A Deterministic Approach to the Spread of Rumors

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Abstract

Ideas, products, and messages spread in ways that resemble the transmission dynamics of viruses. We begin with the same framework as Daley-Kendall, which classifies individuals as susceptibles, "spreaders", and "stiflers", and models rumor spreading as an epidemic. We look at the implications of heterogeneity in the susceptible and spreader classes on the spread of a rumor, an aspect not considered in the Daley-Kendall model. Finally, the dynamics of rumor spreading in chat rooms that are accessible to a large number of groups are explored under the assumption of simple, local (neighborhood) dynamics. The characterization of dynamics is carried out through a combination of analytical and numerical results. Efforts to determine the most effective ways to stop or accelerate the spread of rumors are also discussed.

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1 Introduction

The diffusion of information is important in all of our lives. We read the morning paper, listen to afternoon talk radio, and watch the nightly news to receive information about events, locally and globally. All of these are examples of information passed at a rapid rate to a large number of people in a short period of time. However, it need not be news that is passed along. Clothing fads, winning sports teams, and the latest music artists are all examples of things that follow dynamics similar to the news. More importantly, both viruses and the news are transmitted by some form of contact, and can travel from person to person. Therefore, ideas, products, and messages spread in ways that resemble the transmission dynamics of viruses. This is the motivation behind our project.

A rumor is an unverified proposition of belief that bears topical relevance for persons actively involved in its dissemination. Rumors are first of all *unauthenticated* bits of information in that they are deprived of "secure standards of evidence"[10]. Rumors seem to spread at enormous rates in a relatively short amount of time. Rumor spreading resembles epidemic spread, hence the propagation of rumors is modeled via modification of standard epidemiological models. The definitions for contacts, births, and deaths will have to be revised when modeling the propagation of rumors. Nevertheless, there are three ideas that do link epidemics and rumors very well. First, the idea of infectivity is present in both processes even though the definitions are different. Viruses such as influenza and chickenpox are extremely contagious and easy to transmit, rumors are just as contagious because all that is needed to infect an individual is to transmit the rumor. Once a rumor is started it seems like almost everybody will eventually know it, and the person who started the rumor has caused "infectections" of the rumor "virus". Second, is the idea that little changes have big effects on the population. In the case of the common cold, it is possible for only a few coughs and sneezes to cause infections in many people. The same holds for rumors due to the fact that only a few people need to know the rumor in order to have rapid dissemination. The final similarity is that major events happen in a short amount of time. The potential for an outbreak to occur is present for both epidemics and rumors[11].

We know that some rumors have larger effects than others do, but everybody's life is affected in some general sense by rumors. However, we see a large presence of rumors spread in high school and collegiate environments. This is alarming because of the potential impact that rumors can have on people. It is important to us to study the propagation of rumors because the analysis can lead to insight about factors that affect the dynamics of the rumor spreading.

The modeling of rumor propagation has been proposed by others such as Rapoport (1948), Bartholomew(1967), and Zanette(2001)[9]. Their approach is based on stochastic processes. Our model draws from a previous model created by Daley and Kendall in which there exists three classes: susceptibles, "spreaders", and "stiflers". We assume two distinct attitudes among suceptibles and spreaders: passive and active. The passive people are those who do not have many contacts. We define active people to be those who have many contacts. We do this because we realize that not everybody is the same and there will be different transmission rates of the rumor among different people. The medium through which the rumors are transmitted is also an important factor in the dynamics of spreading. In this day and age, computers are constantly used and are great means for transmitting information as well as rumors. With this in mind, we also created a model to do some analysis on the spread of rumors over the popular Microsoft Network(MSN) Instant Messenger and chat rooms. We examine a social network with two neighborhoods of people. In each neighborhood we consider a mutually exclusive stratification of individuals between Instant Messenger users and non-Instant Messenger users. This model is based on the work of Carlos Castillo-Chávez and Baojun Song[6].

In this paper, we review the Daley-Kendall model and introduce a model with heterogeneity in susceptibility and transmission. In Section 3, we discuss and analyze our general model and interpret the results gained from numerical simulations. In Section 4, we introduce the Neighborhood-Internet model, and examine numerical simulations for the dynamics of rumors spreading in this environment. Finally, we draw conclusions about both models and do a comparison discussion of the results. We discuss the implications of these results and from this, conclude what parameters have the largest impact on each system so that we can come up with suggestions for possible preventative or control methods.

