,a)+\phi_3(a)r_f(t,a)-(\lambda_3(a)+\rho_2(a)+\mu(a))p_r(t,a)
\end{aligned}
$$

where $n_m(t,a)=s_m(t,a)+d_m(t,a)+r_m(t,a)$ and $n_f(t,a)=s_f(t,a)+d_f(t,a)+r_f(t,a)+p_s(t,a)+p_d(t,a)+p_r(t,a)$.

Also, we have $B^(t)$ which includes the incidence rate of the infectious individuals and the probability of having contact with one of them. This is given by:

$$B^(t)=\int_0^\infty \frac{d_m(t,u)}{n_m(t,u)}p(t,a)du, \tag{32}$$

and assuming proportional mixing

$$p(t,a)=\frac{c(a)n_m(t,a)}{\int_0^\infty c(u)n_m(t,u)du} \tag{33}$$

The boundary conditions are:

$$
\begin{aligned}
s_m(t,0) &= \Lambda\\
d_m(t,0) &= 0\\
r_m(t,0) &= 0
\end{aligned}
$$

and initial conditions,

Table 2: Age-Specific Parameter List

Parameters	Description
$\mu(a)$	age-specific mortality rate
$\phi_1(a)$	age-specific rate at which women that are not using drugs become pregnant
$\phi_2(a)$	age-specific rate at which women that are using drugs become pregnant
$\phi_3(a)$	age-specific rate at which women that are in education (rehabilitation) become pregnant
$\lambda_1(a)$	age-specific rate at which women are susceptible to become pregnant again
$\lambda_2(a)$	age-specific rate of which women drug users are susceptible to become pregnant again
$\lambda_3(a)$	age-specific rate of which women in education (rehabilitation) are susceptible to become pregnant again
$\beta_f(a)$	age-specific contact rate of non-pregnant women with drug using men
$\gamma_1(a)$	age-specific rehabilitation rate of non-pregnant women
$\gamma_2(a)$	age-specific rehabilitation rate of pregnant women
$\rho_1(a)$	age-specific relapse rate of non-pregnant women
$\rho_2(a)$	age-specific relapse rate of pregnant women
$\beta_m(a)$	age-specific transmission rate of men with drug using men
$c(a)$	age-specific contact rate for men
$\gamma_m(a)$	age-specific rehabilitation rate of men
$\rho_m(a)$	age-specific relapse rate of men

$$s_m(0,a) = s_{m_0}(a)$$
$$d_m(0,a) = d_{m_0}(a)$$
$$r_m(0,a) = r_{m_0}(a)$$

.

7.1 Stability Analysis

In order to calculate $\mathcal{R}_0^{as}$ of the model we need to first consider the steady state solutions of the system. Assume $n_m(0,a) = \Lambda \exp(-\int_0^a *\mu(u)da)$ (demographic steady state of the total population)

Solving the male system we found the steady states,

$$B^(t) = \int_0^\infty \frac{d_m^*(a)}{n_m(a)} p(a) da$$

$$s_m^*(a) = \exp^{-\int_0^a (\beta_m(u)c(u)B^* + \mu(u))du}$$

$$d_m^*(a) = \exp^{-\int_0^a (\gamma_m(u) - \rho_m(u))du} \left(\int_0^a (\beta_m(u)c(u)B^* s_m^*(u) \exp^{\int_0^a (\gamma_m(u) - \rho_m(u))du} du \right)$$

$$r_m^*(a) = \exp^{-\int_0^a (\rho_m(u) + \mu(u))du} \left(\int_0^a (\gamma_m(u) d_m^*(u) \exp^{\int_0^a (\rho_m(u) + \mu(u))du} du \right)$$

The $\mathcal{R}_0^{as}$ of the system only depends on the male equations since they are the only "infectious" individuals in the system. Women can be infected but once "infected" cannot make any other woman use drugs as per the assumption that we made. In looking for $\mathcal{R}_0^{as}$ we come across function $f(B^*) = 1$ where $f(0) = \mathcal{R}_0^{as}$.

$$f(B^*) = \int_0^\infty \left(\exp^{-\int_0^a (\gamma_m(u) + \rho_m(u))du} \left(\int_0^a \beta_m(u)c(u) \exp^{\int_0^a (\gamma_m(u) + \rho_m(u) - \beta_m(u)c(u)B^* - \mu(u))du} du \right) \times \left(\frac{c(a)}{\int_0^\infty c(u)\Lambda \exp^{-\int_0^a \mu(u)du} du} \right) \right) du = 1. \quad (34)$$

From the function $f(B^*) = 1$ we come across the $\mathcal{R}_0^{as}$ of the system. Since the function $f(B^*)$ is monotone decreasing, then when $\mathcal{R}_0^{as} < 1$ the steady states do not exist and when $\mathcal{R}_0^{as} \geq 1$ the steady states exists.

$$\mathcal{R}_0^{as} = \int_0^\infty \left(\exp^{-\int_0^a (\gamma_m(u) + \rho_m(u))du} \left(\int_0^a \beta_m(u)c(u) \exp^{\int_0^a (\gamma_m(u) + \rho_m(u) - \mu(u))du} du \right) \times \left(\frac{c(a)}{\int_0^\infty c(u)\Lambda \exp^{-\int_0^a \mu(u)du} du} \right) \right) du. \quad (35)$$

.

8 Discussion

There will always be a certain level of drug use in the population according to our model and parameters. Having a successful education program would mean altering many of the parameters that cause women to use drugs. Looking at our deterministic model, when we vary β_f or β_m, the number of women who use drugs in the short run changes significantly, but reaches a steady state in the long run. However, our model is not very accurate in the long run because we assume a constant population. If we increase β_f by a factor of 10, then we assume that our educational program is failing and that men are getting better at causing women to use drugs. With this assumption, in ten years, the number of women who use drugs increases by 280%, and in 20 years the number of women who use drugs increases by 400%, see Figure (20). If we decrease β_f by a factor of 10, then our educational program is increasing the awareness of the detrimental effects of crack-cocaine to women in general. Under these conditions, the number of women who use drugs decreases by 20% in the first ten years, and 44% after 20 years.

