The Effects of Mass Transportation During a Deliberate Release of Smallpox

CORNELL UNIV., DEPT. OF BIOLOGICAL STATISTICS & COMPUTATIONAL BIOLOGY

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1 Abstract

Since the attack on the world trade center, in New York City on September 11^{th}, the possibility of bio-terrorist attacks on major cities has received continuous attention. The deliberate release of smallpox, a deadly virus, still available in selected U.S. and Russian laboratories, is a source of great concern. Ring vaccination is the current official response policy in the event of smallpox deliberate release. Edward H. Kaplan, David L. Craft and, Lawrence M. Wein argue that mass vaccination is a more viable solution that would optimize resource usage and minimize the mortality rate. In this paper, we model the dynamics of a deliberate release of smallpox in an idealized model of public transportation system of a major city. The city is divided into n neighborhoods and stratified by the proportion of individuals who use public transportation. Two levels of mixing are introduced via within neighborhood and between neighborhood activity levels. Transmission between neighborhoods is driven by interactions in the mass transportation system under the assumption of proportionate mixing. We complement our theoretical work with an agent based model for

two populations of individuals with different proportions of subway and non-subway users. We explore the impact of Kaplan, Craft and, Wein vaccination policy on the spread of epidemics for both models and our initial results seem to support their recommendations.

2 Introduction

After the events of September 11^{th}, 2001, researchers in mathematical biology, epidemiology, and other related disciplines have been concerned about the use of viruses as a tool to cause terror in the American community. This concept is known as Bioterrorism, which is defined as the deliberate release of biological agents and their consequences. The goal of this paper is to study the disease dynamics the follow a subway transportation attack in order to test a set of possible responses. The reaction of a population after or during the attack is key to control the situation. The attack could consist of an outbreak of a fatal disease such as ebola, smallpox, rubella, etc. In this paper we consider smallpox as a biological agent, which is spread mainly through the transportation system of a city, and study the efficiency of the Kaplan, Craft and Wein vaccination strategy.

The initial idea was to consider ebola because it is a very fatal disease that has caused up to 88% [11] of the infected population to die in a matter of 2 weeks. Even though ebola has such a high mortality rate, it does not seem as useful agent of biological terror, it is hard to transmit, because it is not airborne. Furthermore, ebola's latent period is very short 2-4 days, which implies early detection. One needs to find a disease that could cause enough damage in a short amount of time and that it is easy to transmit. Smallpox is one of the most serious candidates. Even though smallpox was officially declared "eradicated" in 1980, there still exist two official repositories of smallpox virus in the world: the CDC in Atlanta and the Russian State Research Center of Virology and Biotechnology in Koltsovo, Novosibirsk. Those supplies are used for scientific research and vaccine development. Those two sources may not be the only two sources for the virus. It is known that the year that worldwide vaccination ceased, the Soviet government began growing and stockpiling large quantities of smallpox virus, specially adapted for use in bombs and missiles [12].Thus, a deliberate release of smallpox is feasible. Even before September 11th, interest was rising in how prepared we are to face a bioterrorism attack. Now, the "unthinkable" has happened, and bolstering our smallpox vaccine supply has become a priority. There are currently about 50 million vaccine doses worldwide – with $300 * 10^6$ the U.S. Experts say that even with an all-out manufacturing effort, it would take at least three years before there was sufficient supply to prevent an epidemic.[12]

If used as a biological weapon, smallpox represents a serious threat to civilian populations because of its case-fatality rate of 30 percent or more among unvaccinated persons and the absence of specific treatment. In a city like that of New York City

with a population size of about 8 million, and where there are about 4.3 million people take the subway weekly, it is easy to cause a fast outbreak [7].Today smallpox has a far greater devastation potential, because routine vaccination throughout the United States ceased more than 25 years ago [9]. In a highly susceptible and mobile population like that of the US, smallpox is likely able to spread widely and rapidly. Smallpox is a viral communicable disease, that it is passed from person to person in a continuing chain of infection. It is spread by inhalation of air droplets or aerosols. This can occur whenever an individual is within a six feet radius from the infectious person. Twelve to fourteen days after infection, the patient typically becomes febrile and has severe aching pains and prostration. Some two to three days later, a papular rash develops over the face and spreads to the extremities. The rash soon becomes vesicular and later, pustular. The patient remains febrile throughout the evolution of the rash and customarily experiences considerable pain as the pustules grow and expand. Gradually, scabs form, which eventually separate, leaving pitted scars. Death usually occurs during the second week [8].

An agent-based model was used to compare mass vaccination vs. ring vaccination, and to determine the effects of transportation in a bio-terrorist attack. We divide the population into 4-neighborhoods to assimilate the transportation model. Mass vaccination and ring vaccination are introduced after a predetermined number of infected cases are reported. At time zero, 50 exposed individuals are introduced representing an initial bio-terrorist attack. We found that both vaccination strategies work better when the epidemic is detected early. Yet, it is not clear whether mass vaccination works better than ring vaccination. The difference in the resulting fatality cases with both vaccination methods is not sufficient to determine which method is better. This suggests that more detailed study is necessary before concluding what is the optimal vaccination method to implement. We noticed a non-surprising distribution of fatality cases per neighborhood. These are directly proportional to the percentage of SU per neighborhood.

3 The Model

The main purpose of this paper is to model smallpox transmission in a city with a widely used subway system, particularly when their is a deliberate released. There are many different options on how to model the contact dynamics of populations that use a transportation system. Here, we modify the model found in a paper "El transporte público y la dinámica de la tuberculosis a nivel poblacional" (The public transportation system and the dynamics of tuberculosis at a population level) [1]. A secondary objective of this project is to test various responses strategies to a deliberate release. Kaplan, Craft and Wein argues that mass vaccination is the best option becuase provides the best response for what defines as the worst case scenario. We hope to test his policy suggestion in a dynamic model.

In order to focusse the discussion we use New York City as the basis for our project. The city is divided into n neighborhoods and within each neighborhood the population is subdivided into regular subway users (SU) and regular non-subway users (NSU). It is assumed that SU have contacts with both SU and NSU in their own neighborhood. SU may also have contact with other SU from different neighborhoods, but only when they shared a subway ride. Contacts between SU individuals from different neighborhoods outside the subway are considered negigable int he context of smallpox transmission. It is also assumed that NSU individuals have most of their contacts not only within their own neighborhood but also mostly with those living in the same neighborhood.

If a fixed number of infected individuals is introduced in the subway, then the first cases of infection would occur in the SU population. Newly infected individuals will then take the virus back to their own neighborhoods generating infections in the NSU and SU populations. Once the attack is recognized and smallpox is detected, vaccination starts. The model is constructed in a way that only a proportion of the population is vaccinated per day because the immediate vaccination of the whole population is impossible in one day.From a view point of a deliberate release, it will be assumed (later tested by simulations) that rapid spread is most likely if the disease is introduced in the subway.

Susceptibles that are vaccinated have 0.97 probability of moving into the recovered class and immunity is supposed to fast for about 5 years. Exposed individuals that are vaccinated within the first four days are vaccinated to smallpox can move into the recovered class with the same chance as the susceptible individuals. Otherwise they become infectious. There is not effective treatment for smallpox. Infected individuals have a chance of 70% of recovery [10].

The analysis of this model is complicated, and we have limited results. However we use numerical analysis to gain insight into the impact of smallpox epidemics in this system. We use matlab to run some simulations and an agent-based model which gave us an insight on different case scenarios.

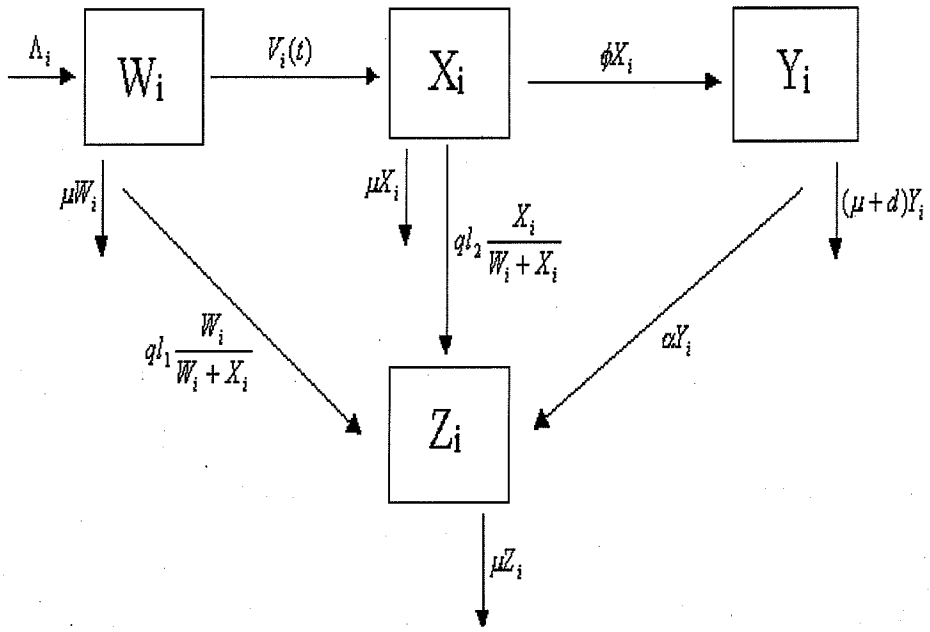

Figure 1: Subway Users Model

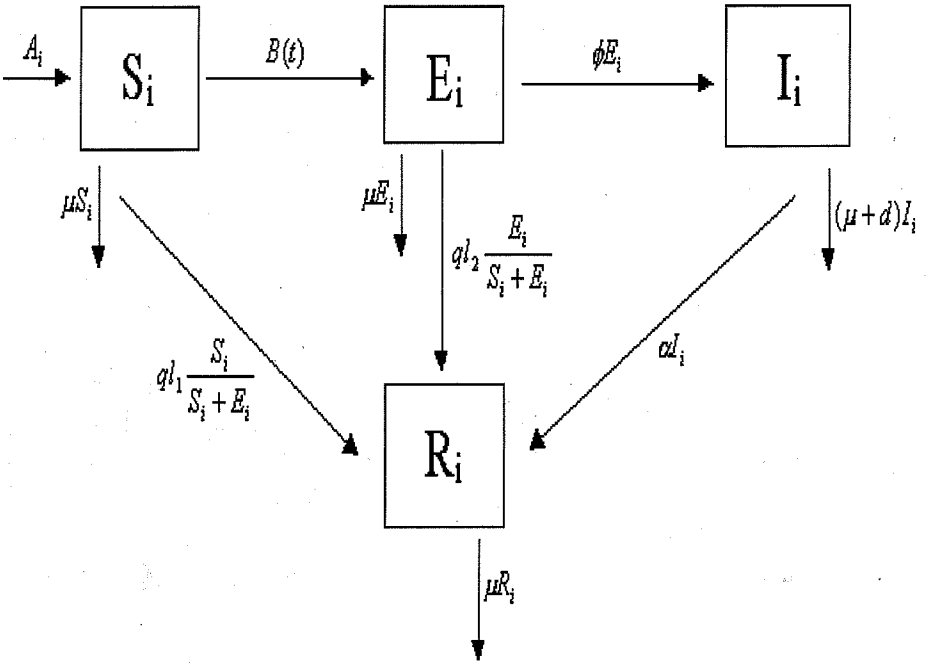

Figure 2: Non-Subway Users Model

4 Model Equations

From the flow diagram Figure 1 we can put down the equations for subway users:

$$\frac{dW_i}{dt} = \Lambda_i - V_i(t) - \left(\mu W_i + ql_1 \frac{W_i}{W_i + X_i}\right), \quad (1)$$

$$\frac{dX_i}{dt} = V_i(t) - \left(\mu X_i + \phi X_i + ql_2 \frac{X_i}{W_i + X_i}\right), \quad (2)$$

$$\frac{dY_i}{dt} = \phi_i X_i - (\mu + \alpha + d)\, Y_i, \quad (3)$$

$$\frac{dZ_i}{dt} = \alpha Y_i - \mu Z_i + ql_1 \frac{W_i}{W_i + X_i} + ql_2 \frac{X_i}{W_i + X_i}, \quad (4)$$

$$T_i(t) = W_i(t) + X_i(t) + Y_i(t) + Z_i(t). \quad (5)$$

For SU: $W(t)_i$ is the number of susceptibles at time t, $X(t)_i$ is the number of exposed at time t, $Y(t)_i$ is the number of infectious at time t, and $Z(t)_i$ is the number of recovered at time t. This means that the total population of subway users is $T(t)_i$= $W(t)_i + X(t)_i + Y(t)_i + Z(t)_i$.

From Figure 2 the non-subways users equations:

$$\frac{dS_i}{dt} = A_i - B_i(t) - \left(\mu S_i + ql_1 \frac{S_i}{S_i + E_i}\right), \quad (6)$$

$$\frac{dE_i}{dt} = B_i(t) - \left(\mu E_i + \phi E_i + ql_2 \frac{E_i}{S_i + E_i}\right), \quad (7)$$

$$\frac{dI_i}{dt} = \phi E_i - (\mu + \alpha + d)\, I_i, \quad (8)$$

$$\frac{dR_i}{dt} = \alpha I_i - \mu R_i + ql_1 \frac{S_i}{S_i + E_i} + ql_2 \frac{E_i}{S_i + E_i}, \quad (9)$$

$$Q_i(t) = S_i(t) + E_i(t) + I_i(t) + R_i(t).. \quad (10)$$

For NSU: $S_i(t)$ is the number of susceptibles at time t, $E(t)_i$ is the number of exposed at time t, $I(t)_i$ is the number of infectious ant time t, and $R(t)_i$ is the number of recovered at time t. This means that the total population of non-subway users is $Q(t)_i$= $S(t)_i + E(t)_i + I(t)_i + R(t)_i$.

where $i = 1, ..., n$ is the number of neighborhoods. Terms and parameters are defined in table 1.

5 Parameters of the model

The following table show the parameters and their meaning. We model interactions between individuals in the subway and non-subway environment using proportionate mixing.