2 Review of Daley-Kendall Framework

In December of 1964, D. J. Daley and D. G. Kendall published a paper aiming to stochastically model the spread of rumors. They considered a closed homogeneously mixing population of $N+1$ individuals. At any time an individual can be classified as belonging to one of three categories:

$X(t)$ Denotes those individuals who are ignorant of the rumor;

$Y(t)$ Denotes those individuals who are actively spreading the rumor; and

$Z(t)$ Denotes those individuals who know the rumor but have ceased spreading it.

Initially, $X(0) = N$, $Y(0) = 1$ and $Z(0) = 0$, while for all t, $X(t) + Y(t) + Z(t) = N + 1$. They referred to these three types of individuals as ignorants, spreaders and stiflers, respectively[9].

The rumor is propagated through the closed population by contact between ignorants and spreaders, following the law of mass action. They assume that any spreader involved in any pairwise meeting 'infects' the 'other'. If the 'other' is an ignorant then he/she will become a spreader; if the 'other' is a spreader or a stifler, then the spreader(s) will become a stifler(s). A stifler will never, under any circumstances, infect a susceptible because the definition of a spreader. Stiflers do not transmit the rumor[9].

Next, Cintrón-Arias and Castillo-Chávez proposed the following deterministic version of the Daley-Kendall model:

$$\begin{aligned} \tfrac{dX}{dt} &= -\lambda X \tfrac{Y}{N} \\ \tfrac{dY}{dt} &= \lambda X \tfrac{Y}{N} - \alpha \tfrac{Y(Y+Z)}{N} \\ \tfrac{dZ}{dt} &= \alpha \tfrac{Y(Y+Z)}{N} \end{aligned} \qquad (1)$$

This model has been extremely useful in the interpretation of the Daley-Kendall because some analytical analysis can be done on this deterministic version of the model. Still, the Daley-Kendall model makes some other assumptions. There is no inflow to the susceptible class or outflow from any of the classes. The model also assumes that everybody is similar and interacts with the same amount of people. Along these same lines, their model does not take into account the personality of the person who is spreading or receiving the rumor. And finally, it does not allow for people who are "ignorant" to hear the rumor and then choose not to spread it. Still, their model was extremely innovative and is still very useful in the modeling and analysis of rumor spreading.

3 Model With Two Attitude Levels

3.1 Background

Comparing the standard S→I→R and Cintrón-Arias' (X→Y→Z) epidemic model's "infected" class of differential equations:

$$\frac{dI}{dt} = \beta S \frac{I}{N} - \gamma I \quad \text{and} \quad \frac{dY}{dt} = \lambda X \frac{Y}{N} - \alpha \frac{Y(Y+Z)}{N}$$

We can see that there is some similarity between the two equations but the difference is in the second term. In the standard model there is linear removal from the infected class at some rate γ. There is removal in Cintrón-Arias' model but it is not linear. Notice that the removal from the spreader class is a result of coming in contact with either another spreader or a stifler. This means that there will not be an average length of time spent in the spreader class. In the standard SIR model a constant proportion of the population, γ, is removed from the infective class, so that the average length of time spent in the infective class is $\frac{1}{\gamma}$. Therefore the average length of time for the spreader class can be much larger than the length of time spent in the infective class. This is because the length of time spent in the spreader class is dependent upon when the next contact with a spreader or stifler is made.

With our model, we tried to go beyond the Daley-Kendall framework. While it would be impossible to account for or address all the assumptions that could be made to model reality, we hope to relax at least some of them. As mentioned before, there are many different personalities in the world. We will make a simplification, as it pertains to rumor spreading, and say that everybody can be divided into two attitudes: passive and active. Passive people we define as those who have fewer contacts in a day and generally do not wish to spread rumors. The active group can be thought of as the "cliques" and popular people who want to pass around gossip. Note however, that passive people will still gossip. They are not as likely to gossip as the active group, but if they do, then it is at a lower rate than the active people. We constructed a model (Figure 1) with two susceptible classes, two gossiper classes, and one stifler class. The passive classes will be S_1, G_1 and Z. The active classes will be S_2 and G_2.