It is interesting to note that if we increase β_f by a factor of 100, then the number of drug users increases by nearly 1000% in 20 years, but if we decrease β_f by a factor of 100, in 20 years the number of drug users decreases by 40%. Clearly, we see that while it is worthwhile to educate women in general and try to decrease the rate that men cause women to use drugs, it is not effective to spend a lot of resources trying to educate the general public about crack-cocaine. Although this does not mean that we should not make an effort to educate the public. Our data clearly shows that if we allow crack-cocaine to be spread more easily, we will have an explosion of drug abuse.

Considering just the population of pregnant women, if we increase β_f by a factor of 10 then, in the first ten years the number of susceptible drug using women will decrease by 15%, the number of drug (crack-cocaine) using women increased by 757%, and the number of women in rehabilitation decreased by 25%, see Figure (21). In twenty years, the number of susceptible drug using women will decrease by 40%, the number of drug (crack-cocaine) using women increased by 550%, and the number of women in rehabilitation does not change. Similarly, if we decrease β_f by a factor of 10, then in the first ten years the number of susceptible drug using women will increase by 1.5%, the number of drug (crack-cocaine) using women decreased by 43%, and the number of women in rehabilitation decreased by 25%. In twenty years, the number of susceptible drug using women will increase by 4.4%, the number of drug (crack-cocaine) using women decreased by 75%, and the number of women in rehabilitation decreases by 20%. However, changing β_f

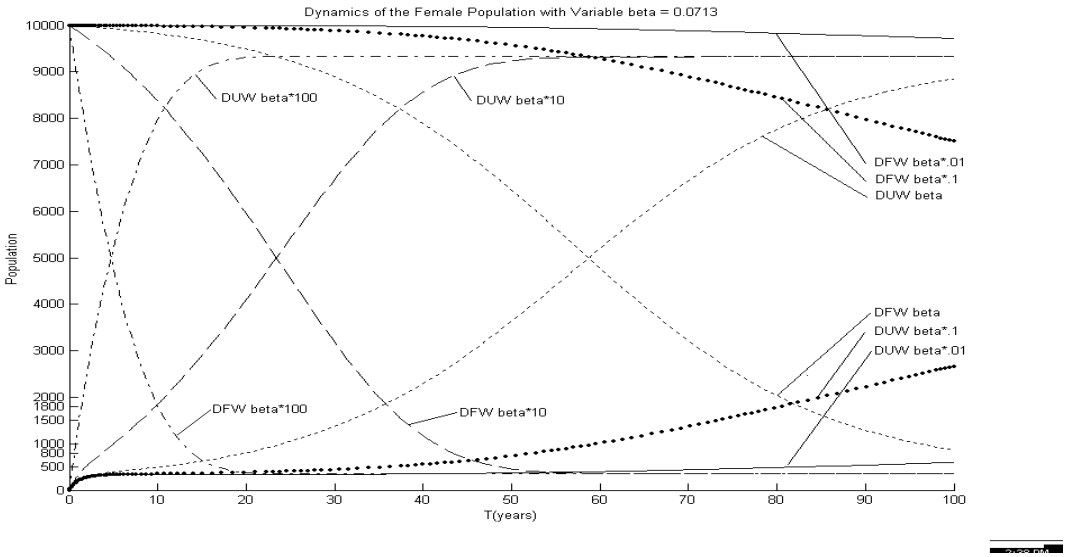

Figure 20: Changing β_f can have major impacts on the population of drug using women. DUW = drug using women including pregnant women. DFW = Drug free women including pregnant women and women in rehabilitation. β is β_f.

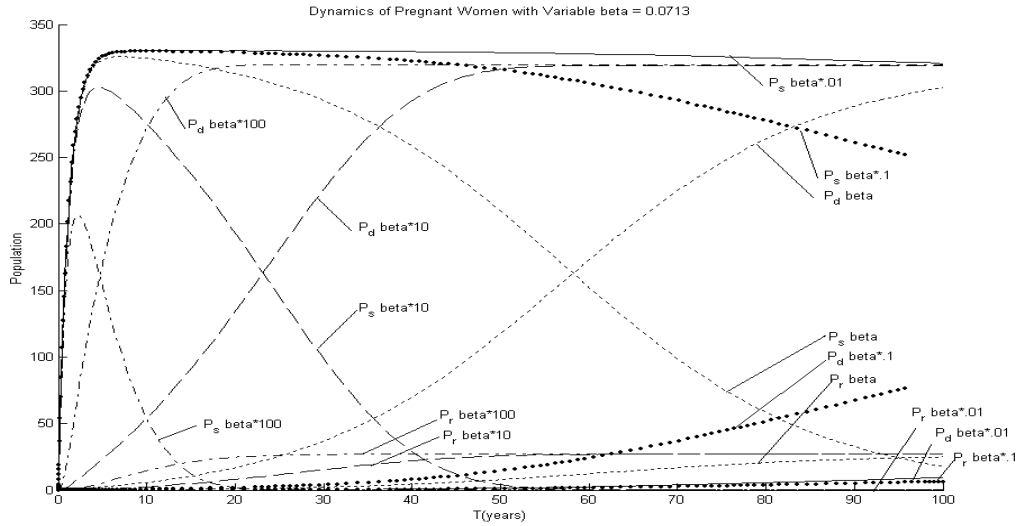

Figure 21: The effects of β_f on the population of pregnant women. P_s is pregnant susceptible women, P_d is pregnant drug (crack-cocaine) abusing women, P_r is pregnant women in a rehabilitation program.