$B_i(t)$	rate of incidence, new number of infections per unit of time for NSU
$V_i(t)$	rate of incidence, new number of infections per unit of time for SU
μ	natutal mortality rate
d	mortality rate due to smallpox
q	amount of people vaccinated at time t
l_i	vaccination efficacy rate
ϕ	rate of infection
α	recovery rate for I
σ	is the rate at which a SU leaves the subway
ρ	is the rate at which a SU takes the subway
a	average number of contacts of the non-subway users per unit of time
b	average number of contacts of the subway users per unit of time
β	transmission rate
$\frac{1}{\rho}$	is the average time spent in the subway
$\frac{\sigma}{\sigma+\rho}$	is the proportion of time spent off the subway for a subway user
$\frac{\rho}{\sigma+\rho}$	is the proportion of time spent in the subway for a subway user

Table 1: Proportionate contacts and their definitions

The rate of infection for non users is

$$B_i(t) = \beta_i a_i S_i \left[\tilde{P}_{a_i} \frac{I_i}{T_i(\frac{\sigma_i}{\rho_i+\sigma_i}) + Q_i} + \tilde{P}_{b_i} \frac{Y_i(\frac{\sigma_i}{\rho_i+\sigma_i})}{T_i(\frac{\sigma_i}{\rho_i+\sigma_i}) + Q_i} \right], \tag{11}$$

and the rate of infection for subway users is

$$V_i(t) = \beta_i b_i W_i \left[\bar{P}_{a_i} \frac{I_i}{T_i(\frac{\sigma_i}{\rho_i+\sigma_i}) + Q_i} + \bar{P}_{b_i} \frac{Y_i(\frac{\sigma_i}{\rho_i+\sigma_i})}{T_i(\frac{\sigma_i}{\rho_i+\sigma_i} + Q_i)} + \sum_{j=1}^{N} \bar{P}_{b_j} \frac{Y_j(\frac{\rho_j}{\rho_j+\sigma_j})}{T_j(\frac{\rho_j}{\rho_j+\sigma_j})} \right], \tag{12}$$

where

$$
\begin{aligned}
P_{a_i a_i} &= \tilde{P}_{a_i} = \frac{a_i Q_i}{a_i Q_i + b_i(\frac{\sigma_i}{\rho_i+\sigma_i})T_i}, \\
P_{a_i b_i} &= \tilde{P}_{b_i} = \frac{b_i(\frac{\sigma_i}{\rho_i+\sigma_i})T_i}{a_i Q_i + b_i(\frac{\sigma_i}{\rho_i+\sigma_i})T_i}, \\
P_{b_i a_i} &= \bar{P}_{a_i} = \frac{a_i Q_i}{a_i Q_i + b_i(\frac{\sigma_i}{\rho_i+\sigma_i})T_i}, \\
P_{b_i b_i} &= \bar{P}_{b_i} = \frac{b_i(\frac{\sigma_i}{\rho_i+\sigma_i})T_i}{a_i Q_i + b_i(\frac{\sigma_i}{\rho_i+\sigma_i})T_i}, \\
P_{b_i b_j} &= \bar{P}_{b_j} = \frac{b_j(\frac{\rho_j}{\rho_j+\sigma_j})T_j}{\sum_{k=1}^{N} b_k(\frac{\rho_k}{\rho_k+\sigma_k})T_k}.
\end{aligned}
$$

$P_{a_i a_i}$	the proportionate mixing between non-subway user from neighborhood i and non-subway users from neighborhood i
$P_{a_i b_i}$	the proportionate mixing between non-subway user from neighborhood i and subway users from neighborhood i
$P_{b_i a_i}$	the proportionate mixing between subway user from neighborhood i and non-subway users from neighborhood i
$P_{b_i b_i}$	the proportionate mixing between subway user from neighborhood i and subway users from neighborhood i
$P_{b_i b_j}$	the proportionate mixing between subway user from neighborhood i and subway users from neighborhood j
$P_{a_i a_j}$	the proportionate mixing between non-subway user from neighborhood i and non-subway users from neighborhood j
$P_{a_i b_j}$	the proportionate mixing between non-subway user from neighborhood i and subway users from neighborhood j

Table 2: Proportionate contacts and their definitions

Notice that
$P_{a_i a_j} = 0$ if i≠j (individuals Q_i and Q_j do not have contact if i≠j),
$P_{a_i b_j} = 0$ if i≠j (individuals Q_i and T_j do not have contact if i≠j),
$P_{a_i a_i} = \tilde{P}_{a_i} > 0$ (individuals Q_i and Q_i have contact),
$P_{a_i b_i} = \tilde{P}_{b_i} > 0$ (individuals Q_i and T_i have contact outside subway),
$P_{b_i a_i} = \bar{P}_{a_i} > 0$ (individuals T_i and Q_i have contact outside subway),
$P_{b_i b_i} = \bar{P}_{b_i} > 0$ (individuals T_i and T_i have contact outside subway),
$P_{b_i b_j} = \bar{P}_{b_j} > 0$ (individuals T_i and T_j have contact in the subway).

The following constraints apply
1) The sum of all conditional probabilities that individuals mix inside the subway is equal to 1

$$\sum_{j=1}^{N} \bar{P}_{b_j} = 1$$

2)The sum of conditional probabilities that individuals from Q_i have contacts with individuals from Q_i or individuals from T_i(outside the subway), in their neighborhood (i=1,2,...N) satisfy

$$\tilde{P}_{a_i} + \tilde{P}_{b_i} = 1$$

, $i = 1, 2, ..., n$
3)The sum of conditional probabilities that individuals from T_i have contacts with Q_i individuals or with T_i individuals off the subway, that is in their neighborhood satisfy

$$\bar{P}_{a_i} + \bar{P}_{b_i} = 1$$

, $i = 1, 2, ..., n$.

6 Calculation of R_0 for one neighborhood

In order to determine the central behavior of the epidemic, we linearize the system at the disease free equilibrium. R_0 the number of secondary infections produced by an infected individual, is compute following [5] and [6] for one neighborhood. R_0 is found using the eigenvalues of the matrix (λI - MD^{-1}) from our system of equations. The disease free equilibrium is:

$$\left(\frac{A - ql_1}{\mu}, 0, 0, \frac{ql_1}{\mu}, \frac{\Lambda - ql_1}{\mu}, 0, 0, \frac{ql_1}{\mu}\right),$$

where $\frac{1}{\mu}$ represent the average life span of the system and ql_1 represent the vaccine efficacy.

The Jacobian matrix evaluate at the disease free equilibrium is:

$$\begin{pmatrix}
-\mu & \frac{\mu q l_1}{A-ql_1} & -\frac{\beta a^2 A(A-ql_1)}{\mu\Phi} & 0 & 0 & 0 & -\frac{\beta a M^2}{\Phi} & 0 \\
0 & -\Gamma & \frac{\beta a^2 A(A-ql_1)}{\mu\Phi} & 0 & 0 & 0 & \frac{\beta a M^2}{\Phi} & 0 \\
0 & \phi & -\Gamma & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\mu q(l_2-l_1)}{A-ql_1} & \alpha & -\mu & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{\beta abA(\Lambda-ql_1)}{\Phi} & 0 & -\mu & \frac{\mu q l_1}{\Lambda-ql_1} & \left(-\frac{\beta b^2 M^2 \Lambda(\Lambda-ql_1)}{\mu\Phi}+\frac{\beta b\mu}{\Lambda}\right) & 0 \\
0 & 0 & \frac{\beta abA(\Lambda-ql_1)}{\Phi} & 0 & 0 & -\left(\mu+\phi+\frac{\mu q l_2}{\Lambda-ql_1}\right) & \left(\frac{\beta b^2 M^2 \Lambda(\Lambda-ql_1)}{\mu\Phi}+\frac{\beta b\mu}{\Lambda}\right) & 0 \\
0 & 0 & 0 & 0 & 0 & \phi & -\Gamma & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \alpha & -\mu
\end{pmatrix}$$

where

$$\begin{aligned}
M &= \frac{\sigma}{\rho+\sigma}, \\
\Phi &= (\Lambda M + A)(aA + bM\Lambda), \\
\Omega &= \left(\mu+\phi+\frac{\mu q l_2}{A-ql_1}\right), \\
\Gamma &= \mu+\alpha+d.
\end{aligned}$$

In this matrix we can see that we have four negative eigenvalue, hence the stability conditions must be computed from the 4 by 4 submatrix.

$$\begin{pmatrix}
-\Omega & \frac{\beta a^2 A(A-ql_1)}{\mu\Phi} & 0 & \frac{\beta a M^2}{\Phi} \\
\phi & -\Gamma & 0 & 0 \\
0 & \frac{\beta abA(\Lambda-ql_1)}{\Phi} & -\Omega & \left(\frac{\beta b^2 M^2 \Lambda(\Lambda-ql_1)}{\mu\Phi}+\frac{\beta b\mu}{\Lambda}\right) \\
0 & 0 & \phi & -\Gamma
\end{pmatrix}$$

In other to obtain R_0 we need to find the largest eigenvalue from the matrix (λI - MD^{-1}):

$$\begin{pmatrix}
\lambda & -\frac{\beta a^2 A(A-ql_1)}{\mu\Gamma\Phi} & 0 & -\frac{\beta a M^2}{\Gamma\Phi} \\
-\frac{\phi}{\Omega} & \lambda & 0 & 0 \\
0 & -\frac{\beta abA(\Lambda-ql_1)}{\Gamma\Phi} & \lambda & -\frac{1}{\Gamma}\left(\frac{\beta b^2 M^2 \Lambda(\Lambda-ql_1)}{\mu\Phi}+\frac{\beta b\mu}{\Lambda}\right) \\
0 & 0 & \frac{\phi}{\Omega} & \lambda
\end{pmatrix}$$

The characteristic equation for the determinant of (λI - MD^{-1}) is a double quartic polynomial of the form $\lambda^4 - G\lambda^2 + H$, where:

$$G = \frac{\Omega}{\Gamma}\left[\left(\frac{\beta^2 M^2 b^2 \Lambda(\Lambda - ql_1)}{\mu\Phi} + \frac{\beta b\mu}{\Lambda}\right) + \frac{\Omega\beta a^2 A(A - ql_1)}{\mu\Phi}\right]$$

$$H = \frac{\Omega}{\Gamma}\left[\left(\frac{\Omega\beta^2 b^2 M^2 \Lambda(\Lambda - ql_1)}{\mu\Phi} + \frac{\beta b\mu}{\Lambda}\right) + \left(\frac{(\beta abA(\Lambda - ql_1))(\beta^2 aM^2)}{\Gamma\Phi^2}\right)\right]$$

Solving this equation for λ positive we obtain:

$$\lambda = \left(\frac{1}{2}\left(G + \sqrt{G^2 - 4H}\right)\right)^{\frac{1}{2}}$$

Since this is the biggest eigenvalue we can define R_0:

$$R_0 = \left(\frac{1}{2}\left(G + \sqrt{G^2 - 4H}\right)\right)^{\frac{1}{2}} \tag{13}$$

7 R_0 For Two Neighborhoods

In order to analize the R_0 for two neighborhoods (calculations are in the appendix) we assume that $q = 0$, $\rho_i = 0$ and $\sigma_i = 0$. For this two neighborhoods R_0= $\max\{R_{01}, R_{02}\}$, where:

$$R_{0i} = \frac{\phi\beta_i[(a_i A_i)^2 + (b_i\Lambda_i)^2]}{(\mu + \phi)(\mu + \alpha + d)(a_i A_i + b_i\Lambda_i)(A_i + \Lambda_i)}$$

Hence R_0 can be rewritten as:

$$R_{0i} = \beta_i\left(\frac{\phi}{\mu + \phi}\right)\left(\frac{1}{\mu + \alpha + d}\right)\left[a_i\left(\frac{\frac{a_i A_i}{\mu}}{\frac{a_i A_i + b_i\Lambda_i}{\mu}}\right)\left(\frac{\frac{A_i}{\mu}}{\frac{A_i + \Lambda_i}{\mu}}\right) + b_i\left(\frac{\frac{b_i A_i}{\mu}}{\frac{a_i A_i + b_i\Lambda_i}{\mu}}\right)\left(\frac{\frac{\Lambda_i}{\mu}}{\frac{A_i + \Lambda_i}{\mu}}\right)\right]$$

where:

$\frac{\phi}{\mu+\phi}$: Proportion of individuals that goes from the exposed to the infected class.

$\frac{1}{\mu+\alpha+d}$: Average infectious period.

$\frac{\frac{A_i}{\mu}}{\frac{A_i+\Lambda_i}{\mu}}$: Proportion of non-subway users in neighborhood i.

$\frac{\frac{\Lambda_i}{\mu}}{\frac{A_i+\Lambda_i}{\mu}}$: Proportion of subway users in neighborhood i.

$\frac{\frac{a_i A_i}{\mu}}{\frac{a_i A_i+b_i \Lambda i}{\mu}}$: Proportion of contacts of non-subway users the total contacts in the neighborhood i.

$\frac{\frac{b_i \Lambda_i}{\mu}}{\frac{a_i A_i+b_i \Lambda i}{\mu}}$: Proportion of contacts of subway users to the total contacts in the neighborhood i.

R_0 has two sources, one from the non-subway users and the other from subway users.

8 Estimation of Parameters

The data was obtained from different organizations but mainly from the Center For Disease Control(CDC), and various websites which hold New York City Visitor Information.

Parameter	μ	d	l_1	l_2	ϕ	α	σ	ρ	β
Value	0.0065	0.3	0.97	0.97	0.083	0.7	19-22 hrs	2-5 hrs	0.7

Table 3: Estimation of Parameters

9 Simulations

9.1 Numerical Simulations

Since our model is an attempt to describe the spread of smallpox in a subway system from individuals of different neighborhoods in New York City, it is important to examine the behavior of the population as time progresses. A good approach to this is the use of computer simulations. In our project we use two different programs matlab, and Starlogo to write an agent based model. We found that the total population of NYC is 8 million. In mathlab, we rescaled this population to about 8,000. According to the data we decide to make the time scale in days, with a period of 30 days. We consider two neighborhoods, where each has a NSU and a SU population. The first neighborhood deals with the 8,000 residents of NYC and the second neighborhod has 1000 individuals, which represent the tourist of NYC 1 million per day. In our programs we assign the values of the different parameters. We found that the parameters

that affect the model most are q, which is the amount of people vaccinated, a which is average number of contacts of the non-subway users per unit of time and b, which is the average number of contacts of subway users per unit of time.

From Figure 3 we can se that with no vaccination, and low contact rates, smallpox is almost non-existant, especially for the NSU population. For both neighborhoods, the suceptibles almost constitute of the entire population. For the subway users, the exposed class increased and for the non-subway users it stayed almost constant. In the majority of the cases, the quantity of recovered individuals does not grow because there is no vaccination.

In the case where we have no vaccination, and higher contact rates (Figure 4), the number of susceptibles is low, and the number of exposed increases rapidly. The majority of the population becomes recovered, which is due to the fact that 30% of those who have died of smallpox. Thus, 70% survive, but depending on the population size, 30% could be a huge number, especially within a city of NYC.

Analizing the case were the population gets vaccinated, and where we have low contact rates (Figure 5), we found that the population is divided into susceptibles and recovered. The amount of exposed and infected, for the non-subway users, is considered low and constant. However, for the subway users, the majority of the population is exposed. For the first neighborhood, the number of susceptibles is very high, but eventually decreases after the twentieth day for the SU. This shows that the virus propagates more quickly and easily in the population of SU. Having vaccination for smallpox, decreases the chances of having an uncontrolled epidemic.