3.2 The Model

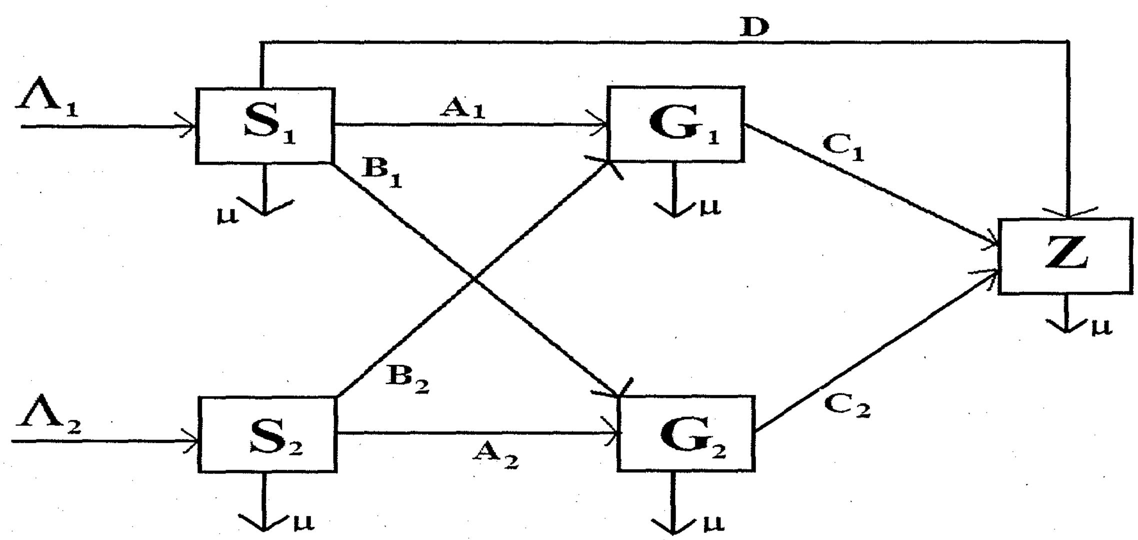

Figure 1: Two Attitude Model

From the model one obtains the system of ODE's:

$$
\begin{aligned}
\dot{S}_1 &= \Lambda_1 - \mu S_1 - A_1 - B_1 - D, \\
\dot{S}_2 &= \Lambda_2 - \mu S_2 - A_2 - B_2, \\
\dot{G}_1 &= A_1 + B_2 - C_1 - \mu G_1, \\
\dot{G}_2 &= A_2 + B_1 - C_2 - \mu G_2, \\
\dot{Z} &= C_1 + C_2 + D - \mu Z,
\end{aligned} \tag{2}
$$

Where the incidence rates are defined as:

$$
\begin{aligned}
A_1 &= c_1 S_1 \left(\frac{c_1 G_1 q_1}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2 q_2}{c_1 N_1 + c_2 N_2} \right), \\
B_1 &= c_1 S_1 \left(\frac{c_1 G_1 q_3}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2 q_4}{c_1 N_1 + c_2 N_2} \right), \\
C_1 &= c_1 G_1 \left(\frac{c_1 (G_1 + Z)}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2}{c_1 N_1 + c_2 N_2} \right), \\
D &= c_1 S_1 \left(\frac{c_1 G_1 q_5}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2 q_6}{c_1 N_1 + c_2 N_2} \right), \\
A_2 &= c_2 S_2 \left(\frac{c_1 G_1 q_7}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2 q_8}{c_1 N_1 + c_2 N_2} \right), \\
B_2 &= c_2 S_2 \left(\frac{c_1 G_1 q_9}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2 q_{10}}{c_1 N_1 + c_2 N_2} \right), \\
C_2 &= c_2 G_2 \left(\frac{c_1 (G_1 + Z)}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2}{c_1 N_1 + c_2 N_2} \right),
\end{aligned} \tag{3}
$$

Parameter	Description
Λ_i	Number of MSN accounts created per unit of time for the ith class
μ	Number of accounts canceled or voided per unit of time
c_1	The average number of effective contacts a person with a passive attitude has per unit of time
c_2	The average number of effective contacts a person with a passive attitude has per unit of time Also defined as kc_1 where $k > 1$ so that c_2 is directly proportional to c_1
q_i	$i = \{1...10\}$ The proportion of people leaving either suceptable classes