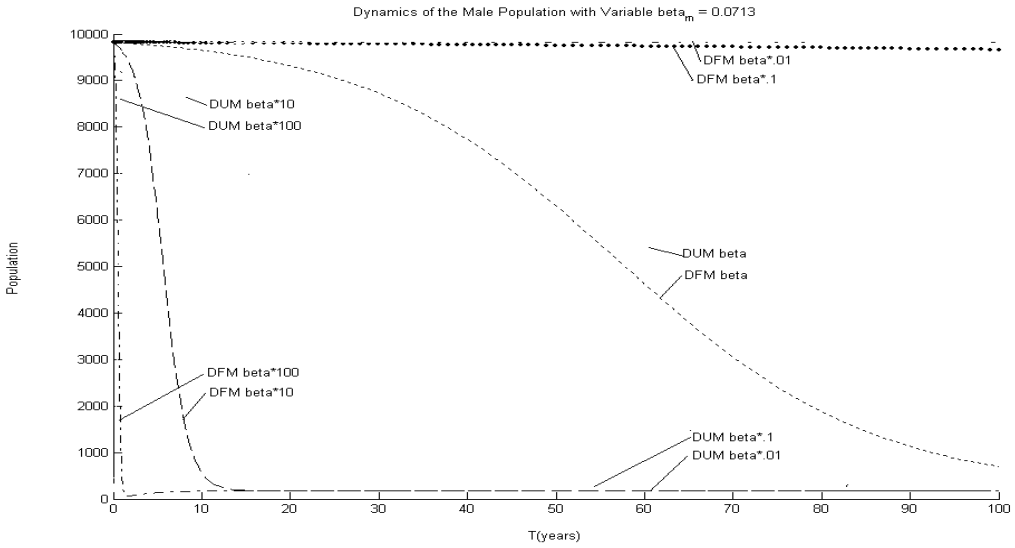

Figure 22: The effects of β_m on the male population. DUM are drug (crack-cocaine) abusing men. DFM are men who do not abuse crack-cocaine. β is β_m.

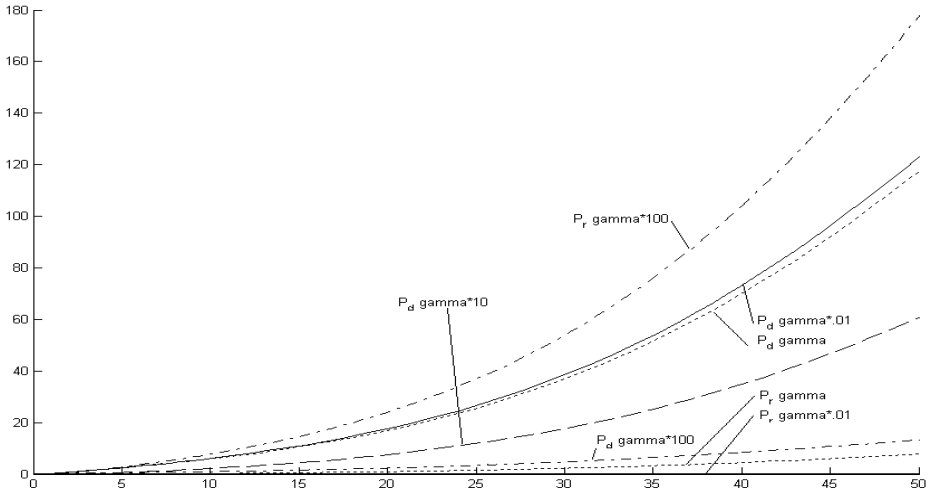

Figure 23: The effects of γ on pregnant women.

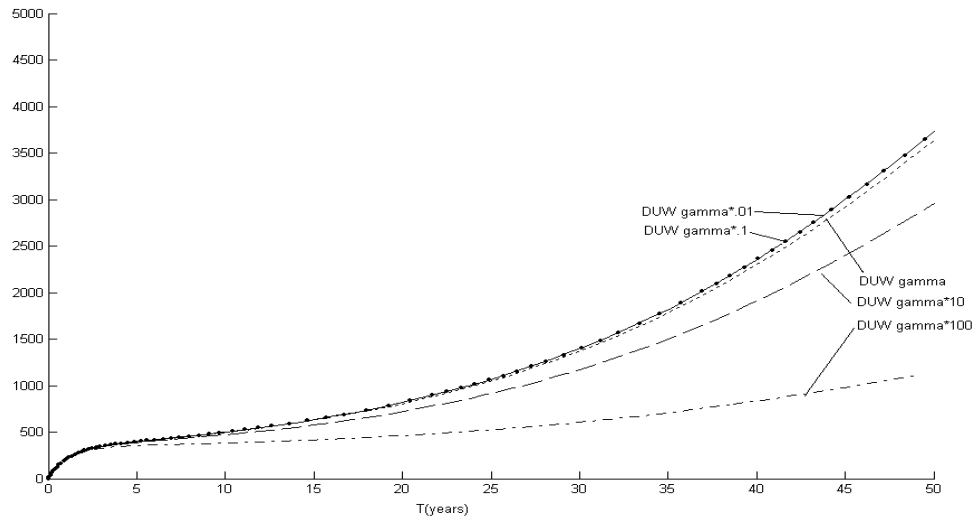

Figure 24: The effects of γ on the total female population.

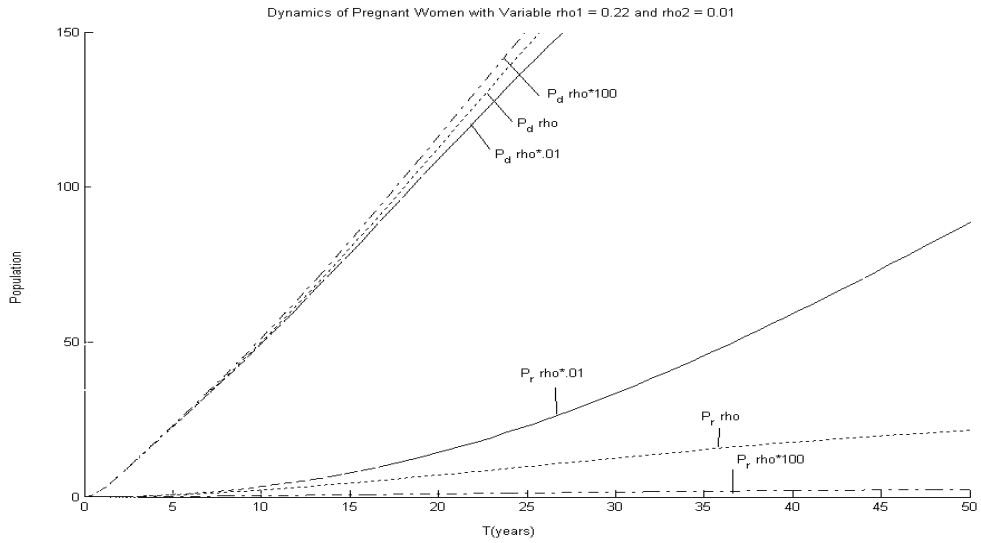

Figure 25: The effects of ρ on pregnant women.