In Figure 6, vaccination and higher contact rates are given. From these results, we found that even if the contact rates are high, the vaccination is effective. For both neighborhoods, there exists a low number of people infected with smallpox, and a low susceptible population. Almost everybody becomes immune, and the recovered class grows dramatically fast, due to vaccination. Finally, the exposed class grows rapidly, but after a short period of time, it reaches a steady state.

In general, we can conclude that vaccination is effective, even if the contact rates are high. It seems that vaccination is the most effective solution, but these simulation do not illustrate wether mass vaccination is better or worst than ring vaccination. Ring vaccination versus mass vaccination are more deeply discussed in the agent bases-model simulations.

In our second part of the numerical simulation we let $q = 0$ which means that we don't have vaccination and, $\rho_1 = \rho_2$, $\sigma_1 = \sigma_2$, $\beta_1 = \beta_2$, for R_0 of two neighborhoods.If $b_i > a_i$ (figure 7), this is if the contact rate of the subway users is greater than the contact rate of non-subway users, the basic reproductive number $R_0(0)$ when $\rho = 0$ is greater than $R_0(\rho)$ when $\rho \neq 0$. In this case the terrorist have a big impact on the spread of smallpox. On the other hand if $b_i < a_i$ the terrorist have less activity (figure 8), then the basic reproductive number $R_0(0)$ when $\rho = 0$ is less than $R_0(\rho)$ when $\rho \neq 0$.