Table 1: Definitions of Parameters

Therefore the fully expanded equations are:

$$\begin{aligned}
\dot{S_1} &= \Lambda_1 - \mu S_1 - c_1 S_1 \left(\frac{c_1 G_1 q_1}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2 q_2}{c_1 N_1 + c_2 N_2} \right) \\
&\quad - c_1 S_1 \left(\frac{c_1 G_1 q_3}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2 q_4}{c_1 N_1 + c_2 N_2} \right) - c_1 S_1 \left(\frac{c_1 G_1 q_5}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2 q_6}{c_1 N_1 + c_2 N_2} \right) \qquad (4) \\
\dot{S_2} &= \Lambda_2 - \mu S_2 - c_2 S_2 \left(\frac{c_1 G_1 q_7}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2 q_8}{c_1 N_1 + c_2 N_2} \right) \\
&\quad - c_2 S_2 \left(\frac{c_1 G_1 q_9}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2 q_{10}}{c_1 N_1 + c_2 N_2} \right) \qquad (5) \\
\dot{G_1} &= c_1 S_1 \left(\frac{c_1 G_1 q_1}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2 q_2}{c_1 N_1 + c_2 N_2} \right) + c_2 S_2 \left(\frac{c_1 G_1 q_9}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2 q_{10}}{c_1 N_1 + c_2 N_2} \right) \\
&\quad - c_1 G_1 \left(\frac{c_1 (G_1 + Z)}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2}{c_1 N_1 + c_2 N_2} \right) - \mu G_1 \qquad (6) \\
\dot{G_2} &= c_2 S_2 \left(\frac{c_1 G_1 q_7}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2 q_8}{c_1 N_1 + c_2 N_2} \right) + c_1 S_1 \left(\frac{c_1 G_1 q_3}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2 q_4}{c_1 N_1 + c_2 N_2} \right) \\
&\quad - c_2 G_2 \left(\frac{c_1 (G_1 + Z)}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2}{c_1 N_1 + c_2 N_2} \right) - \mu G_2 \qquad (7) \\
\dot{Z} &= c_1 G_1 \left(\frac{c_1 (G_1 + Z)}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2}{c_1 N_1 + c_2 N_2} \right) + c_2 G_2 \left(\frac{c_1 (G_1 + Z)}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2}{c_1 N_1 + c_2 N_2} \right) \\
&\quad + c_1 S_1 \left(\frac{c_1 G_1}{c_1 N_1 + c_2 N_2} + \frac{c_2 G_2}{c_1 N_1 + c_2 N_2} \right) - \mu Z \qquad (8)
\end{aligned}$$

3.3 Assumptions

The model described is different from the Daley-Kendal model because it has different activity rates for each population, which we denote as c_1 and c_2. This is somewhat similar to a model that considers a core group of very active people, but in this case, our second group is more active spreading a rumor. The purpose of having only one Z (stifler) class is that we assume that once you become a stifler it does not matter what type of susceptible you were originally. You are simply not going to spread the rumor. Also we include Z in the passive population, denoted by N_1, because we assume once a person is a stifler, his or her activity level will become that of a passive person since he or she is no longer willing to pass along the rumor.

We also have added inflow and outflow to the different classes. While usually defined as births and deaths, we define these to be the number of new internet accounts created ("births") and the number of internet accounts that are canceled or become void ("deaths"). The total population, N_T, is constant since:

$$
\begin{aligned}
\dot{N_T} &= \Lambda_1 + \Lambda_2 - \mu S_1 + \mu S_2 + \mu G_1 + \mu G_2 + \mu Z = 0. \quad \text{Hence} \\
\Lambda_1 + \Lambda_2 &= \mu S_1 + \mu S_2 + \mu G_1 + \mu G_2 + \mu Z \\
N_T &= N_1 + N_2 \qquad (9) \\
N_1 &= S_1 + G_1 + Z \\
N_2 &= S_2 + G_2
\end{aligned}
$$

In our model, the only way to have contact is to convey the words of the rumor. We assume that all contacts are effective with probability of 100%. There are a few ways to interpret this transmission. One way is that you are not told the rumor during a conversation, in which case you are still not aware of the rumor and remain susceptible. The other case is that an individual is told the rumor but either does not understand the rumor or did not hear it when he or she was told. In both of these cases, the person is at least aware of the rumor so they must leave the susceptible class; however, this individual is probably not going to be spreading the rumor. We consider these individuals to be stiflers. Essentially, we consider a contact to occur when the rumor is told, whether it is heard or not is irrelevant.

For our model, we assume that contact between two gossipers, of either activity level, or contact between a gossiper and a stifler, he or she moves directly to the stifler class at some rate C_i. The reason is that if a person is a gossiper and he or she encounters either another gossiper or a stifler he or she realizes the rumor is no longer current news and decides to stop spreading it.