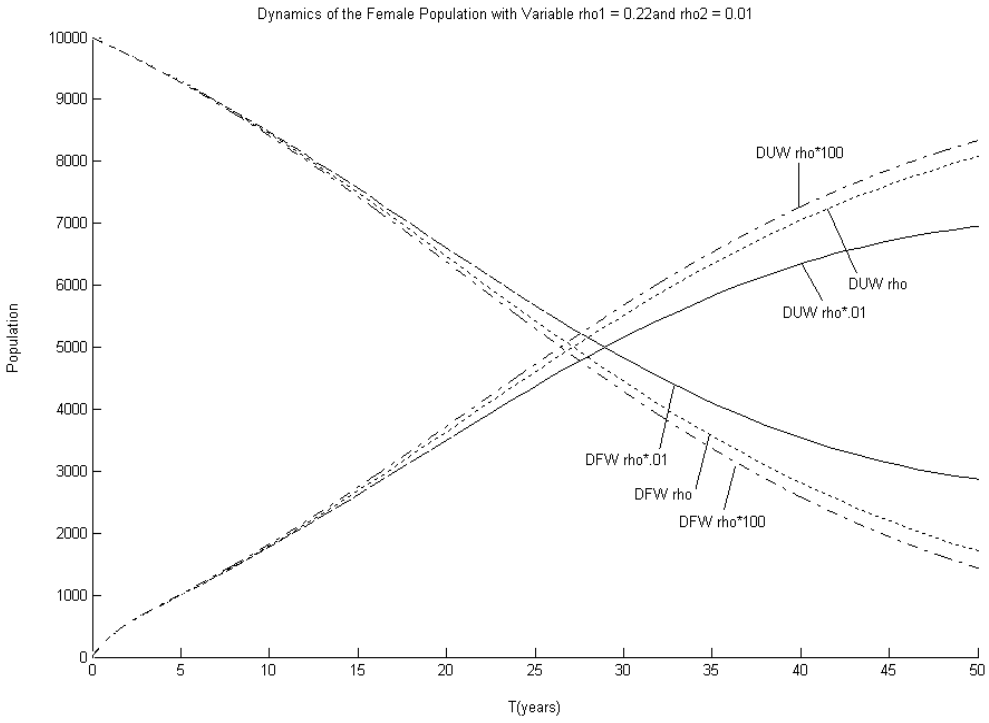

Figure 26: The effects of ρ on the total female population.

by two orders of magnitude makes little difference in the short run, with less than a percent difference from those values obtained by changing β_f by only one order of magnitude.

If we look at the men, changing β_m produces the same qualitative behaviors observed in the case with the female population, see Figure (22).

In the case with pregnant women, if we decrease γ_i by a factor of 10 then there are virtually no pregnant women in rehabilitation during our time scale. If we increase γ_i by a factor of 10, then there is a 600% change in the number of pregnant women in rehabilitation, see Figure (23). Thus the population of pregnant women is very sensitive to the γ_i.

However, the population of pregnant women is small relative to the total population of women. Therefore our deterministic model is relatively insensitive to changes in γ_i. If we decrease γ_i by two orders of magnitude, there is less than a 1% change for the first 60 years, see Figure (24). If we increase γ_i by a factor of 10, then it takes over 30 years before we get a 10% difference in the number of women who will abuse crack-cocaine.

Another important parameter to any rehabilitation program is the rate of relapse. Ideally, one would want women to never use drugs again after finishing a rehabilitation program. In the case of pregnant women, we found that if we increased ρ_1 and ρ_2 by a factor of 100, the number of women in rehabilitation decreases by 20% in ten years, see Figure (25). This is due to the direct effect of women leaving rehabilitation programs. However the total number of drug (crack-cocaine) using women changes by only 1%, see Figure (26). This is because the number of drug using women is large relative to the number of women in rehabilitation. If we decrease ρ_1 and ρ_2 by a factor of 100, then the number of pregnant women in rehabilitation programs increases by 40% in ten years. Likewise, the number of drug using women changes by only 2%. In 20 years, if you decrease ρ_1 and ρ_2 by a factor of 100, then the number of women in rehab increases by 94%, and the number of drug using women decreases by 3%. If you increase ρ_1 and ρ_2 by a factor of 100, then the number of women in rehab decreases by 17% and the number of drug using women increases by 3%.

Although there are other parameters that may have an impact on the dynamics of our model, namely λ_i and ϕ_i, they will be considered as constants in our system. The mechanics behind altering λ_i involves changing the gestation period of women or the rate that they have miscarriages and is outside of the scope of our project. Adjusting ϕ_i, the rate that women get pregnant is another consideration that is outside the scope of our project.

Population	β_f	γ_i	ρ_i	β_f&γ_i	β_f&ρ_i	γ_i&ρ_i	β_f&γ_i&ρ_i
P_s	0.17	0	0	0.17	0.17	0.019	0.15
P_d	−0.47	−0.27	0	−0.67	−0.53	−0.4	−0.73
P_r	−0.6	6	0.8	3	0	10	5
D_f	−0.44	−0.2	0.04	−0.52	−0.44	−0.28	−0.6
$S_f + R_f$	0.18	0.05	0.03	0.20	0.18	0.11	0.25

Figure 27: Percent change due to variation of parameters: β_f decreased by a factor of 10, γ_i increased by a factor of 10, ρ_i decreased by a factor of 10

The three most important parameters are β_f, γ_i and ρ_i. We have already seen the changes that result from altering just one parameter at a time. The next step is to see whether or not altering two or all three parameters can cause significant changes:

From Figure (27) we see that changing β_f is the most effective way to reduce the amount of drug abuse in our model with γ_i as the next most effective parameter and ρ_i as the least effective. Combining parameters is always beneficial, but sometimes the amount of change achieved is not much compared to changing just one variable. For example, changing β_i causes an 18% reduction in the number of drug free women while changing β_i and γ_i only creates a 19% reduction and changing β_i and ρ_i does not cause significant change. While it is clear that β_i is the most important parameter, changing all three parameters brings about the most change.