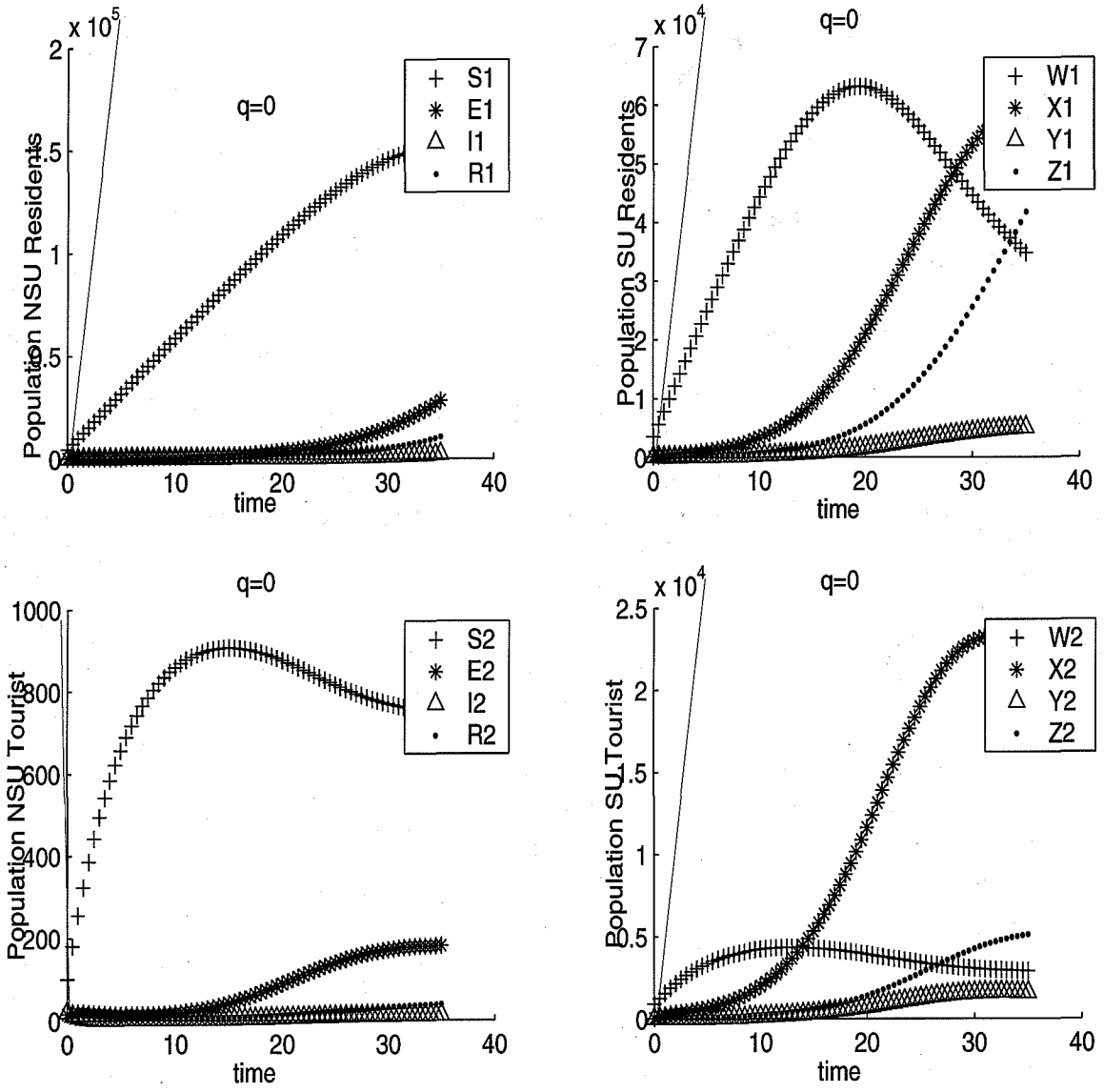

Figure 3: Parameters Varying $q = 0, a_1 = 5, a_2 = 3, b_1 = 7, b_2 = 2$

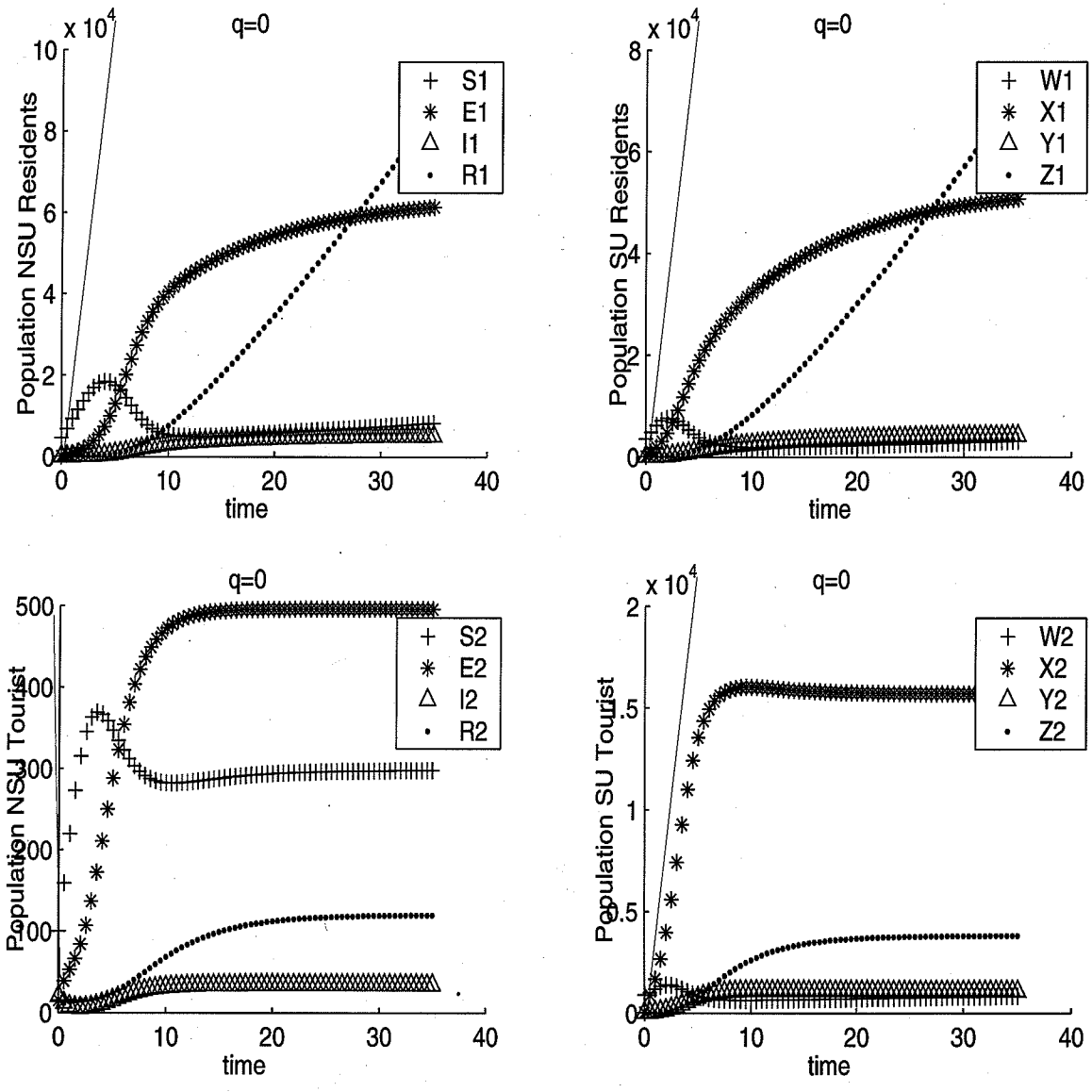

Figure 4: Parameters Varying $q = 0, a_1 = 30, a_2 = 20, b_1 = 40, b_2 = 20$

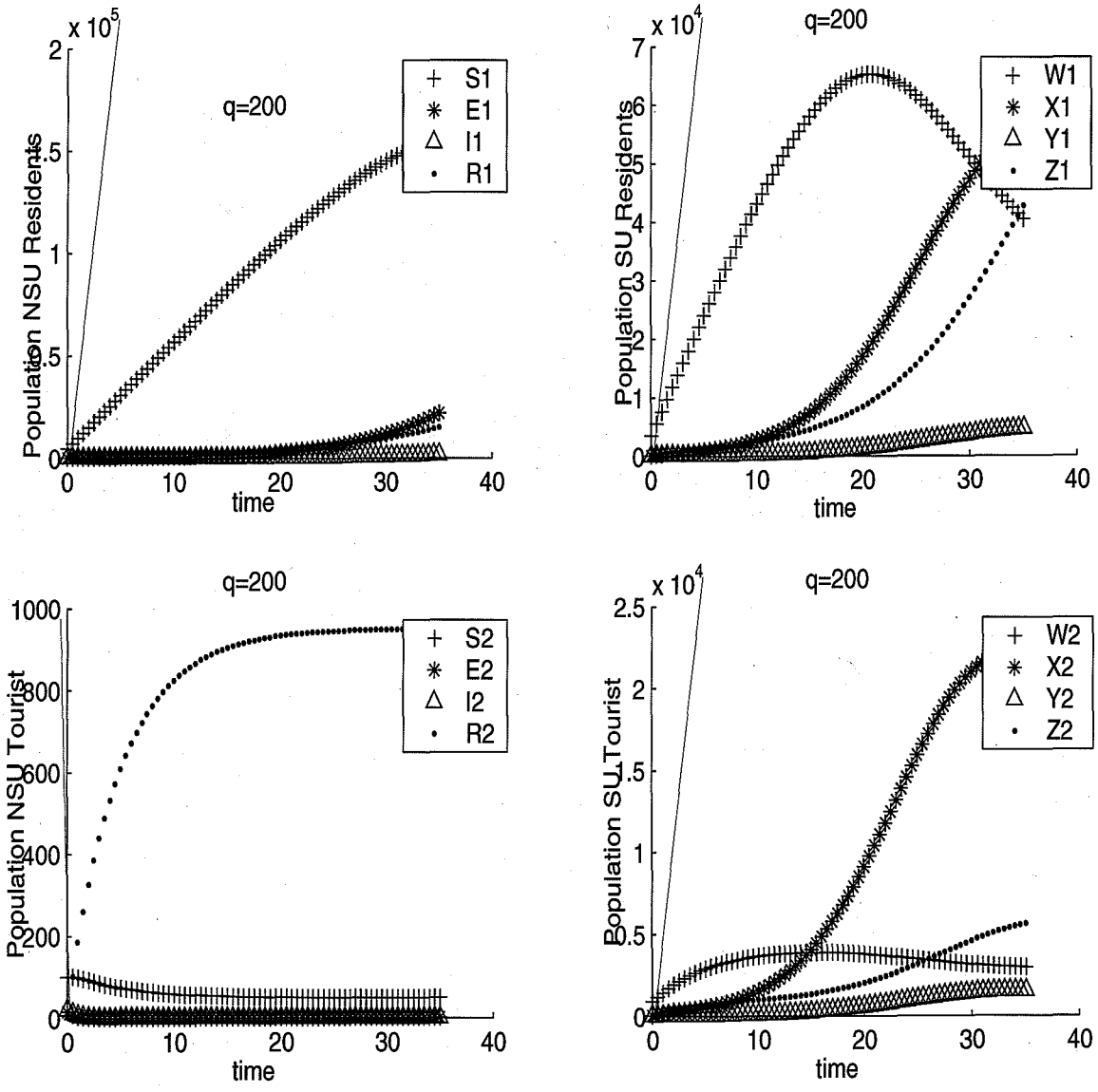

Figure 5: Parameters Varying $q = 200, a_1 = 5, a_2 = 3, b_1 = 7, b_2 = 2$

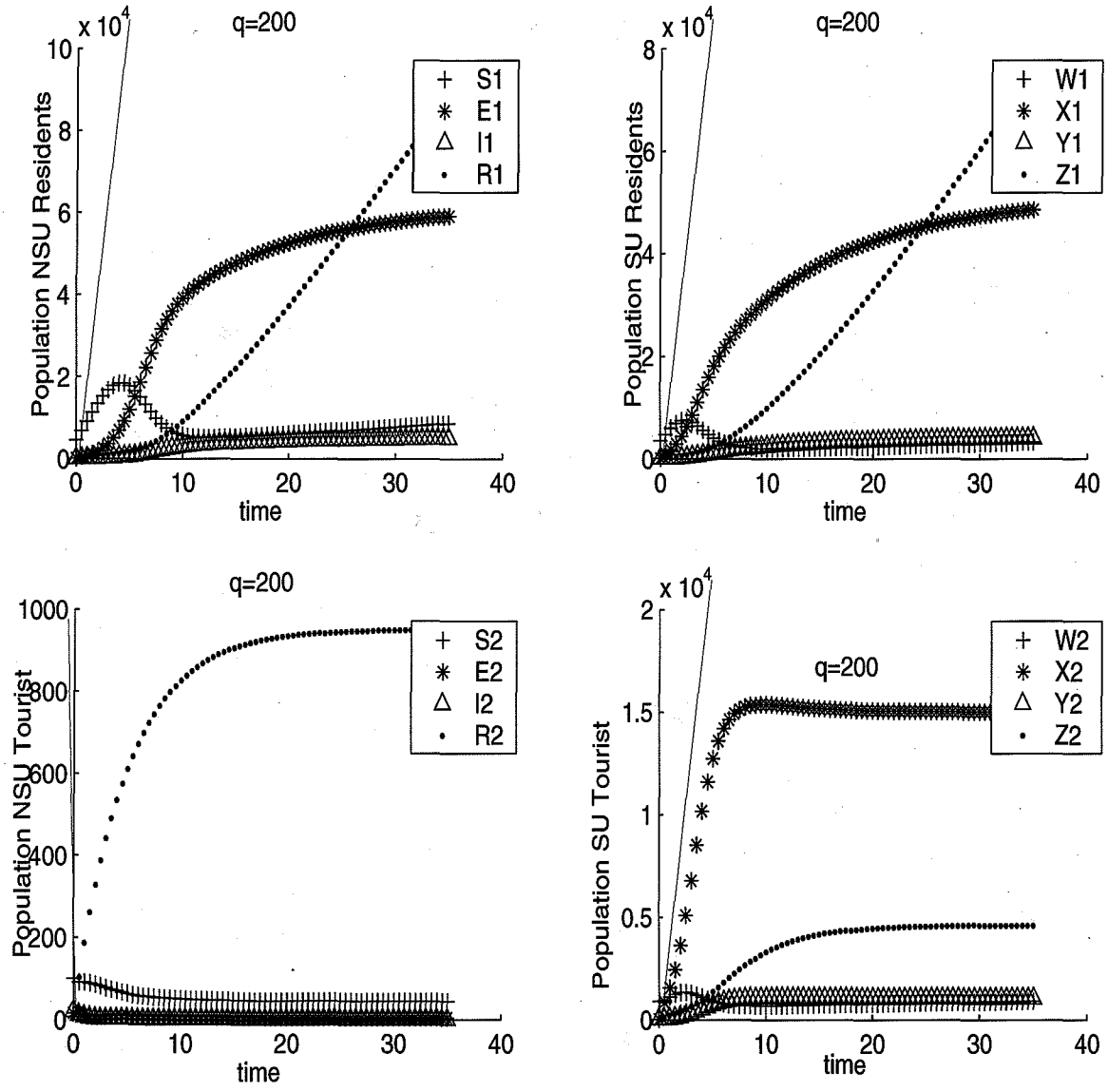

Figure 6: Parameters Varying $q = 200, a_1 = 30, a_2 = 20, b_1 = 40, b_2 = 20$

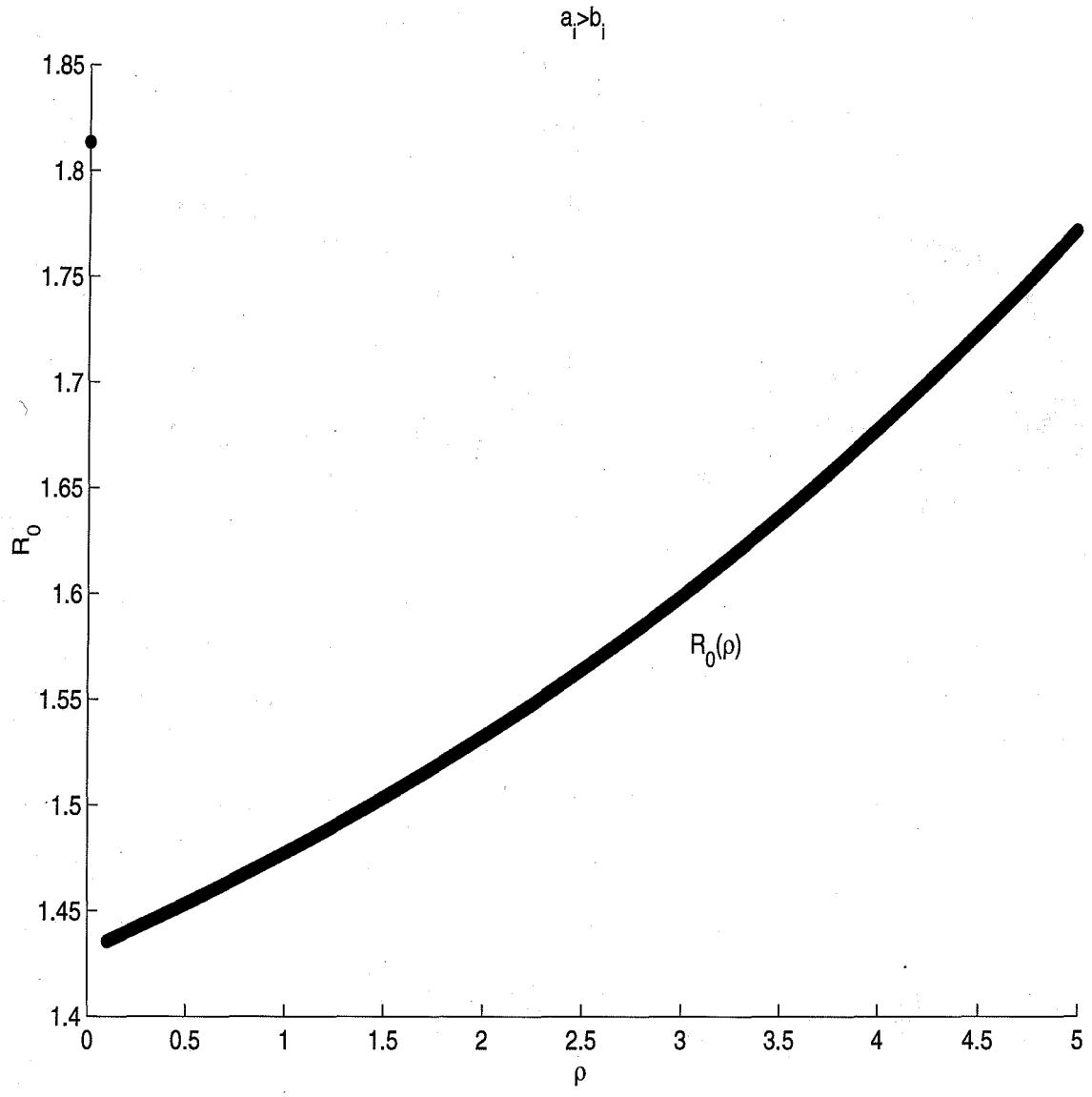

Figure 7: Graphic of R_0 in function of ρ when $a_i > b_i$

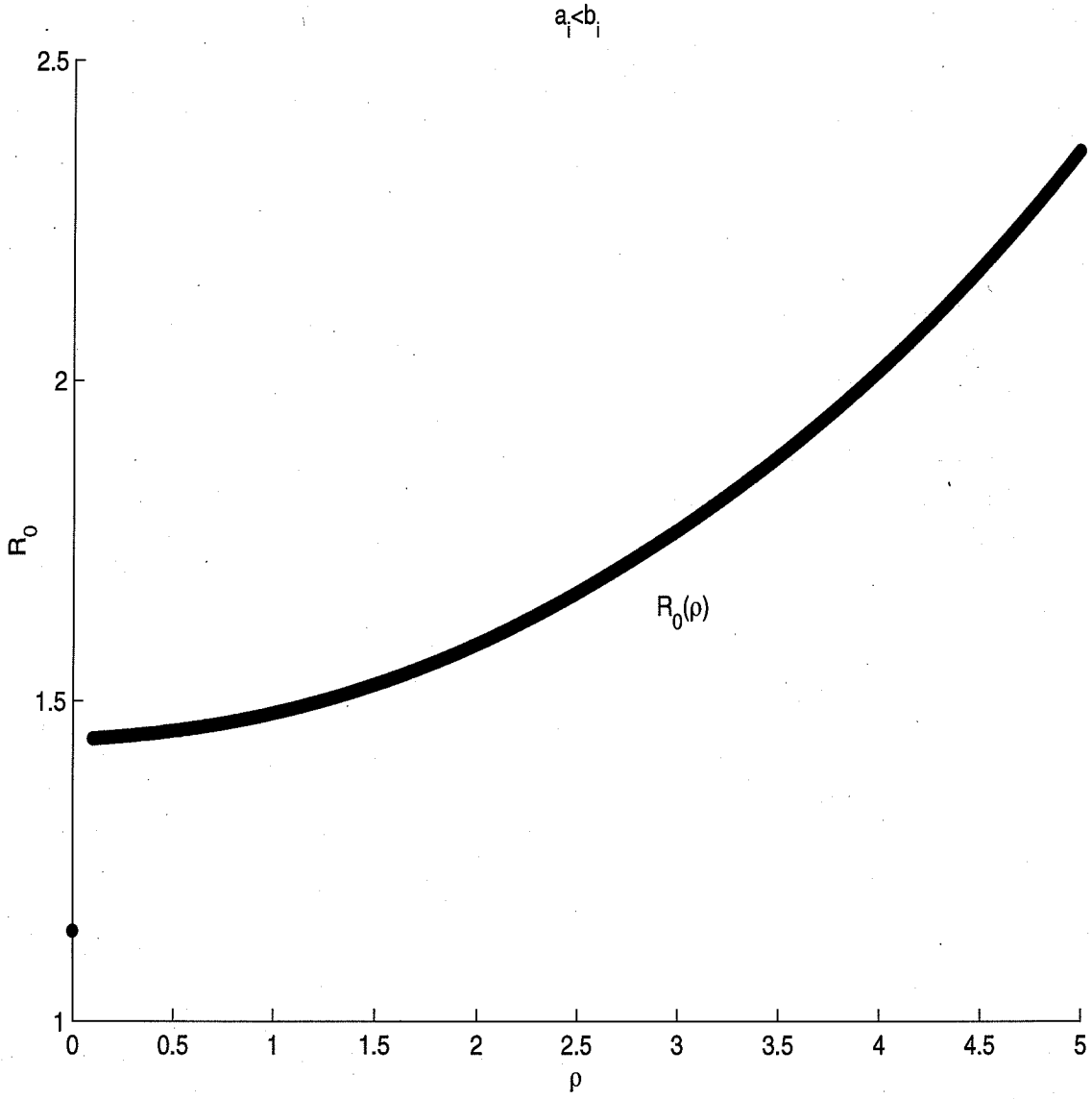

Figure 8: Graphic of R_0 in function of ρ when $a_i < b_i$

9.2 Agent Based Model

An agent-based model was used to compare mass vaccination vs. ring vaccination, and to determine the effects of transportation in a bio-terrorist attack. In the case of a deliberate release of smallpox, the StarLogo software was used to write two codes to model the behavior of subway and non-subway users. The bio-terrorist attack consists of introducing 50 exposed individuals into a total population of 4000 individuals. We are interested in obtainig results that either verify or question Kaplan's conclusion that mass vaccination should be an alternative to the current ring vaccination policy. Both vaccination methods were implemented when a predetermined number of infected cases has been reported. The values assumed for this number are 50, 100 and 400.

In the case of two neighborhoods (fig.9), 90% of the population were assumed to be subway users in one neighborhood, and 44% in the other, representing the populations of tourists and locals respectively. Mass vaccination was found to be a better method than ring vaccination in the sense that it gets rid of the epidemic faster. Results suggests that if the attack is detected promptly, ring vaccination is a better method than mass vaccination, otherwise ring vaccination not only takes longer to control the epidemic, but also results in more fatalities. These apparently contradicting results question Kaplan's conclusion, since under different conditions and assumptions, both vaccination methods work effectively.

In the case of four neighborhoods (fig.10, fig.11), we consider the percentages for subway users per neighborhood to be 90%, 10%, 44%, and 70%. Again, mass vaccination and ring vaccination are introduced as control measures after a predetermined value of infected cases has been reported.

Similar to the two neighborhood case, both vaccination methods work well when the epidemic is detected early. We observe that the results are optimal when mass vaccination is implemented after 50 infected cases have been reported, yet the number of fatalities was higher than when ring vaccination was implemented. We attribute this to the fact that the vaccine is assumed to have a 1% chance of killing the individuals by its side-effects. It is not clear whether mass vaccination works better than ring vaccination. This suggests that more detailed study is necessary before concluding what is the best vaccination method to be used. If the epidemic is not detected early, the casualties double when ring vaccination is used instead of mass vaccination. Hence, in this case, mass vaccination is indeed a better method.

The percentage of subway users per neighborhood affects the amount of infection that takes place in each neighborhood. In general, in the case of four neighborhoods, we found the distribution of fatalities per neighborhood to be directly proportional to the percentage of SU per neighborhood (fig.11). This implies that transportation plays a major role in the spread of smallpox.

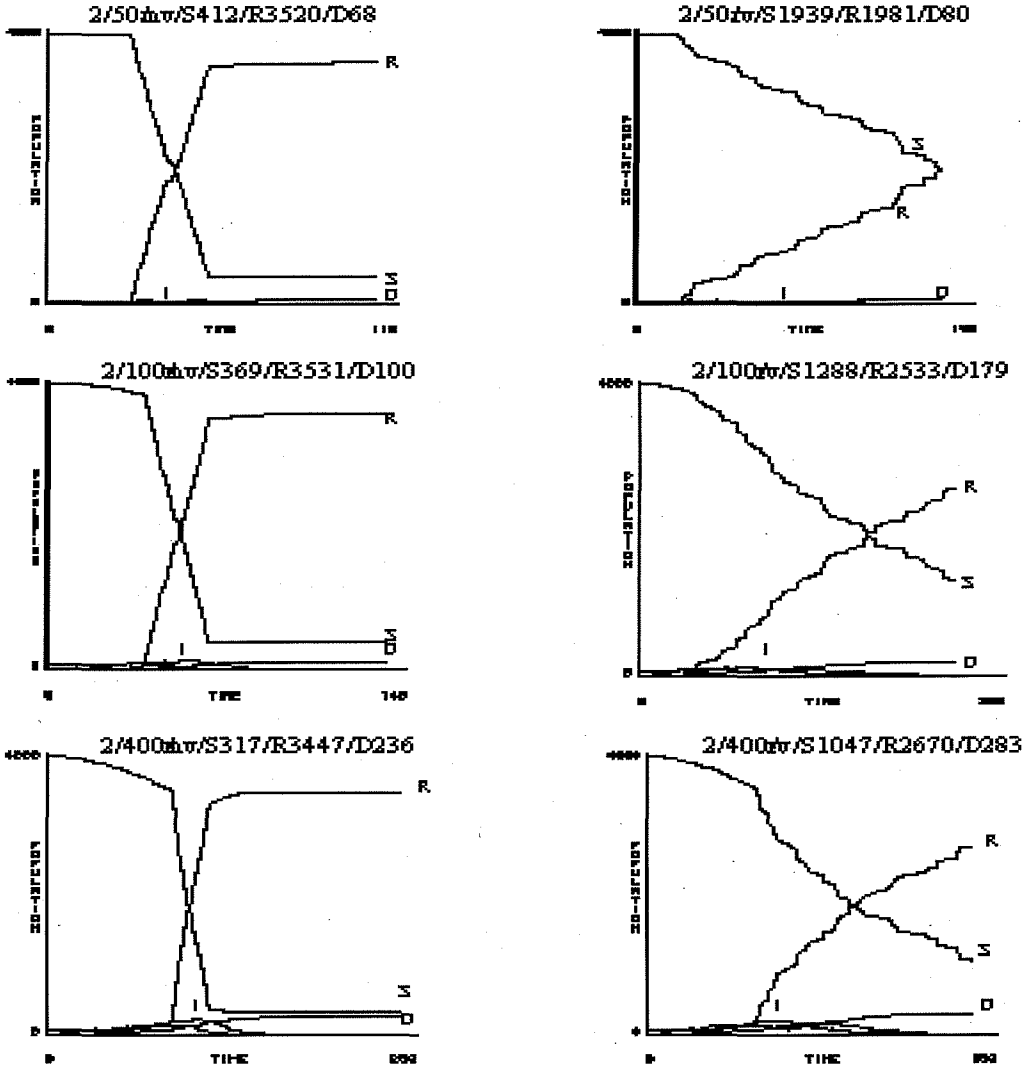

Figure 9: Graphs for 2-neighborhoods varying the population size to start mass vaccination or ring vaccination