The q_i's represent the proportion of individuals that change from the susceptible classes to the spreader classes. Since our model assumes that a contact will automatically lead to the transmission of the rumor, there are some properties that the q_i's have. Since a contact between someone in S_1 and G_1 will result in an individual leaving the S_1 class, all of the q_i's in $\dot{S}$ that are multiplied by S_1 and G_1 must add up to 1. So $q_1 + q_3 + q_5 = 1$. Similarly $q_2 + q_4 + q_6 = q_7 + q_9 = q_8 + q_{10} = 1$. So the q_i's will determine the proportion of individuals that will enter different spreader classes from each susceptible classes. It should also be noted that q_5 and q_6 will determine the fraction of individuals that will leave the S_1 class to enter the Z class. These q_i's are going to be dependent upon the particular quality of the rumor, as a rumor can have a definite effect upon the mentality, and therefore activity, of a population. Thus we will have to consider some different cases when we attempt to assign values to these q's.

Finally, another difference from the Daley-Kendall model is that we allow for movement from the passive susceptible class directly into the stifler class. This allows for susceptibles to hear the rumor and decide not to spread if from the beginning. However, we do not allow active people to move directly to the stifler class because these are people who are searching for and want to spread gossip.

3.4 Equilibrium and the Basic Reproductive Number

For our model, we find the rumor free equilibrium (RFE) to be:

$$\left(S_1 = \frac{\Lambda_1}{\mu}, S_2 = \frac{\Lambda_2}{\mu}, G_1 = 0, G_2 = 0, Z = 0\right)$$

The RFE consists of a single point, unlike similar SIR models whose locus of equilibria can consist of a line of equilibria or a hyperplane. We were unable to find an analytical representation of the endemic equilibria. However, we suspect the existence of at least one endemic equilibria, a view that is supported by the results of our numerical simulations.

The basic reproductive number, or R_0, is defined as the average number of secondary transmissions of the rumor produced when a typical spreader is introduced into a population where everyone is ignorant[13]. Thus R_0 is often considered as the threshold quantity that determines when an infection can invade and persist in a new host population. Thus, our model used R_0 to determine the average number of secondary transmissions of the rumor. If a person, on average, will tell more than one other person before they stop transmitting the rumor, then $R_0 > 1$ and the RFE will be unstable. If this should occur then there will be a continuous presence of the rumor in the population.

Following the Next Generation Operator (NGO) approach, as outlined in a paper by Castillo-Chávez and others[4], we see that there are no latent classes and proceed to find the the matrix of the partial derivatives of our differential equations for the spreader classes with respect to each spreader class variable. We are then left with the "Mini-Jacobian" evaluated at the disease free equilibrium:

$$\begin{pmatrix} \frac{c_1^2\Lambda_1 q_1}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_1c_2\Lambda_2 q_9}{c_1\Lambda_1+c_2\Lambda_2} - \mu & \frac{c_1c_2\Lambda_1 q_2}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_2^2\Lambda_2 q_{10}}{c_1\Lambda_1+c_2\Lambda_2} \\ \frac{c_1c_2\Lambda_2 q_7}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_1^2\Lambda_1 q_3}{c_1\Lambda_1+c_2\Lambda_2} & \frac{c_2^2\Lambda_2 q_8}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_1c_2\Lambda_1 q_4}{c_1\Lambda_1+c_2\Lambda_2} - \mu \end{pmatrix} \tag{10}$$

Next, we separate the diagonals and find the inverse of the removal from the infected classes, this provides us with a matrix MD^{-1}:

$$\begin{pmatrix} \frac{c_1(c_1\Lambda_1 q_1+c_2\Lambda_2 q_9)}{\mu(c_1\Lambda_1+c_2\Lambda_2)} & \frac{c_2(c_1\Lambda_1 q_2+c_2\Lambda_2 q_{10})}{\mu(c_1\Lambda_1+c_2\Lambda_2)} \\ \frac{c_1(c_1\Lambda_1 q_3+c_2\Lambda_2 q_7)}{\mu(c_1\Lambda_1+c_2\Lambda_2)} & \frac{c_2(c_1\Lambda_1 q_4+c_2\Lambda_2 q_8)}{\mu(c_1\Lambda_1+c_2\Lambda_2)} \end{pmatrix} \tag{11}$$