Changing β_m, γ_m and ρ_m brings about a similar quantitative percentage change for the males, but our discussion is limited to the female case. We assume that women are much more susceptible to efforts to keep them off crack-cocaine and can benefit the most from such efforts. In addition, changing these parameters will only have marginal effects on women because they will be the object of secondary aid due to the interaction with a reduced D_m. If the parameters β_i, γ_i and ρ_i are changed, then that makes women the primary target of aid and they receive the most benefit.

Comparing our deterministic model without drug-induced death and our deterministic model with drug-induced death produced very different results. In the case without drug-induced death, we never reached a steady state in the short run; the population of drug users is constantly increasing. With drug-induced death, the number of drug users would often peak within the first 20 years, and quickly reduce to a steady level within the next hundred years. Although our model is not very accurate in the long term, it is interesting to analyze the deterministic model in this time scale and see its

behavior. It is clear from these simulations that the endemic solution exists at a much lower level than in the case without drug-induced death sometimes half of what we would get if we did not consider drug induced death after the first ten years, see Figures (3a,c). This qualitative phenomenon is paralleled in the case with women in rehabilitation and pregnant women in both classes. Death through cocaine abuse is a very serious problem, with nearly $20,000$ people a year dying through drug related causes [20]. Our data indicates that including drug-induced death is significant in that it changes the dynamics of our system and makes it more accurate to real life.

Looking at the case where men go to jail instead of rehabilitation, we see that the dynamics are similar to the case with rehabilitation, but the system is slower. That is, it takes a longer amount of time for the number of drug users to peak and fall off to a steady state by a factor of 2. Also, because it takes longer for the number of drug users to peak, there are a larger number of drug users at the peak.

A preliminary examination of the case with jail term and rehabilitation indicates faster dynamics of the system. For certain parameters, the number of drug users dies off relatively quickly. This tells us that a combination of strict jail sentencing combined with an effort to remove people from the drug-abusing class is an effective way of combating crack-cocaine abuse, but further investigation is necessary before any strong conclusions can be drawn from this model.

9 Conclusions

Our first model is very simple to analyze and provides a nice starting point for our investigations. Unfortunately, the model is too simplistic to draw any realistic conclusions from. However, we can make some general observations: an endemic solution exists and the level of that solution depends on the parameters. We know that β_f is the most important parameter in determining the level of crack-cocaine abuse in the population of pregnant and non-pregnant women. The driving force behind our system is the nonlinear $\frac{D_m}{N_m}$ term, and β_f is responsible for scaling how much influence that term has. We have also shown that γ and ρ are also important parameters. If we were to introduce an educational program, then we would want that program to alter these parameters. Which parameters we decided to alter depends on the intent of our educational program. If we want to reduce the total number of drug (crack-cocaine) using women, then we would want to focus on altering β_f. If we wanted to focus on the population of pregnant

women in rehabilitation, we would focus on reducing ρ_i, the rate that women relapse back into drug use.

Including a drug-induced mortality rate is a very important alteration to our system. It can have a dramatic impact on the dynamics and make the model more realistic. Unfortunately the added complexity make the system more difficult to analyze and we were unable to analytically find the stability of an endemic solution. From this model we were able to conclude that it was feasible to reduce the amount of crack-cocaine abuse to very low levels, although not necessarily to eliminate it completely. This is encouraging in that it indicates that it may be able to reduce if not eliminate the prevalence of drug abuse in real life.

The jail system has slower dynamics with results that do not show steady state behavior until after over 100 years in some cases. Not only is increasing the jail sentence for drugs not as effective as having a rehabilitation program, but it is also more expensive. This seems to indicate that going to jail is not enough to reduce the amount of drug abuse in our society. It is vital to have a rehabilitation system set up to keep people off of drugs. According to our model with a jail term, it would take an average sentence for drug abuse of 200 years before the prevalence of crack-cocaine would die out. Of course this is unrealistic, but that is the extreme to which we would have to go to in order to eliminate drug abuse from our society.

Now we need to consider which is more cost-effective, putting men in jail or supporting drug rehabilitation programs. With the parameters we have gotten from different governmental sources, we know that putting men in jail is not as effective as a rehabilitation program. Furthermore, the cost of keeping drug offenders in jail has almost tripled in five years from $8 billion in 1993 to $21 billion in 1998 [21]; The cost of rehabilitation programs is $4.4 billion a year [22]. Clearly it is more cost effective to concentrate on public programs that focus on the rehabilitation of people who abuse crack-cocaine than to try and incarcerate these drug-offenders. Not only would it be necessary to increase the jail sentence to these individuals, but also it would perpetuate the problem of skyrocketing corrections facilities costs. On the other hand, rehabilitation programs are far more effective at keeping men, women, and especially pregnant women off of drugs. It is evident that we should shift our allocation of resources and spend greater efforts at increasing the scope of various drug rehabilitation and education programs. Our money would be much better spent if we tried to help people, not lock them behind bars.

10 Future Work

We will leave numerical simulations for the age structure model and stability analysis of the drug abuse free equilibrium will be left for future study. Solutions to partial differential equations pose many challenges, and even numerical solutions could take many more months of effort to attain any useful results. Stability analysis for the endemic equilibria of the drug induced mortality models will also be left for future analysis. We did not have time to fully investigate the dynamics of our model with a jail term, or the model with a jail and rehabilitation term. Further investigation of these models and simulations could provide potentially interesting results regarding periodic states that may or may not be relevant to real life applications.