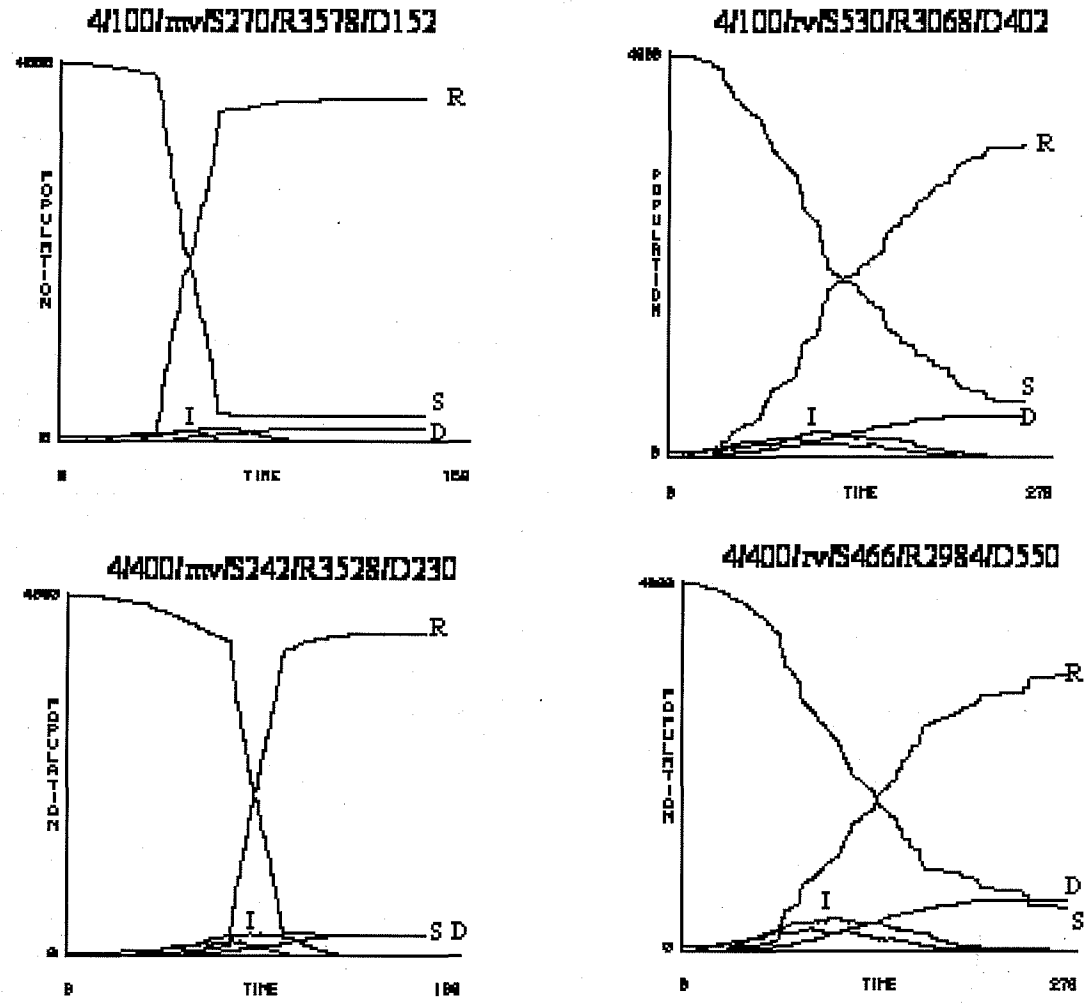

Figure 10: Graphs for 4-neighborhoods varying the population size to start mass vaccination or ring vaccination

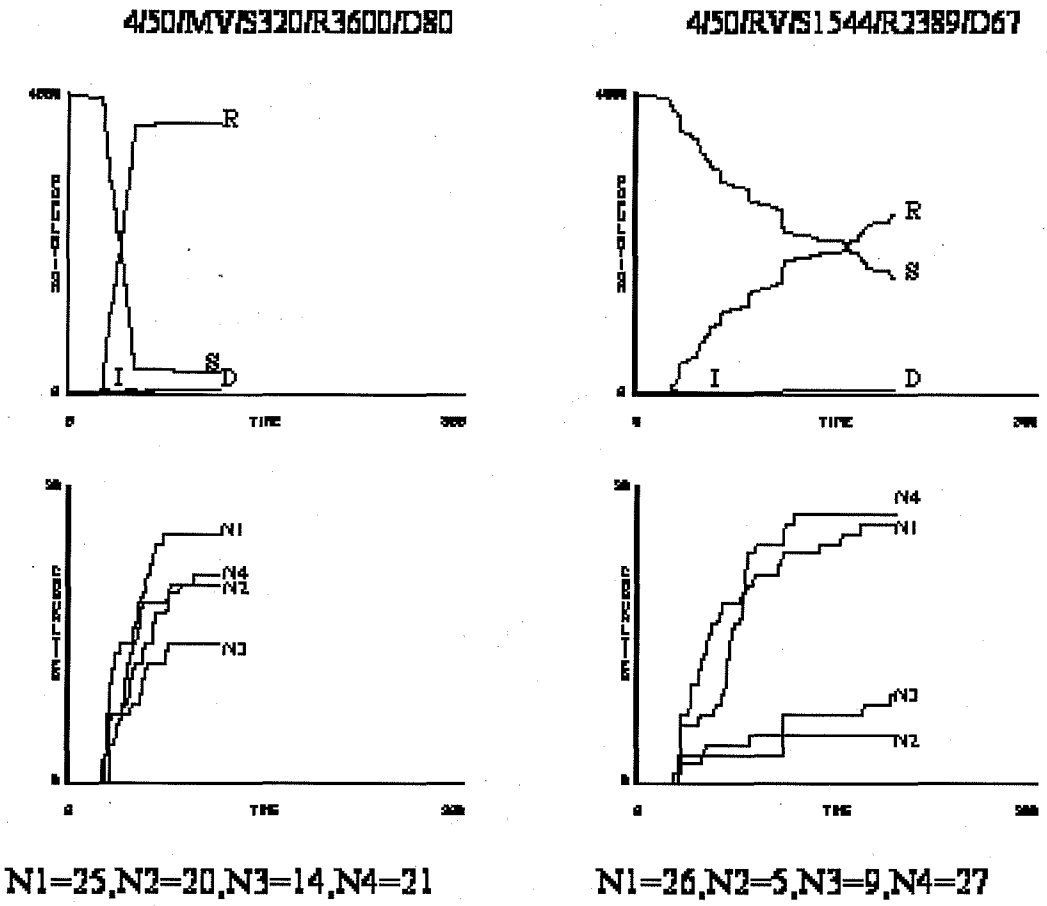

Figure 11: Graphs for 4-neighborhoods with population size to start vaccination is 50. Fatalities divided per neighborhood

10 Results and Discussion

The results obtained from this paper, do not answer the initials questions completely, but do give an insight, and create more open questions. For instance, it is true that vaccination has a major impact on the spread of the virus, but which is the adequate vaccination strategy is still an open question. Ed Kaplan suggested that for the worst case scenario, mass vaccination is the most effective strategy. However, the agent based-model created for this problem, offers different results. For example, it is not so obvious that mass vaccination is more effective, and in some instances, ring vaccination seemed to be more effective. It all depends on different factors, such as the time that it takes to detect the attack. The reaction time is important because the faster the attack is detected, the faster people get vaccinated.

In the simulations, the time it takes to start vaccinating, depends on recognizing a certain number of infected individuals. The question becomes a different, and we ask, how many people need to be infected to announce an outbreak? Also, the number of individuals infected at time zero is very important. The amount of people that get vaccinated each day, can also have a different effect. The number of contacts for a NSU and a SU can make a huge difference because that number increases the chances of infection, and later, the chances of infecting susceptible individuals. Another major difference is the amount of time spent in the subway and outside the subway. The population of SU are infected more rapidly, and in this group, the epidemic spreads more quickly than that of the NSU. Depending on the different initial values, on the different parameters, and on the vaccination strategy, results vary. The problem is more complex, and depending on the different circumstances, either mass or ring vaccination can be effective.

There are some differences between Kaplan, Craft and Wein's model and in this model, but that only shows that their result is incomplete. His assumption about the worst case scenario is not necessarely the worst because he did not incorporate many possible situations. There are many factors that have some impact on the final results, and in this case, transportation is a very important factor, because in the subway, individuals have a major potential of infection. There are many other circumstances that are not taken into consideration by them, and as a result, their conclusions are somewhat handicapped. Kaplan, Craft and Wein's results are not incorrect, but there is a lot more work that can be done to ameliorate the quality of his results, which in turn will give a more holistic view in case of smallpox being deliberatly released.

11 Future Work

There are still many open questions that require more research. We need to provide a better interpretation of the R_0 for multiple neighborhoods. The next question will be

to try to find if there are multiple endemic equilibria, and analized their stability. It is possible to study the case of multiple neighborhoods via numerical simulations. The study of bifurcations is another futher consideration. Another good aproach will be to create a different transportation model that allows more space for flexibility; more subways, and allowing more interactions with the different neighborhoods. Finally, not many biological problems deal with systems of this type using proportionate mixing and futher studies related with the topic should be considered.

12 Acknowledgements

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13 Appendix

13.1 Calculation of R_0 for two neighborhoods

We compulete R_0 following the procedure suggested by Dieckman et al. 1990 [5] and Heesterbeek 1992 [6] only for two neighborhoods

To reduce the calculation, the total take-subway population size is $T_i(t) = W_i(t) + X_i(t) + Y_i(t) + Z_i(t)$ which is governed by the differential equation $\frac{dT_i}{dt} = \Lambda_i - \mu T_i - dY_i$. The total non-take subway population size is $Q_i(t) = S_i(t) + E_i(t) + I_i(t) + R_i(t)$ which is governed by the differential equation $\frac{dQ_i}{dt} = A_i - \mu Q_i - dI_i$. We use T_i and Q_i as variables in place of variables W_i and S_i, respectively. Then, the model becomes the following system.

Equations for subway users

$$\begin{aligned}
\frac{dX_i}{dt} &= V_i(t) - (\mu+\phi)X_i - \frac{ql_2X_i}{T_i-Y_i-Z_i} \\
\frac{dY_i}{dt} &= \phi X_i - (\mu+\phi+d)Y_i \\
\frac{dZ_i}{dt} &= \alpha Y_i - \mu Z_i + ql_1 + \frac{q(l_2-l_1)X_i}{T_i-Y_i-Z_i} \\
\frac{dT_i}{dt} &= \Lambda_i - \mu T_i - dY_i
\end{aligned}$$

Equations for non-subway users

$$\begin{aligned}
\frac{dE_i}{dt} &= B_i(t) - (\mu+\phi)E_i - \frac{ql_2E_i}{Q_i-E_i-R_i} \\
\frac{dI_i}{dt} &= \phi E_i - (\mu+\phi+d)I_i \\
\frac{dR_i}{dt} &= \alpha I_i - \mu R_i + ql_1 + \frac{q(l_2-l_1)E_i}{Q_i-I_i-R_i} \\
\frac{dQ_i}{dt} &= A_i - \mu Q_i - dI_i
\end{aligned}$$

where

$$V_i(t) = \beta_i b_i (T_i - X_i - Y_i - Z_i)\left(\frac{a_iQ_iI_i + b_i\delta_i^2T_iY_i}{G_i} + \frac{\sum_{i=1}^{2} b_i\theta_iY_i}{H}\right),$$

$$B_i(t) = \beta_i a_i (Q_i - E_i - I_i - R_i)\left(\frac{a_iQ_iI_i + b_i\delta_i^2T_iY_i}{G_i}\right),$$

and $\delta_i = \frac{\sigma_i}{\sigma_i+\rho_i}$, $\theta_i = \frac{\rho_i}{\sigma_i+\rho_i}$, $G_i = (a_iQ_i + b_i\delta_iT_i)(Q_i + \delta_iT_i)$, $H = \sum_{i=1}^{2} b_i\theta_iT_i$.

The disease-free equilibrium (DFE) of our model is $P_0 = (X_{i0}, Y_{i0}, Z_{i0}, E_{i0}, I_{i0}, R_{i0}, T_{i0}, Q_{i0})$, where

$X_{i0} = Y_{i0} = 0$, $Z_{i0} = \frac{ql_i}{\mu}$, $T_{i0} = \frac{\Lambda_i}{\mu}$,

$E_{i0} = I_{i0} = 0$, $R_{i0} = \frac{ql_i}{\mu}$, $Q_{i0} = \frac{A_i}{\mu}$.