We find the eigenvalues of this matrix and the NGO guarantees us that the largest eigenvalue will be R_0 or

$$R_0 = \left(\frac{c_1(c_1\Lambda_1 q_1 + c_2\Lambda_2 q_9)}{\mu(c_1\Lambda_1 + c_2\Lambda_2)} + \frac{c_2(c_1\Lambda_1 q_4 + c_2\Lambda_2 q_8)}{\mu(c_1\Lambda_1 + c_2\Lambda_2)}\right)$$
$$-\left(\frac{c_1(c_1\Lambda_1 q_1 + c_2\Lambda_2 q_9)}{\mu(c_1\Lambda_1 + c_2\Lambda_2)}\frac{c_2(c_1\Lambda_1 q_4 + c_2\Lambda_2 q_8)}{\mu(c_1\Lambda_1 + c_2\Lambda_2)} - \frac{c_2(c_1\Lambda_1 q_2 + c_2\Lambda_2 q_{10})}{\mu(c_1\Lambda_1 + c_2\Lambda_2)}\frac{c_1(c_1\Lambda_1 q_3 + c_2\Lambda_2 q_7)}{\mu(c_1\Lambda_1 + c_2\Lambda_2)}\right) \tag{12}$$

We can see that these terms are the linear terms for transmission from the susceptible to spreader classes with respect to the variables represented by the spreader classes, multiplied by the average amount of time a person spends in either infected class: $\frac{1}{\mu}$. Thus, our threshold value for R_0 includes terms that affect the number of individuals entering the:

1. G_1 class due to contacts with individuals in G_1 by q_1 and q_9.

2. G_1 class due to contacts with individuals in G_2 by q_2 and q_{10}.

3. G_2 class due to contacts with individuals in G_1 by q_3 and q_7.

4. G_2 class due to contacts with individuals in G_2 by q_4 and q_8.

3.5 Parameter Estimation

In the model with two levels of activity, we propose to model interaction between people over the internet. We chose the internet because we have first hand knowledge of using the internet and chatting on instant messengers. We also found a sufficient amount of data online about internet and messenger usage, which allowed us to approximate parameter values. In particular, we use Microsoft Network (MSN) and an online study that made telephone surveys to determine levels of activity over the internet. We were able to use population sizes, proportions, rate of growth, and time of activity on the internet due to these two sources; however, there are some additional parameters that are dependent on more than what the data provides. In our deterministic model the q values represent the proportion of people who go to either the G_1, G_2, or Z classes after they leave the susceptible class. These parameters can vary depending on the type of person, the situation in which the rumor is transmitted, and even the quality of the rumor itself.

However, in our model, since we assume that the only difference between the two populations is the level of activity, we assume that this is similar to that found in internet communication. From this, we can say that the only difference we need to consider are the types of rumors. We consider four cases, each with a different type of rumor.

First, we will look at a *frivolous* rumor, where susceptibles are likely to stay within their own levels of activity. We choose the values:

$$\{q_1 = .5, q_2 = .4, q_3 = .3, q_4 = .4, q_5 = .2,\\ q_6 = .2, q_7 = .5, q_8 = .34, q_9 = .5, q_{10} = .66\}$$

The second is an *interesting* rumor where both classes of susceptibles, upon hearing the rumor, decide that they would rather spread the rumor at a higher level of activity.

Here, the parameter values are set to be:

$$\{q_1 = .14, q_2 = .14, q_3 = .66, q_4 = .66, q_5 = .2,$$
$$q_6 = .2, q_7 = .34, q_8 = .34, q_9 = .66, q_{10} = .66\}$$

Third, we look at a *borning* rumor, where most of the susceptibles still decide to spread the rumor, but don't spread it at a high level of activity. In this case:

$$\{q_1 = .5, q_2 = .5, q_3 = .3, q_4 = .3, q_5 = .2,$$
$$q_6 = .2, q_7 = .67, q_8 = .67, q_9 = .33, q_{10} = .33\}$$

Finally, we look at an *unbelievable* rumor, in which the second class of susceptibles is more likely to go to the G_1 class, and the first class of susceptibles is going to be more likely to become stiflers. For this case:

$$\{q_1 = .3, q_2 = .3, q_3 = .2, q_4 = .2, q_5 = .5,$$
$$q_6 = .5, q_7 = .67, q_8 = .67, q_9 = 0.33, q_{10} = .33\}$$

The population sizes, N_1 and N_2, we gather from MSN Hotmail data. We consider two initial populations, people over 55 year of age, and people between the ages of 18 and 34. We will assume the elderly population to be less active on the internet, especially considering the lower fraction of the population that they represent. We also use information from the web study [16] to look at how many new accounts are created on average to determine the parameters necessary for the inflow to be determined. However, this fixes the death rate for this particular model and it would be more difficult to gather information from a population that is not constant. We represent our assumptions numerically as seen in Table 2.