11 Acknowledgements

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Appendix

```
%Here is the system of equations for an ode solver:

function dr=ugs(t,x)

%Sf in our system is x(1)
%Df in our system is x(2)
%Rf in our system is x(3)
%Ps in our system is x(4)
%Pd in our system is x(5)
%Pr in our system is x(6)
%Sm in our system is x(7)
%Dm in our system is x(8)
%Rm in our system is x(9)

global beta1 beta2 beta_m lambda1 lambda2 lambda3 gamma1 gamma2...
gamma_m Nf Nm mu phi1 phi2 phi3 rho1 rho2 rho_m

%r=state of system;

dr=[mu*Nf+lambda1.*x(4)-beta1.*x(1).*(x(8)/Nm)-(mu+phi1).*x(1);
    beta1.*x(1).*(x(8)/Nm)+rho1.*x(3)+lambda2.*x(5)-(mu+phi2+gamma1).*x(2);
```

```
    gamma1.*x(2)+lambda3.*x(6)-(rho1+phi3+mu).*x(3);
    phi1.*x(1)-beta2.*x(4).*(x(8)/Nm)-(lambda1+mu).*x(4);
    beta2.*x(4).*(x(8)/Nm)+phi2.*x(2)+rho2.*x(6)-(lambda2+gamma2+mu).*x(5);
    gamma2.*x(5)+phi3.*x(3)-(rho2+lambda3+mu).*x(6);
    mu*Nm-beta_m.*x(7).*(x(8)/Nm)-mu.*x(7);
    beta_m.*x(7).*(x(8)/Nm)+rho_m.*x(9)-(gamma_m+mu).*x(8);
    gamma_m.*x(8)-(rho_m+mu).*x(9);];

% Plotugs plots the forward history of a system of drug use on pregnant women
% Takes as input parameters tf,b1,b2,bm,L1,L2,L3,g1,g2,gm,totNf,totNm,p1,p2,p3,m,
% r1,r2,rm, and dm
% varies the parameter b1 and b2
% tf = final time, all rates are per year
% b1 = rate at which susceptible women will interact with drug using men
% b2 = rate at which pregnant women will interact with drug using men
% bm = rate at which men will interact with drug using men
% L1 = rate at which susceptible pregnant women become susceptible women
% L2 = rate at which drug using pregnant women become drug using women
% L3 = rate at which rehab pregnant women become rehab women
% g1 = rate at which drug using women enter rehab
% g2 = rate at which pregnant drug using women enter rehab
% gm = rate at which drug using men enter rehab
% totNf = total population of women
% totNm = total population of men
% p1 = rate at which susceptible women get pregnant
% p2 = rate at which drug using women get pregnant
% p3 = rate at which rehab women get pregnant
% m = universal death rate
% r1 = rate at which drug using women in rehab relapse
% r2 = rate at which pregnant drug using women in rehab relapse
% rm = rate at which men enter rehab
% dm = number of drug using males

function y=plotugs(tf,b1,b2,bm,L1,L2,L3,g1,g2,gm,totNf,...
totNm,p1,p2,p3,m,r1,r2,rm,dm)

tic;

global beta1 beta2 beta_m lambda1 lambda2 lambda3 gamma1 gamma2...
gamma_m Nf Nm phi1 phi2 phi3 mu rho1 rho2 rho_m

%redifintion of variables
```

```
beta1=b1;
beta2=b2;
beta_m=bm;
lambda1=L1;
lambda2=L2;
lambda3=L3;
gamma1=g1;
gamma2=g2;
gamma_m=gm;
Nf=totNf;
Nm=totNm;
phi1=p1;
phi2=p2;
phi3=p3;
mu=m;
rho1=r1;
rho2=r2;
rho_m=rm;
Dm=dm;

%Df=df;Rf=rf;Ps=ps;Pd=pd;Pr=pr;Rm=rm
%Intrinsic reproduction number of our system

Ro=beta_m*(rho_m+mu)/(mu*(mu+gamma_m+rho_m))

beta1=b1*.01; % Changing beta by order of magnitude
beta2=b2*.01;
tspan=[0,tf];

[t,z1]=ode45('ugs',tspan,[Nf;0;0;0;0;0;Nm-Dm;Dm;0]);

beta1=b1*.1;
beta2=b2*.1;

tspan=[0,tf];
[t,z2]=ode45('ugs',tspan,[Nf;0;0;0;0;0;Nm-Dm;Dm;0]);

beta1=b1;
beta2=b2;

tspan=[0,tf];

[t,z3]=ode45('ugs',tspan,[Nf;0;0;0;0;0;Nm-Dm;Dm;0]);
```

```
beta1=b1*10;
beta2=b2*10;

tspan=[0,tf];

[t,z4]=ode45('ugs',tspan,[Nf;0;0;0;0;0;Nm-Dm;Dm;0]);

beta1=b1*100;
beta2=b2*100;

tspan=[0,tf];
[t,z5]=ode45('ugs',tspan,[Nf;0;0;0;0;0;Nm-Dm;Dm;0]);

Figure

subplot(211)
hold on
plot(t,(z1(:,3)+z1(:,6)+z1(:,1)+z1(:,4)),'k'); % Total drug free women
plot(t,(z1(:,2)+z1(:,4)),'m') % Total drug using women
plot(t,(z2(:,3)+z2(:,6)+z2(:,1)+z2(:,4)),'k.');
plot(t,(z2(:,2)+z2(:,4)),'m.')
plot(t,(z3(:,3)+z3(:,6)+z3(:,1)+z3(:,4)),'k:');
plot(t,(z3(:,2)+z3(:,4)),'m:')
plot(t,(z4(:,3)+z4(:,6)+z4(:,1)+z4(:,4)),'k--');
plot(t,(z4(:,2)+z4(:,4)),'m--')
plot(t,(z5(:,3)+z5(:,6)+z5(:,1)+z5(:,4)),'k-.');
plot(t,(z5(:,2)+z5(:,4)),'m-.')
h = legend('DFW beta = ' num2str(b1*.01),'DUW beta = ' num2str(b1*.01), ...
   'DFW beta = ' num2str(b1*.1),'DUW beta = ' num2str(b1*.1), ...
   'DFW beta = ' num2str(b1*1),'DUW beta = ' num2str(b1*1), ...
   'DFW beta = ' num2str(b1*10),'DUW beta = ' num2str(b1*10), ...
   'DFW beta = ' num2str(b1*100),'DUW beta = ' num2str(b1*100));
title('Dynamics of the Female Population with Variable beta')
xlabel('T(years)')
ylabel('Population')

subplot(212)
Figure

hold on
plot(t,(z1(:,7)+z1(:,9)),'k'); % Drug free men
plot(t,z1(:,8),'m'); % Drug using men