The linearization matrix at DFE is

$$\begin{pmatrix}
-a_{11} & a_{12} & 0 & a_{15} & 0 & a_{18} & 0 & 0 \\
\phi & -(\mu+\alpha+d) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_{42} & -a_{44} & a_{45} & 0 & 0 & 0 & 0 \\
0 & 0 & \phi & -(\mu+\alpha+d) & 0 & 0 & 0 & 0 \\
0 & a_{72} & 0 & 0 & -a_{77} & a_{78} & 0 & a_{7\times11} \\
0 & 0 & 0 & 0 & \phi & -(\mu+\alpha+d) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a_{10\times8} & -a_{10\times10} & a_{10\times11} \\
0 & 0 & 0 & 0 & 0 & 0 & \phi & -(\mu+\alpha+d)
\end{pmatrix},$$

where

$$a_{11} = \mu + \phi + \frac{ql_2\mu}{F_1}\ ,\ a_{44} = \mu + \phi + \frac{ql_2\mu}{C_1}\ ,$$

$$a_{77} = \mu + \phi + \frac{ql_2\mu}{F_2}\ ,\ a_{10\times10} = \mu + \phi + \frac{ql_2\mu}{C_2}\ ,$$

$$a_{12} = \beta_1 b_1 F_1\left(\frac{b_1\delta_1^2\Lambda_1}{\bar{G}_1} + \frac{b_1\theta_1}{\bar{H}}\right),\ a_{15} = \beta_1 b_1 F_1\left(\frac{a_1 A_1}{\bar{G}_1}\right),$$

$$a_{18} = \beta_1 b_1 F_1\left(\frac{b_2\theta_2}{\bar{H}}\right),\ a_{42} = \beta_1 a_1 C_1\left(\frac{b_1\delta_1^2\Lambda_1}{\bar{G}_1}\right),$$

$$a_{45} = \beta_1 a_1 C_1\left(\frac{a_1 A_1}{\bar{G}_1}\right),\ a_{78} = \beta_2 b_2 F_2\left(\frac{b_2\delta_2^2\Lambda_2}{\bar{G}_2} + \frac{b_2\theta_2}{\bar{H}}\right),$$

$$a_{7\times 11} = \beta_2 b_2 F_2\left(\frac{a_2 A_2}{\bar{G}_2}\right),\ a_{72} = \beta_2 b_2 F_2\left(\frac{b_1\theta_1}{\bar{H}}\right),$$

$$a_{10\times 8} = \beta_2 a_2 C_2\left(\frac{b_2\delta_2^2\Lambda_2}{\bar{G}_2}\right),\ a_{10\times 11} = \beta_2 a_2 C_2\left(\frac{a_2 A_2}{\bar{G}_2}\right),$$

with

$$F_i = \Lambda_i - ql_1\ , C_i = A_i - ql_1,\ \bar{G}_i = (a_i A_i + b_i\delta_i\Lambda_i)(A_i + \delta_i\Lambda_i)\ ,\ \bar{H} = \sum_{i=1}^{2} b_i\theta_i\Lambda_i.$$

From Dieckman et.al. 1990 and Heesterbeek 1992, the characteristic polynomail of the matrix MD^{-1} is

$$f(\lambda) = |MD^{-1} - \lambda I| = \begin{vmatrix} -\lambda & b_{12} & 0 & b_{14} & 0 & b_{16} & 0 & 0 \\ b_{21} & -\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_{32} & -\lambda & b_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{43} & -\lambda & 0 & 0 & 0 & 0 \\ 0 & b_{52} & 0 & 0 & -\lambda & b_{56} & 0 & a_{58} \\ 0 & 0 & 0 & 0 & b_{65} & -\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{76} & -\lambda & b_{78} \\ 0 & 0 & 0 & 0 & 0 & 0 & b_{87} & -\lambda \end{vmatrix},$$

where $b_{21} = \frac{\phi}{a_{11}}$, $b_{43} = \frac{\phi}{a_{44}}$, $b_{65} = \frac{\phi}{a_{77}}$, $b_{87} = \frac{\phi}{a_{10\times 10}}$, $b_{12} = \frac{a_{12}}{\alpha+\mu+d}$, $b_{32} = \frac{a_{42}}{\alpha+\mu+d}$, $b_{52} = \frac{a_{72}}{\alpha+\mu+d}$, $b_{14} = \frac{a_{15}}{\alpha+\mu+d}$, $b_{34} = \frac{a_{45}}{\alpha+\mu+d}$, $b_{16} = \frac{a_{18}}{\alpha+\mu+d}$, $b_{56} = \frac{a_{78}}{\alpha+\mu+d}$, $b_{76} = \frac{a_{10\times 8}}{\alpha+\mu+d}$, $b_{58} = \frac{a_{7\times 11}}{\alpha+\mu+d}$, $b_{78} = \frac{a_{10\times 11}}{\alpha+\mu+d}$.
Hence,

$$\begin{aligned} f(\lambda) = & \Big((\lambda^2 - b_{12}b_{21})(\lambda^2 - b_{34}b_{43}) - b_{14}b_{21}b_{32}b_{43}\Big)\Big((\lambda^2 - b_{56}b_{65})(\lambda^2 - b_{78}b_{87}) - b_{58}b_{65}b_{76}b_{87}\Big) \\ & -b_{16}b_{21}b_{52}b_{65}(\lambda^2 - b_{34}b_{43})(\lambda^2 - b_{78}b_{87}) \\ = & \ \lambda^8 + e_2\lambda^6 + e_1\lambda^4 + e_0\lambda^2 + e \end{aligned}$$

where

$$
\begin{aligned}
e_2 &= -(b_{12}b_{21} + b_{34}b_{43} + b_{56}b_{65} + b_{78}b_{87})\,, \\
e_1 &= b_{12}b_{21}b_{34}b_{43} + (b_{12}b_{21} + b_{34}b_{43})(b_{56}b_{65} + b_{78}b_{87}) + b_{56}b_{65}b_{78}b_{87} \\
&\quad -b_{58}b_{65}b_{76}b_{87} - b_{14}b_{21}b_{32}b_{43} - b_{16}b_{21}b_{52}b_{65}\,, \\
e_0 &= -b_{21}b_{43}(b_{12}b_{34} - b_{14}b_{32})(b_{56}b_{65} + b_{78}b_{87}) \\
&\quad -b_{65}b_{87}(b_{56}b_{78} - b_{58}b_{76})(b_{12}b_{21} + b_{34}b_{43}) + b_{16}b_{21}b_{52}b_{65}(b_{34}b_{43} + b_{78}b_{87})\,, \\
e &= b_{21}b_{43}(b_{12}b_{34} - b_{14}b_{32})b_{65}b_{87}(b_{56}b_{78} - b_{58}b_{76}) - b_{16}b_{21}b_{34}b_{43}b_{52}b_{65}b_{78}b_{87}.
\end{aligned}
$$

Noting that

$$
\begin{aligned}
b_{12}b_{34} - b_{14}b_{32} &= \tfrac{a_{12}}{\mu+\alpha+d}\tfrac{a_{34}}{\mu+\alpha+d} - \tfrac{a_{15}}{\mu+\alpha+d}\tfrac{a_{42}}{\mu+\alpha+d} \\
&= \tfrac{a_{45}}{(\mu+\alpha+d)^2}\beta_1 b_1 F_1 \tfrac{b_1\theta_1}{H} = b_{34}\tfrac{\beta_1 b_1 F_1 b_1\theta_1}{H(\mu+\alpha+d)},
\end{aligned}
$$

and

$$
\begin{aligned}
b_{56}b_{78} - b_{58}b_{76} &= \tfrac{a_{78}}{\mu+\alpha+d}\tfrac{a_{10\times 11}}{\mu+\alpha+d} - \tfrac{a_{7\times 11}}{\mu+\alpha+d}\tfrac{a_{10\times 8}}{\mu+\alpha+d} \\
&= \tfrac{a_{10\times 11}}{(\mu+\alpha+d)^2}\beta_2 b_2 F_2 \tfrac{b_2\theta_2}{H} = b_{78}\tfrac{\beta_2 b_2 F_2 b_2\theta_2}{H(\mu+\alpha+d)} \\
b_{16}b_{52} &= \tfrac{a_{18}a_{72}}{(\mu+\alpha+d)^2} = \tfrac{1}{(\mu+\alpha+d)^2}\tfrac{\beta_1 b_1 F_1 b_2\theta_2}{H}\tfrac{\beta_2 b_2 F_2 b_1\theta_1}{H}
\end{aligned}
$$

We known $e_0 < 0$ and $e = 0$. In fact,

$$
\begin{aligned}
e_0 &= -b_{21}b_{34}b_{43}(b_{56}b_{65} + b_{78}b_{87})\tfrac{\beta_1 b_1^2 F_1\theta_1}{H(\mu+\alpha+d)} \\
&\quad -b_{65}b_{78}b_{87}(b_{12}b_{21} + b_{34}b_{43})\tfrac{\beta_2 b_2^2 F_2\theta_2}{H(\mu+\alpha+d)} \\
&\quad +b_{16}b_{21}b_{52}b_{65}(b_{34}b_{43} + b_{78}b_{87}) \\
&= -b_{21}b_{34}b_{43}b_{65}\left(b_{56}\tfrac{\beta_1 b_1^2 F_1\theta_1}{H(\mu+\alpha+d)} - b_{16}b_{52}\right) \\
&\quad -b_{12}b_{65}b_{78}b_{87}\left(b_{12}\tfrac{\beta_2 b_2^2 F_2\theta_2}{H(\mu+\alpha+d)} - b_{16}b_{52}\right) \\
&\quad -b_{34}b_{43}b_{78}b_{87}\left(b_{21}\tfrac{\beta_1 b_1^2 F_1\theta_1}{H(\mu+\alpha+d)} + b_{65}\tfrac{\beta_2 b_2^2 F_2\theta_2}{H(\mu+\alpha+d)}\right) \\
&= -b_{21}b_{65}\tfrac{(\beta_1 b_1 F_1)(\beta_2 b_2 F_2)b_1 b_2}{D_2 H(\mu+\alpha+d)^2}(b_{34}b_{43}\theta_1\delta_2^2\Lambda_2 + b_{78}b_{87}\theta_2\delta_1^2\Lambda_1) \\
&\quad -b_{34}b_{43}b_{78}b_{87}\left(b_{21}\tfrac{\beta_1 b_1^2 F_1\theta_1}{H(\mu+\alpha+d)} + b_{65}\tfrac{\beta_2 b_2^2 F_2\theta_2}{H(\mu+\alpha+d)}\right)
\end{aligned}
$$

Then the characteristic polynomail $f(\lambda)$ of the matrix MD^{-1} becomes

$$f(\lambda) = \lambda^2(\lambda^6 + e_2\lambda^4 + e_1\lambda^2 + e_0) \triangleq xg(x),$$

where, $x = \lambda^2$ and $g(x) = (x^3 + e_2x^2 + e_1x + e_0)$

$$\begin{aligned} x_1 &= -\tfrac{1}{3}e_2 + (S+T), \\ x_2 &= -\tfrac{1}{3}e_2 - \tfrac{1}{2}(S+T) + \tfrac{\sqrt{3}}{2}i(S-T), \\ x_2 &= -\tfrac{1}{3}e_2 - \tfrac{1}{2}(S+T) - \tfrac{\sqrt{3}}{2}i(S-T), \end{aligned}$$

where $S = (R+\sqrt{D})^{\frac{1}{3}}$, $T = (R-\sqrt{D})^{\frac{1}{3}}$, $D = Q^3 + R^2$. Q and R are given by

$$Q = \frac{1}{9}(3e_1 - e_2^2),\ R = \frac{1}{54}(9e_1e_2 - 27e_0 - 2e_2^3).$$

Because $g(0) = e_0 < 0$ and $g(+\infty) = +\infty$, it follows from mean value Theorem, that, there exists at least one $x^* > 0$ such that $f(x^*) = 0$. We have to determine the largest of x_1, x_2 and x_3

If the discriminant $D > 0$, the first root x_1 is a positive real root and other two roots (x_2 and x_3) are complex conjugates. Then the basic reproduction number R_0 is equal to $\sqrt{x_1}$. If $D \leq 0$, all roots are real (when $D = 0$, at least two roots are equal), define

$$\theta = \cos^{-1}\Big(\frac{R}{\sqrt{-Q^3}}\Big),\ 0 \leq \theta \leq \pi$$

In terms of θ, S and T can be rewritte as $S = \sqrt{-Q}(\cos\frac{\theta}{3} - i\sin\frac{\theta}{3})$, $T = \sqrt{-Q}(\cos\frac{\theta}{3} + i\sin\frac{\theta}{3})$ Hence,

$$\begin{aligned} x_1 &= -\tfrac{1}{3}e_2 + (S+T) = -\tfrac{1}{3}e_2 + 2\sqrt{-Q}\cos\tfrac{\theta}{3}, \\ x_2 &= -\tfrac{1}{3}e_2 - \tfrac{1}{2}(S+T) + \tfrac{\sqrt{3}}{2}i(S-T) = -\tfrac{1}{3}e_2 + 2\sqrt{-Q}\cos\Big(\tfrac{\theta+2\pi}{3}\Big), \\ x_3 &= -\tfrac{1}{3}e_2 - \tfrac{1}{2}(S+T) - \tfrac{\sqrt{3}}{2}i(S-T) = -\tfrac{1}{3}e_2 + 2\sqrt{-Q}\cos\Big(\tfrac{\theta+4\pi}{3}\Big), \end{aligned}$$

$$x_1 - x_2 = 2\sqrt{-Q}\Big(\cos\frac{\theta}{3} - \cos\frac{\theta+2\pi}{3}\Big) = 4\sqrt{-Q}\sin\frac{\theta+\pi}{3}\sin\frac{\pi}{3} = 2\sqrt{-3Q}\sin\frac{\theta+\pi}{3},$$

$$x_1 - x_3 = 2\sqrt{-Q}\Big(\cos\frac{\theta}{3} - \cos\frac{\theta+4\pi}{3}\Big) = 4\sqrt{-Q}\sin\frac{\theta+2\pi}{3}\sin\frac{2\pi}{3} = 2\sqrt{-3Q}\sin\frac{\theta+2\pi}{3},$$

Noting that $0 \leq \theta \leq \pi$, then $0 \leq \frac{\theta+\pi}{3} \leq \frac{2\pi}{3}$ and $0 \leq \frac{\theta+2\pi}{3} \leq \pi$, then $x_1 - x_2 \geq 0$ and $x_1 - x_3 \geq 0$. In both cases ($D \geq 0$ and $D < 0$), we conclude that x_1 is the largest. Thus the basic reproduction number R_0 is $\sqrt{x_1}$, that is

$$R_0 = \sqrt{-\frac{1}{3}e_2 + (S+T)}\,.$$

13.2 First Matlab Programs for the Simulations

```
tspan=[0:.5:35];
x0=[4500 40 50 10 3500 50 60 10 100 15 20 7 900 30 40 5]';

d=.3;
q=210 ;
L1=.95;
L2=.95;
phi=0.083;
alpha=.7;
Lambda1=4500;
A1=5500;
Lambda2=1000;
A2=200;
mu1=.00025;
mu2=.2;
sigma1=2;
sigma2=3;
rho1=22;
rho2=21;
a1=30;
a2=20;
b1=40;
b2=20;
beta1=0.77;
beta2=0.77;

global ppp
we wrote all the parameters as a vector
ppp = [d q L1 L2 phi alpha Lambda1 A1 Lambda2 A2 mu1 mu2 sigma1 sigma2 rho1
rho2 a1 a2 b1 b2 beta1 beta2]';

[t,x]=ode45('elmejor1',tspan,x0);

figure
subplot(221)
hold on
plot(t,x(:,1),'r+')
plot(t,x(:,2),'b*')
plot(t,x(:,3),'g')
plot(t,x(:,4),'k.')
```

```
title(['q=',num2str(q)]);
legend('S1', 'E1', 'I1', 'R1')
xlabel('time')
ylabel('Population NSU Residents')

subplot(222)
hold on
plot(t,x(:,5),'r+')
plot(t,x(:,6),'b*')
plot(t,x(:,7),'ĝ')
plot(t,x(:,8),'k.')
title(['q=',num2str(q)]);
legend('W1','X1','Y1','Z1')
xlabel('time')
ylabel('Population SU Residents')

subplot(223)
hold on
plot(t,x(:,9),'r+')
plot(t,x(:,10),'b*')
plot(t,x(:,11),'ĝ')
plot(t,x(:,124),'k.')
title(['q=',num2str(q)]);
legend('S2','E2','I2','R2')
xlabel('time')
ylabel('Population NSU Tourist')

subplot(224)
hold on
plot(t,x(:,13),'r+')
plot(t,x(:,14),'b*')
plot(t,x(:,15),'ĝ')
plot(t,x(:,16),'k.')
title(['q=',num2str(q)]);
legend('W2','X2','Y2','Z2')
xlabel('time')
ylabel('Population SU Tourist')
```