```

```
h = legend('Total Drug Free Men','Total Drug Using Men');
title('Dynamics of the Male Population')
xlabel('T(years)');
ylabel('Population');

Figure

hold on
plot(t,z1(:,4),'m')
plot(t,z1(:,5),'c')
plot(t,z1(:,6),'y')
plot(t,z2(:,4),'m.')
plot(t,z2(:,5),'c.')
plot(t,z2(:,6),'y.')
plot(t,z3(:,4),'m:')
plot(t,z3(:,5),'c:')
plot(t,z3(:,6),'y:')
plot(t,z4(:,4),'m--')
plot(t,z4(:,5),'c--')
plot(t,z4(:,6),'y--')
plot(t,z5(:,4),'m-.')
plot(t,z5(:,5),'c-.')
plot(t,z5(:,6),'y-.')
h=legend('P_s beta = ' num2str(b1*.01),'P_d beta = ' num2str(b1*.01),...
                      'P_r beta = ' num2str(b1*.01), ...
         'P_s beta = ' num2str(b1*.1),'P_d beta = ' num2str(b1*.1),...
                      'P_r beta = ' num2str(b1*.1), ...
         'P_s beta = ' num2str(b1*1),'P_d beta = ' num2str(b1*1),...
                      'P_r beta = ' num2str(b1*1), ...
         'P_s beta = ' num2str(b1*10),'P_d beta = ' num2str(b1*10),...
                      'P_r beta = ' num2str(b1*10), ...
         'P_s beta = ' num2str(b1*100),'P_d beta = ' num2str(b1*100),...
                      'P_r beta = ' num2str(b1*100));

title('Dynamics of Pregnant Women')
xlabel('T(years)');
ylabel('Population');

Figure

hold on
plot(t,z(:,1),'r') % Susceptible pregnant women
plot(t,z(:,2),'b') % Drug using pregnant women
```

```
plot(t,z(:,3),'g') % Pregnant women in rehab
plot(t,z(:,4),'m') % Susceptible pregnant women
plot(t,z(:,5),'c') % Drug using pregnant women
plot(t,z(:,6),'y') % Pregnant women in rehab
plot(t,z(:,7),'r:') % Susceptible pregnant women
plot(t,z(:,8),'b:') % Drug using pregnant women
plot(t,z(:,9),'g:') % Pregnant women in rehab
title(['R_0 = ' num2str(Ro)])
h=legend('S_f','D_f','R_f','P_s','P_d','P_r','S_m','D_m','R_m');
xlabel('T(Years)');
ylabel('Population');

toc

% The code for the stochastic simulations
% Drugs2 plots the Deterministic and Stochastic versions of the model.
% Takes as input parameters tfinal,b1,b2,bm,L1,L2,L3,g1,g2,gm,p1,p2,p3,m,
% r1,r2,rom, and HM
% varies the parameter b1 and b2
% tfinal = final time, all rates are per year
% b1 = rate at which susceptible women will interact with drug using men
% b2 = rate at which pregnant women will interact with drug using men
% bm = rate at which men will interact with drug using men
% L1 = rate at which susceptible pregnant women become susceptible women
% L2 = rate at which drug using pregnant women become drug using women
% L3 = rate at which rehab pregnant women become rehab women
% g1 = rate at which drug using women enter rehab
% g2 = rate at which pregnant drug using women enter rehab
% gm = rate at which drug using men enter rehab
% p1 = rate at which susceptible women get pregnant
% p2 = rate at which drug using women get pregnant
% p3 = rate at which rehab women get pregnant
% m =  natural mortality rate
% r1 = rate at which drug using women in rehab relapse
% r2 = rate at which pregnant drug using women in rehab relapse
% rom = rate at which men enter rehabilitation
% HM = number of iterations

function y=drugs2(tfinal,L1,L2,L3,p1,p2,p3,r1,r2,rom,b1,bm,g1,g2,gm,m,HM)

lambda1=L1;lambda2=L2;lambda3=L3;phi1=p1;phi2=p2;phi3=p3;rho1=r1;...
rho3=r2;rho_m=rom;beta1=b1;beta_m=bm;
gamma1=g1;gamma2=g2;gamma_m=gm;mu=m;HM;
```

```
tfinal;                                 % Redefintion of variables
Nf=10000/1000;                          % Initial conditions
NM=10000/1000;
d=977/2;
pd=20/2;
dm=1840/2;
rh=0;
pr1=0;
rm=0;
ps=0;
Dh=0;

s=Nf-d-rh-ps-pd-pr1;
sm=NM-dm-rm;

tic;

R0 = beta_m*(rho_m+mu)/(mu*(rho_m+mu+gamma_m))
k=0;
ban=0;

for i = 1 : HM                          % Stochastic Calculations

   if ban==1
      break
   end

   t=0;
   S=s; D=d; RH=rh; PS=ps; PD=pd; PR1=pr1; SM=sm; DM=dm; RM=rm;
   Nf=s+d+rh+ps+pd+pr1;
   NM=sm+dm+rm;

   state=[t S D RH PS PD PR1 SM DM RM];

   TR=1;

   state_block=zeros(10000,10);
   state=state_block;

   while (t < tfinal) & (D+RH+PD+PR1+DM+RM > 0 & ban==0) %& ban==0

      k=k+1;
```

```
Nf=S+D+RH+PS+PD+PR1;
NM=SM+DM+RM;
Bh=mu*Nf;
S_Dh=mu*S;
S_PS=phi1*S;
S_D=beta1*S*(DM/NM);
D_PD=phi2*D;
D_RH=gamma1*D;
D_Dh=mu*D;
RH_D=rho1*RH;
RH_PR1=phi3*RH;
RH_Dh=mu*RH;
PS_S=lambda1*PS;
PS_PD=beta1*PS*(DM/NM);
PS_Dh=mu*PS;
PD_D=lambda2*PD;
PD_PR1=gamma2*PD;
PD_Dh=mu*PD;
PR1_PD=rho3*PR1;
PR1_RH=lambda3*PR1;
PR1_Dh=mu*PR1;
Bm=mu*NM;
SM_DM=beta_m*SM*(DM/NM);
SM_Dh=mu*SM;
DM_RM=gamma_m*DM;
DM_Dh=mu*DM;
RM_DM=rho_m*RM;
RM_Dh=mu*RM;
%RM_S=epsilon*RM;

 R = [Bh S_Dh S_PS S_D D_PD D_RH D_Dh RH_D RH_PR1 RH_Dh PS_S PS_PD...
     PS_Dh PD_D PD_PR1 PD_Dh PR1_PD PR1_RH PR1_Dh...
     Bm SM_DM SM_Dh DM_RM DM_Dh RM_DM RM_Dh];

% Total rate
TR = sum(R);
% the vector of probabilities
pr = R/TR;

prcum = cumsum(pr);
prcum=[0 prcum];

r = rand;
```

```
 slot= sum(r>prcum);
if slot == 1
    S=S+1;
elseif slot == 2
    S=S-1; Dh=Dh+1;
elseif slot == 3
    S=S-1; PS=PS+1;
elseif slot == 4
    S=S-1; D=D+1;
elseif slot == 5
    D=D-1; PD=PD+1;
elseif slot == 6
    D=D-1; RH=RH+1;
elseif slot == 7
    D=D-1; Dh=Dh+1;
elseif slot == 8
    RH=RH-1; D=D+1;
elseif slot == 9
    RH=RH-1; PR1=PR1+1;
elseif slot == 10
    RH=RH-1; Dh=Dh+1;
elseif slot == 11
    PS=PS-1; S=S+1;
elseif slot == 12
    PS=PS-1; PD=PD+1;
elseif slot == 13
    PS=PS-1; Dh=Dh+1;
elseif slot == 14
    PD=PD-1; D=D+1;
elseif slot == 15
    PD=PD-1; PR1=PR1+1;
elseif slot == 16
    PD=PD-1; Dh=Dh+1;
elseif slot == 17
    PR1=PR1-1; PD=PD+1;
elseif slot == 18
    PR1=PR1-1; RH=RH+1;
elseif slot == 19
    PR1=PR1-1; Dh=Dh+1;
elseif slot == 20
    SM=SM+1;
elseif slot == 21
```

```
          SM=SM-1; DM=DM+1;
      elseif slot == 22
          SM=SM-1; Dh=Dh+1;
      elseif slot == 23
          DM=DM-1; RM=RM+1;
      elseif slot == 24
          DM=DM-1; Dh=Dh+1;
      elseif slot == 25
          RM=RM-1; DM=DM;
      else
          RM=RM-1; Dh=Dh+1;
       end

       t = t - log(rand)/TR;
       a=min(t,tfinal);
       state(k,:) = [a S D RH PS PD PR1 SM DM RM];
       if k==10000
           state=[state;state_block];
       end
  end

   lastrow(i,:)=state(end,:);
   x = state(1:k,1);
   yS = state(1:k,2);
   yD = state(1:k,3);
   yRH = state(1:k,4);
   yPS = state(1:k,5);
   yPD = state(1:k,6);
   yPR1 = state(1:k,7);
   ySM = state(1:k,8);
   yDM = state(1:k,9);
   yRM = state(1:k,10);

save stoch.txt state -ASCII

i=i+1;
end

tf=tfinal;
b2=beta1;
rm=rom;
                    % Deterministic Calculations
plotugs1(tf,b1,b2,bm,L1,L2,L3,g1,g2,gm,Nf,NM,p1,p2,p3,m,r1,r2,rm,dm)
```

```
global t1 z1

f1=Figure;
f2=Figure;
f3=Figure;
                    % Comparitive Plot
Figure(f1)
subplot(211)
hold on
plot(x,yD,'b')
plot(x,yPD,'b:')
plot(t1,z1(:,2),'k--')
plot(t1,z1(:,5),'k-.')
title(['Drug Using Women: beta1= ' num2str(beta1) ', rho1= ' num2str(rho1) ',...
      rho2= ' num2str(rho3) ', Ro= ' num2str(R0)]);
xlabel('Time(years)')
ylabel('Population(Women)')
h=legend('Non-Pregnant Women D_f','Pregnant Women P_d',...
         'Deterministic D_f','Deterministic P_d',0);

subplot(212)
hold on
plot(x,yRH,'r')
plot(x,yPR1,'m:')
plot(t1,z1(:,3),'k--')
plot(t1,z1(:,6),'k-.')
title(['Recovered Women: gamma1= ' num2str(gamma1) ',...
       gamma2= ' num2str(gamma2) ', Ro= ' num2str(R0)]);
xlabel('Time(years)')
ylabel('Population(Women)')
h=legend('Non-Pregnant Women R_f','Pregnant Women P_r',...
         'Deterministic R_f','Deterministic P_r',0);

Figure(f2)
hold on
plot(x,yDM,'b')
plot(x,yRM,'r:')
plot(t1,z1(:,8),'k--')
plot(t1,z1(:,9),'k-.')
title(['Drug Using and Recovered Men: gamma_m= ' num2str(gamma_m) ',...
        rho_m= ' num2str(rho_m) ', beta_m= ' num2strbeta_m)...
       ', Ro= ' num2str(R0)]);
xlabel('Time(years)')
```

```
ylabel('Population(Men)')
h=legend('Drug Using Men D_m','Recovered Men R_m','Deteministic D_m',...
        'Deterministic R_m',0);

Figure(f3)
subplot(211)
hold on
plot(x,yD,'b')
plot(x,yDM,'b:')
plot(t1,z1(:,2),'k--')
plot(t1,z1(:,8),'k-.')
title(['Drug Users: beta1= ' num2str(beta1) ', rho1= ' num2str(rho1) ',...
       beta_m= ' num2str(beta_m) ', rho_m= ' num2strrho_m)...
       ', Ro= ' num2str(R0)])
xlabel('Time(years)')
ylabel('Total Population')
h=legend('Women D_f','Men D_m','Deterministic D_f','Deterministic D_m',0);

subplot(212)
hold on
plot(x,yRH,'r')
plot(x,yPR1,'r.')
plot(x,yRM,'r:')
plot(t1,z1(:,3),'k--')
plot(t1,z1(:,6),'k')
plot(t1,z1(:,9),'k-.')
title(['Recovered Men and Women: gamma1= ' num2str(gamma1) ',...
        gamma2= ' num2str(gamma2) ', gamma_m= ' num2str(gamma_m)...
        ', Ro= ' num2str(R0)])
xlabel('Time(years)')
ylabel('Total Population')
h=legend('Women R_f','Pregnant Women P_r','Men R_m','Deterministic R_f',...
         'Deterministic P_r','Deterministic R_m',0);

fD=lastrow(:,3);   % I1
fPD=lastrow(:,6);  % I2
fDM=lastrow(:,9);  % T1
toc;

%drugs2(100,0.8,0.7,0.75,0.028489,0.023313,0.049089,0.22,0.01,0.32,...
        0.0714,0.0714,0.007108,0.014967,0.00578,0.00004,1)
```