13.3 Second Matlab Program for Simulations

```
function retval=nuevo1(t,x)

global d q L1 L2 phi alpha Lambda1 A1 Lambda2 A2 mu1 B V sigma1 sigma2 rho1
rho2 a1 a2 b1 b2 beta1 beta2

global ppp

j = 1;
p =ppp;

d = p(j); j=j+1;
q= p(j); j=j+1; varies
L1= p(j); j=j+1;
L2= p(j); j=j+1;
phi= p(j); j=j+1;
alpha= p(j); j=j+1;
Lambda1= p(j); j=j+1;
A1= p(j); j=j+1;
Lambda2= p(j); j=j+1;
A2= p(j); j=j+1;
mu1= p(j); j=j+1;
mu2= p(j); j=j+1;
sigma1= p(j); j=j+1;
sigma2= p(j); j=j+1;
rho1= p(j); j=j+1;
rho2= p(j); j=j+1;
a1= p(j); j=j+1;
a2= p(j); j=j+1;
b1= p(j); j=j+1;
b2= p(j); j=j+1;
beta1= p(j); j=j+1;
beta2= p(j); j=j+1;

For the initial conditions
S1=x(1);
E1=x(2);
K1=x(3);
R1=x(4);
W1=x(5);
X1=x(6);
```

```
Y1=x(7);
Z1=x(8);
S2=x(9);
E2=x(10);
K2=x(11);
R2=x(12);
W2=x(13);
X2=x(14);
Y2=x(15);
Z2=x(16);

Q1=S1+E1+L1+R1; Total Population for non-user (Residents)
T1=W1+X1+Y1+Z1; Total Population for user (Residents)

kc1=a1*Q1; Average number of contact for non-user
kc2=b1*T1*((sigma1)./(rho1+sigma1)); Average number of contact between users
off the subway

Proportionate Mixing C1=(kc1)./(kc1+kc2); non-user with non-user same neighbor-
hood
D1=(kc2)./(kc1+kc2); non-user with user same neighborhood
H1=(kc1)./(kc1+kc2); user with non-user same neighborhood
F1=(kc2)./(kc1+kc2); user with user same neighborhood

Q2=S2+E2+L2+R2; Total Population for non-users (Tourist)
T2=W2+X2+Y2+Z2; Total Population for users (Tourist)

ec3=b1*T1*((rho1)./(rho1+sigma1));
ec4=b2*T2*((rho2)./(rho2+sigma2));

Pb1=(ec3)./(ec3+ec4);
Pb2=(ec4)./(ec3+ec4);

G1=((Pb1).*(Y1./T1))+((Pb2).*(Y2./T2));

Rates of Infections
bb1=(T1*((sigma1)./(sigma1+rho1)))+(Q1);
bb2=Y1*((sigma1)./(sigma1+rho1));

B1(t)-transmission rate NSU
B1=(beta1*a1*S1).*(((C1.*K1)./(bb1))+((D1.*bb2)./(bb1)));
```

```
V1(t)-transmission rate for SU
V1=(beta1*a1*W1).*(((H1.*K1)./(bb1))+((F1.*bb2)./(bb1))+(G1));

derivative of S1
t1=A1-B1;
t2=mu1*S1;
t3=(q*L1*S1)./(S1+E1);
cS1=t1-t2-t3;

derivative of E1
e1=B1;
e2=mu1*E1;
e3=phi*E1;
e4=(q*L2*E1)./(S1+E1);
cE1=e1-e2-e3-e4;

derivative of K1
k1=phi*E1;
k2=(mu1+alpha+d)*K1;
cK1=k1-k2;

derivative of R1
r1=alpha*K1;
r2=mu1*R1;
r3=(q*L1*S1)./(S1+E1);
r4=(q*L2*E1)./(S1+E1);
cR1=r1-r2+r3+r4;

derivative of W1
w1=Lambda1-V1;
w2=mu1*W1;
w3=(q*L1*W1)./(W1+X1);
cW1=w1-w2-w3;

derivative of X1
x1=mu1*X1;
x2=phi*X1;
x3=(q*L2*X1)./(W1+X1);
cX1=V1-x1-x2-x3;
```

derivative of Y1
y1=phi*X1;
y2=(mu1+alpha+d)*Y1;
cY1=y1-y2;

derivative of Z1
z1=alpha*Y1;
z2=mu1*Z1;
z3=(q*L1*W1)./(W1+X1);
z4=(q*L2*X1)./(W1+X1);
cZ1=z1-z2+z3+z4;

ec1=a2*Q2; Average number of contact for non-user
ec2=b2*T2*((sigma2)./(rho2+sigma2)); Average number of contact between users off the subway

Proportionate Mixing
C2=(ec1)./(ec1+ec2); non-user with non-user same neighborhood
D2=(ec2)./(ec1+ec2); non-user with user same neighborhood
H2=(ec1)./(ec1+ec2); user with non-user same neighborhood
F2=(ec2)./(ec1+ec2); user with user same neighborhood
G2=(ec3)./(ec3+ec2); user with user in the bus within different neighborhood

Rates of Infections
bb3=(T2*((sigma2)./(sigma2+rho2)))+(Q2);
bb4=Y2*((sigma2)./(sigma2+rho2));

B2(t)-transmission rate NSU
B2=(beta2*a2*S2).*(((C2.*K2)./(bb3))+((D2.*bb4)./(bb3)));

V2(t)-transmission rate for SU
V2=(beta2*a2*W2).*(((H2.*K2)./(bb3))+((F2.*bb4)./(bb3))+(G1));

derivative of S2
s1=A2-B2;
s2=mu2*S2;
s3=(q*L1*S2)./(S2+E2);
cS2=s1-s2-s3;

derivative of E2
m1=B2;

```
m2=mu2*E2;
m3=phi*E2;
m4=(q*L2*E2)./(S2+E2);
cE2=m1-m2-m3-m4;

derivative of K2
h1=phi*E2;
h2=(mu2+alpha+d)*K2;
cK2=h1-h2;

derivative of R2
r5=alpha*K2;
r6=mu2*R2;
r7=(q*L1*S2)./(S2+E2);
r8=(q*L2*E2)./(S2+E2);
cR2=r5-r6+r7+r8;

derivative of W2
w4=Lambda2-V2;
w5=mu2*W2;
w6=(q*L1*W2)./(W2+X2);
cW2=w4-w5-w6;

derivative of X2
x4=mu2*X2;
x5=phi*X2;
x6=(q*L2*X2)./(W2+X2);
cX2=V1-x4-x5-x6;

derivative of Y2
y3=phi*X2;
y4=(mu2+alpha+d)*Y2;
cY2=y3-y4;

derivative of Z2
z5=alpha*Y2;
z6=mu2*Z2;
z7=(q*L1*W2)./(W2+X2);
z8=(q*L2*X2)./(W2+X2);
cZ2=z5-z6+z7+z8;
```

retval=[cS1,cE1,cK1,cR1,cW1,cX1,cY1,cZ1,cS2,cE2,cK2,cR2,cW2,cX2,cY2,cZ2]';

13.4 Codes for StarLogo Simulations

13.5 Code for 2-neighborhoods

```
;; Model for Population Dynamics in 2-neighborhoods and one subway station
;; People are divided in two populations bustakers and nonbustakers
;; An initial attack occurs in the subway station with n-exposed individuals
where only bustakers from all neighborhoods meet and as they move back into
their neighrborhoods
;; they spread the disease
;; We assume that the probability of contacting a nonbustaker is the same
for all nonbustakers and bustakers, as well as
;; the probablity of contacting a bustaker is the same for all nonbustakers
and bustakers
;; green - suceptible population, yellow - exposed population, red -
infected population
;; As people recover they exit turn blue and do not contribute to the
contact process


patches-own [vacpatch vac? vacpatchx vacpatchy neighborhood neighborhoodx
neighborhoody old-pc]
turtles-own [t-neighborhood bustaker? clock]
globals [n v suceptible exposed infected recovered numdead infect d1 d2]

;; PATCHES
;; vacpatch, vacpatchx, vacpatchy define the vaccination sectors
;; vac? is boolean variable for random patch vaccination purposes
;; neighborhood is the neighborhood the patch is in.
;; neighborhoodx and neighborhoody are used to calculate neighborhood
;; old-pc keeps track of the original color of the patch so it can change
back.

;;TURTLES
;; t-neighborhood is the neighborhood the turtle is in
;; bustaker is to control the percentage of the population that will take
the subway
;; clock is to control both  the infection time and the time of exposure to
```

```
smallpox

;; GLOBALS
;; n is used as a dummy variable
;; v is a dummie variable for vaccination procedures
;; suceptible, exposed, infected and recovered are self-explanatory
;; numdead keeps track of the casualties due to the attack
;; infect is to initialize vaccination procedures
;; d1, d2 keep dead count per neighborhood


;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;  THE PROGRAM
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
to clear
ca                    ;; clears all
setsuceptible 0
setnumdead 0
setrecovered 0
setinfected 0
setexposed 0
setn 0
setv 0
end


to setup
ca                    ;; clear all
define-vaccinepatch   ;; creates vaccination sectors
define-neighborhood   ;; defines neighborhoods and subway
create-people         ;; creates n peolpe
setclock 0            ;; starts clock on exposed individuals
setsuceptible inipop  ;; suceptible population at time 0
setnumdead 0          ;; dead count at time 0
setrecovered 0        ;; recovered count at time 0
setinfected 0         ;; infected count at time 0
setexposed nexposed   ;; exposed at time 0
init-graph            ;; initializes graphics
setv 0                ;; loop clock for vaccination
setvac? FALSE
setinfect 0
```

```
setd1 0
setd2 0
end

to go
if (time > 0) [wait time]
ifelse ((stop-bus = 0) or (infect <= ninfected)) [move-people]
[move-people2]     ;; defines movement of peolpe
up-clock
get-infectious                                          ;; infection
procedure
get-exposed                                             ;; smallpox exposure
procedure

if (vaccinate-1 = 1) [if (infect > ninfected) [patch-vaccinate1]]
if (vaccinate-no = 1) [if (infect > ninfected) [patch-vaccinate2]]
if (vaccinate-3 = 1) [if (infect > ninfected) [patch-vaccinate3]]

;;if (vaccinate-1 = 1) [patch-vaccinate1]                ;; vaccinates
everybody per vacpatch horizontally once
;;if (vaccinate-2 = 1) [patch-vaccinate2]                ;; vaccinates
everybody per randomly selected vacpatch once
;;if (vaccinate-3 = 1) [patch-vaccinate3]                ;; vaccinates
everybody in patch when first infected case is detected

count
update-graph

if (infected + exposed < 1) [stopbutton2]
end

to count
setsuceptible count-turtles-with [color = green]
setexposed count-turtles-with [color = yellow]
setinfected count-turtles-with [color = red]
end

to count-dead
if (t-neighborhood = 1) [tsetd1 d1 + 1]
if (t-neighborhood = 2) [tsetd2 d2 + 1]
end
```

```
to define-neighborhood
if (ycor > 0) [setneighborhood 1]      ;; defines neighborhood 1
if (ycor <= 0) [setneighborhood 2 ]    ;; defines neighborhood 2
if (xcor > -1 * n-var) and (xcor < n-var) and (ycor > -1 * n-var) and (ycor
< n-var) [setpc grey setneighborhood 3] ;; defines neighborhood 3
end

to up-clock
if (color = yellow) [setclock clock + 1] ;; tic toc
if (color = red) [setclock clock + 1]    ;; tic toc
end

to move-people
  rt random 180
  lt random 180
  ifelse ((t-neighborhood = (neighborhood-at dx dy )) or ((bustaker?) and
((neighborhood-at dx dy ) = 3))) [fd 1] [rt 180]
end

to move-people2
  rt random 180
  lt random 180
  ifelse ((neighborhood-at 0 0) = (neighborhood-at dx dy)) [fd 1] [lt 180]
end

to create-people
crt inipop             ;; creates n people at time 0
fd random 100          ;; all people randomly move forward up to 100 steps at
time 0
setc green             ;; all green for suceptible

repeat 100 [if ((neighborhood-at 0 0) = 3 ) [fd random 100]]

if (ycor > 0) [sett-neighborhood 1]  ;; defines neighborhood 1
if (ycor <= 0) [sett-neighborhood 2 ]  ;; defines neighborhood 2

repeat 100 [if ((neighborhood-at 0 0) != t-neighborhood ) [fd random 100]]

;;;;;;;;;;;;;;;;CONTROL THE % OF SUBWAY TAKERS PER
NEIGHBORHOOD;;;;;;;;;;;;;;;;;;;;;;
```

```
setbustaker? FALSE
if ((random 100) < per-bt-1) and (t-neighborhood = 1) [setbustaker? TRUE]
if ((random 100) < per-bt-2) and (t-neighborhood = 2) [setbustaker? TRUE]
if who < nexposed [setc yellow setbustaker? true setxcor 0 setycor 0 rt
random 360 fd random 15] ;;make exposed people by an initial attack of
smallpox

end

;;;;;;;;;; Individuals have 85% chance of becoming exposed due to contacts
with infected individuals

to get-exposed
if (((color-at 0 0) = red) and ((random 100) < 85))  [if (color = green)
[setc yellow]]
end


to get-infectious
if (clock > latent-time and color = yellow) [tsetinfect (infect + 1) setc
red setclock 0]
if (clock > infectious-time and color = red) [ifelse ((random 100) < 30)
[tsetnumdead numdead + 1 die] [tsetrecovered recovered + 1 setc blue]]
end

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; Vaccination Procedures
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
to patch-vaccinate1
if (v <= vaccinepatch ^ 2) [if (vacpatch = v) [to-vaccinate] setv (v + 1)]
end

to patch-vaccinate2
setv (random (1 + vaccinepatch ^ 2) + 1)
if (vacpatch = v)[if (vac? = FALSE) [setvac? TRUE to-vaccinate]]
end

to patch-vaccinate3
if (color = red and random 4000 = who) [tsetv (vacpatch-at 0 0)]
if (vacpatch = v)[if (vac? = FALSE) [setvac? TRUE to-vaccinate]]
```

```
end

to to-vaccinate
if ((vacpatch-at 0 0) = v) [
if ((color = green) and ((random 100) < 97)) [ifelse ((random 100) < 1)
[tsetnumdead numdead + 1 count-dead die] [tsetrecovered recovered + 1 setc
blue]]     ;; vaccine has 97% chance of immunizing suceptible individuals 1%
chance of killing individuals due to secondary effects of vaccine
if ((color = yellow) and (clock <= 4) and ((random 100) < 97)) [ifelse
((random 100) < 1) [tsetnumdead numdead + 1 count-dead die] [tsetrecovered
recovered + 1 setc blue]]  ;; vaccine has 97% chance of immunizing exposed
individuals if time of exposure is less than 4 days (loops),1% chance of
killing individuals due to secondary effects of vaccine

]

end


to define-vaccinepatch

setn -1 * vaccinepatch      ;;start negative

repeat vaccinepatch [if (xcor >= ((screen-edge-x / vaccinepatch) * n))
[setvacpatchx (n + vaccinepatch) * vaccinepatch / 2] setn n + 2] ;; this
marks off the vaccination sectors horizontally.

setn -1 * vaccinepatch
repeat vaccinepatch [if (ycor >= ((screen-edge-y / vaccinepatch) * n))
[setvacpatchy ((n + vaccinepatch)  / 2)] setn n + 2] ;; this marks off the
the vaccineation sectors vertically
setvacpatch vacpatchx + vacpatchy + 1  ;;add them up and you get a unique
neighborhood (uses powers to make this work)

scale-pc brown (vacpatch + 1) vaccinepatch (vaccinepatch ^ 2) ;; shade them
blue to tell them apart
setold-pc pc ;; set the old color to this color (the color we come back to
if we change back from yellow)
end
```

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;  Graphics
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

to init-graph
pw1
clearplot
setplot-yrange 0 inipop
auto-plot-on
setplotwindow-name "Population_Dynamics
setplot-xlabel "TIME
setplot-ylabel "POPULATION

pp1
setppc green
setpen-name "Suscpetible

pp2
setppc yellow
setpen-name "Exposed

pp3
setppc red
setpen-name "Infected

pp4
setppc blue
setpen-name "Recovered

pp5
setppc violet
setpen-name "Casualties

pw2
clearplot
setplot-yrange 0 20
auto-plot-on
setplotwindow-name "Casualties_per_Neighborhood
setplot-xlabel "TIME
setplot-ylabel "Casualties
```

```
pp1
setppc green
setpen-name "N1

pp2
setppc pink
setpen-name "N2

end


to update-graph
pw1
pp1
plot suceptible
pp2
plot exposed
pp3
plot infected
pp4
plot recovered
pp5
plot numdead


pw2
pp1
plot d1
pp2
plot d2

end


;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;  Debugers
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

to show-neighborhoods
        if (neighborhood = 1) [setpc white]
```

```
        if (neighborhood = 2) [setpc black]
        if (neighborhood = 3) [setpc gray]
end

to show-vacpatch
scale-pc brown (vacpatch + 1) vaccinepatch (vaccinepatch ^ 2) ;; shade them
blue to tell them apart
if (neighborhood = 3) [setpc gray]
end

to mark-people
        if (t-neighborhood = 1) [setc pink]
if (t-neighborhood = 2) [setc green]
end

to mark-bustakers
if bustaker? [setc orange]
end
```

13.6 Code for 4-neighborhoods

```
;; Model for Population Dynamics in 4-neighborhoods and one subway station
;; People are divided in two populations bustakers and nonbustakers
;; An initial attack occurs in the subway station with n-exposed individuals
where only bustakers from all neighborhoods meet and as they move back into
their neighrborhoods
;; they spread the disease
;; We assume that the probability of contacting a nonbustaker is the same
for all nonbustakers and bustakers, as well as
;; the probablity of contacting a bustaker is the same for all nonbustakers
and bustakers
;; green - suceptible population, yellow - exposed population, red -
infected population
;; As people recover they exit turn blue and do not contribute to the
contact process


patches-own [vacpatch vac? vacpatchx vacpatchy neighborhood neighborhoodx
neighborhoody old-pc]
turtles-own [t-neighborhood bustaker? clock]
```

```
globals [n v suceptible exposed infected recovered numdead infect d1 d2 d3
d4]

;; PATCHES
;; vacpatch, vacpatchx, vacpatchy define the vaccination sectors
;; vac? is boolean variable for random patch vaccination purposes
;; neighborhood is the neighborhood the patch is in.
;; neighborhoodx and neighborhoody are used to calculate neighborhood
;; old-pc keeps track of the original color of the patch so it can change
back.

;;TURTLES
;; t-neighborhood is the neighborhood the turtle is in
;; bustaker is to control the percentage of the population that will take
the subway
;; clock is to control both  the infection time and the time of exposure to
smallpox

;; GLOBALS
;; n is used as a dummy variable
;; v is a dummie variable for vaccination procedures
;; suceptible, exposed, infected and recovered are self-explanatory
;; numdead keeps track of the casualties due to the attack
;; d1, d2, d3, d4 keep track of dead count per neighborhood


;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;   THE PROGRAM
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

to clear
ca                      ;; clears all
setsuceptible 0
setnumdead 0
setrecovered 0
setinfected 0
setexposed 0
setn 0
setv 0
end
```

```
to setup
ca                      ;; clear all
define-vaccinepatch     ;; creates vaccination sectors
define-neighborhood     ;; defines neighborhoods and subway
create-people           ;; creates n peolpe
setclock 0              ;; starts clock on exposed individuals
setsuceptible inipop    ;; suceptible population at time 0
setnumdead 0            ;; dead count at time 0
setrecovered 0          ;; recovered count at time 0
setinfected 0           ;; infected count at time 0
setexposed nexposed     ;; exposed at time 0
init-graph              ;; initializes graphics
setv 0                  ;; loop clock for vaccination
setvac? FALSE
setinfect 0
setd1 0
setd2 0
setd3 0
setd4 0
end


to go
if (time > 0) [wait time]
ifelse ((stop-bus = 0) or (infect <= ninfected)) [move-people]
[move-people2]      ;; defines movement of peolpe
up-clock
get-infectious                                          ;; infection
procedure
get-exposed                                             ;; smallpox exposure
procedure

if (vaccinate-1 = 1) [if (infect > ninfected) [patch-vaccinate1]]
if (vaccinate-no = 1) [if (infect > ninfected) [patch-vaccinate2]]
if (vaccinate-3 = 1) [if (infect > ninfected) [patch-vaccinate3]]

;;if (vaccinate-1 = 1) [patch-vaccinate1]                ;; vaccinates
everybody per vacpatch horizontally once
;;if (vaccinate-2 = 1) [patch-vaccinate2]                ;; vaccinates
everybody per randomly selected vacpatch once
;;if (vaccinate-3 = 1) [patch-vaccinate3]                ;; vaccinates
```

```
everybody in patch when first infected case is detected

count
update-graph

if (infected + exposed < 1) [stopbutton2]
end

to count
setsuceptible count-turtles-with [color = green]
setexposed count-turtles-with [color = yellow]
setinfected count-turtles-with [color = red]
end

to count-dead
if (t-neighborhood = 1) [tsetd1 d1 + 1]
if (t-neighborhood = 2) [tsetd2 d2 + 1]
if (t-neighborhood = 3) [tsetd3 d3 + 1]
if (t-neighborhood = 4) [tsetd4 d4 + 1]
end

to define-neighborhood
if (ycor > 0 and xcor > 0) [setneighborhood 1]     ;; defines neighborhood 1
if (ycor > 0 and xcor <= 0) [setneighborhood 2]    ;; defines neighborhood 2
if (ycor <= 0 and xcor <= 0) [setneighborhood 3]   ;; defines neighborhood 3
if (ycor <= 0 and xcor > 0) [setneighborhood 4]    ;; defines neighborhood 4
if (xcor > -1 * n-var) and (xcor < n-var) and (ycor > -1 * n-var) and (ycor
< n-var) [setpc grey setneighborhood 5] ;; defines neighborhood 5
end

to up-clock
if (color = yellow) [setclock clock + 1] ;; tic toc
if (color = red) [setclock clock + 1]    ;; tic toc
end

to move-people
  rt random 180
  lt random 180
  ifelse ((t-neighborhood = (neighborhood-at dx dy )) or ((bustaker?) and
((neighborhood-at dx dy ) = 5))) [fd 1] [rt 180]
end
```

```
to move-people2
  rt random 180
  lt random 180
  ifelse ((neighborhood-at 0 0) = (neighborhood-at dx dy)) [fd 1] [lt 180]
end

to create-people
crt inipop              ;; creates n people at time 0
fd random 100           ;; all people randomly move forward up to 100 steps at
time 0
setc green              ;; all green for suceptible

repeat 100 [if ((neighborhood-at 0 0) = 5 ) [fd random 100]]

if (ycor > 0 and xcor > 0) [sett-neighborhood 1]     ;; defines neighborhood
1
if (ycor > 0 and xcor <= 0) [sett-neighborhood 2]    ;; defines neighborhood
2
if (ycor <= 0 and xcor <= 0) [sett-neighborhood 3]   ;; defines neighborhood
3
if (ycor <= 0 and xcor > 0) [sett-neighborhood 4]     ;; defines
neighborhood 4

repeat 100 [if ((neighborhood-at 0 0) != t-neighborhood ) [fd random 100]]

;;;;;;;;;;;;;;;;CONTROL THE % OF SUBWAY TAKERS PER
NEIGHBORHOOD;;;;;;;;;;;;;;;;;;;;;;;

setbustaker? FALSE
if ((random 100) < per-bt-1) and (t-neighborhood = 1) [setbustaker? TRUE]
if ((random 100) < per-bt-2) and (t-neighborhood = 2) [setbustaker? TRUE]
if ((random 100) < per-bt-3) and (t-neighborhood = 3) [setbustaker? TRUE]
if ((random 100) < per-bt-4) and (t-neighborhood = 4) [setbustaker? TRUE]

if who < nexposed [setc yellow setbustaker? true setxcor 0 setycor 0 rt
random 360 fd random n-var] ;;make exposed people by an initial attack of
smallpox

end
```

```
;;;;;;;;;; Individuals have 85% chance of becoming exposed due to contacts
with infected individuals

to get-exposed
if (((color-at 0 0) = red) and ((random 100) < 85))  [if (color = green)
[setc yellow]]
end


to get-infectious
if (clock > latent-time and color = yellow) [tsetinfect (infect + 1) setc
red setclock 0]
if (clock > infectious-time and color = red) [ifelse ((random 100) < 30)
[tsetnumdead numdead + 1 count-dead die] [tsetrecovered recovered + 1 setc
blue]]
end

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;; Vaccination Procedures
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

to patch-vaccinate1
if (v <= vaccinepatch ^ 2) [if (vacpatch = v) [to-vaccinate] setv (v + 1)]
end

to patch-vaccinate2
setv (random (1 + vaccinepatch ^ 2) + 1)
if (vacpatch = v)[if (vac? = FALSE) [setvac? TRUE to-vaccinate]]
end

to patch-vaccinate3
if (color = red and random 4000 = who) [tsetv (vacpatch-at 0 0)]
if (vacpatch = v)[if (vac? = FALSE) [setvac? TRUE to-vaccinate]]
end

to to-vaccinate
if ((vacpatch-at 0 0) = v) [
if ((color = green) and ((random 100) < 97)) [ifelse ((random 100) < 1)
[tsetnumdead numdead + 1 count-dead die] [tsetrecovered recovered + 1 setc
blue]]     ;; vaccine has 97% chance of immunizing suceptible individuals 1%
```

```
chance of killing individuals due to secondary effects of vaccine
if ((color = yellow) and (clock <= 4) and ((random 100) < 97)) [ifelse
((random 100) < 1) [tsetnumdead numdead + 1 count-dead die] [tsetrecovered
recovered + 1 setc blue]]  ;; vaccine has 97% chance of immunizing exposed
individuals if time of exposure is less than 4 days (loops),1% chance of
killing individuals due to secondary effects of vaccine

]

end


to define-vaccinepatch

setn -1 * vaccinepatch      ;;start negative

repeat vaccinepatch [if (xcor >= ((screen-edge-x / vaccinepatch) * n))
[setvacpatchx (n + vaccinepatch) * vaccinepatch / 2] setn n + 2] ;; this
marks off the vaccination sectors horizontally.

setn -1 * vaccinepatch
repeat vaccinepatch [if (ycor >= ((screen-edge-y / vaccinepatch) * n))
[setvacpatchy ((n + vaccinepatch)  / 2)] setn n + 2] ;; this marks off the
the vaccineation sectors vertically
setvacpatch vacpatchx + vacpatchy + 1  ;;add them up and you get a unique
neighborhood (uses powers to make this work)

scale-pc brown (vacpatch + 1) vaccinepatch (vaccinepatch ^ 2) ;; shade them
blue to tell them apart
setold-pc pc ;; set the old color to this color (the color we come back to
if we change back from yellow)
end



;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;  Graphics
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

to init-graph
pw1
```

```
clearplot
setplot-yrange 0 inipop
auto-plot-on
setplotwindow-name "Population_Dynamics
setplot-xlabel "TIME
setplot-ylabel "POPULATION

pp1
setppc green
setpen-name "Suscpetible

pp2
setppc yellow
setpen-name "Exposed

pp3
setppc red
setpen-name "Infected

pp4
setppc blue
setpen-name "Recovered

pp5
setppc violet
setpen-name "Casualties

pw2
clearplot
setplot-yrange 0 30
auto-plot-on
setplotwindow-name "Casualties_per_Neighborhood
setplot-xlabel "TIME
setplot-ylabel "Casualties

pp1
setppc green
setpen-name "N1

pp2
setppc pink
```

```
setpen-name "N2

pp3
setppc violet
setpen-name "N3

pp4
setppc brown
setpen-name "N4

end


to update-graph
pw1
pp1
plot suceptible
pp2
plot exposed
pp3
plot infected
pp4
plot recovered
pp5
plot numdead

pw2
pp1
plot d1
pp2
plot d2
pp3
plot d3
pp4
plot d4

end


;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;  Debugers
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
```

```
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

to show-neighborhoods
        if (neighborhood = 1) [setpc white]
        if (neighborhood = 2) [setpc black]
  if (neighborhood = 3) [setpc white]
        if (neighborhood = 4) [setpc black]
  if (neighborhood = 5) [setpc gray]
end

to show-vacpatch
scale-pc brown (vacpatch + 1) vaccinepatch (vaccinepatch ^ 2) ;; shade them
blue to tell them apart
if (neighborhood = 5) [setpc gray]
end

to mark-people
        if (t-neighborhood = 1) [setc green]
  if (t-neighborhood = 2) [setc pink]
        if (t-neighborhood = 3) [setc violet]
  if (t-neighborhood = 4) [setc brown]
end

to mark-bustakers
if bustaker? [setc orange]
end
```