Parameter	Value
k	2
c_1	0.1
μ	$\frac{1}{200}$
d	0.5

Table 2: Other parameter values for simulations

We will just say that for a short while, we will look at a constant population size on the internet. To look at the activity rates of each of the populations, we use the web study to look at the average length of time that the average person will spend online. From this, we make an approximation as to the average number of contacts the older population will have, which we take to be about .1. So every ten days the less active part of the population will tell one new person the rumor. Then we approximate the higher level of activity to be twice that of the lower activity. We discuss later the impact of having a higher level of activity, and what this does to the model.

3.6 Cases: Frivolous, Boring, Interesting, & Unbelievable

Numerical integration of each of the 4 cases gives the following curves:

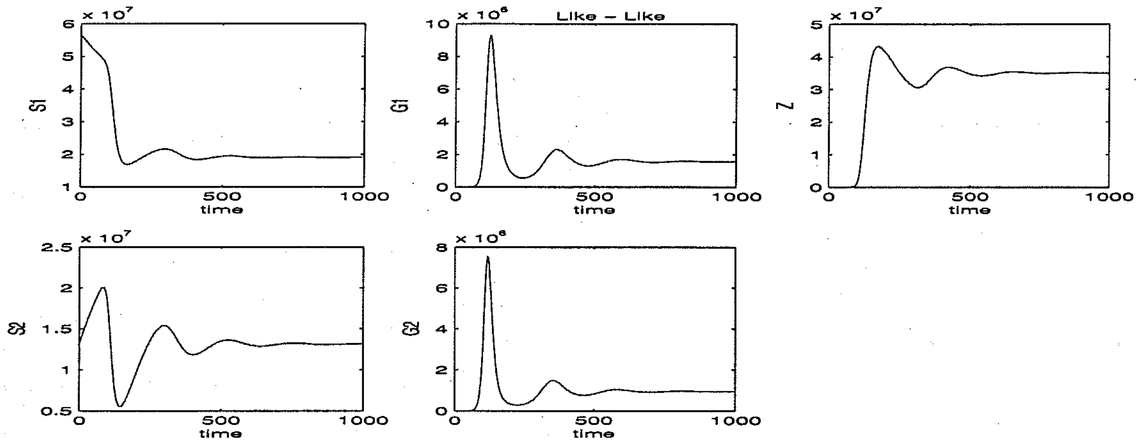

Figure 2: Frivolous Rumor Simulations

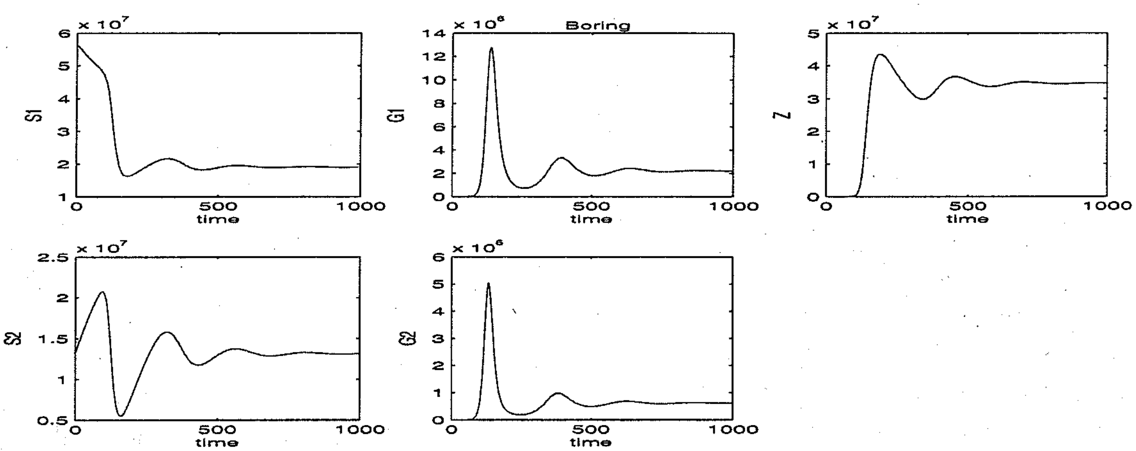

Figure 3: Boring Rumor Simulations

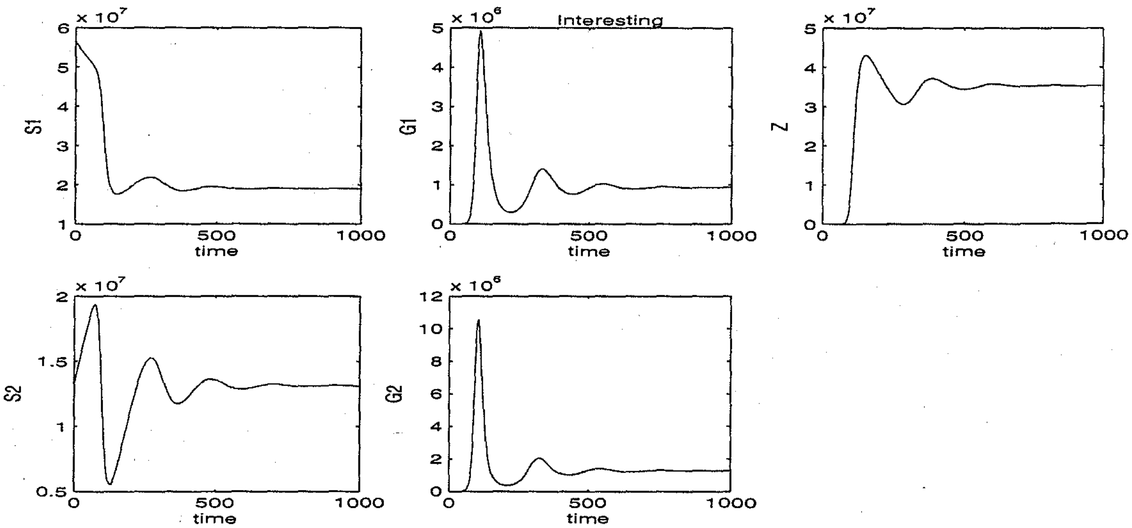

Figure 4: Interesting Rumor Simulations

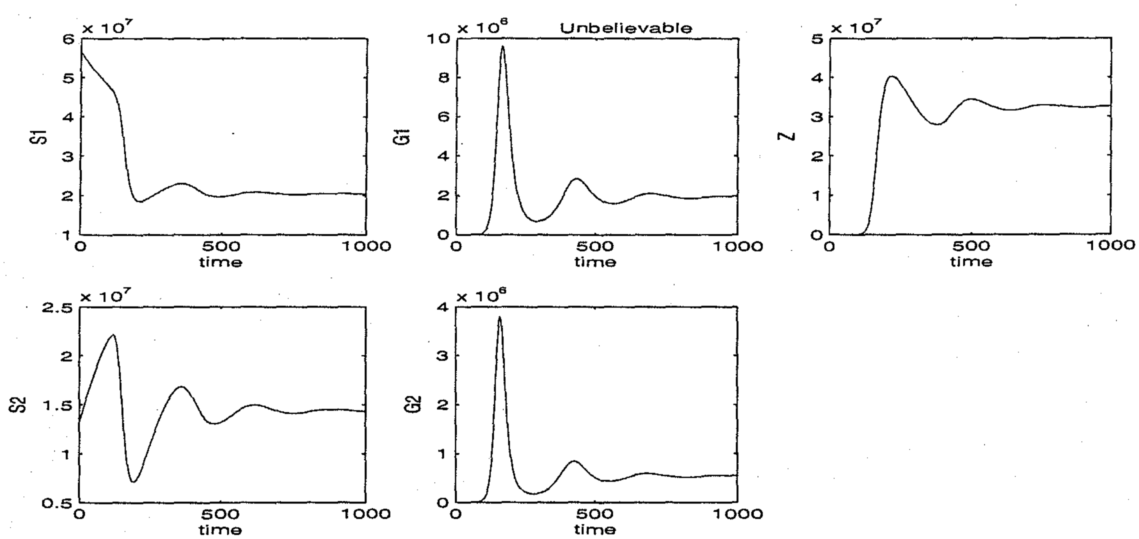

Figure 5: Unbelievable Rumor Simulations

One can see from Figures 2-5, that there is an endemic equilibrium that we were unable to solve for analytically. All of the equilibria seem to be similar, that is, the same level in the susceptible and stifler classes, with almost non-existent spreader classes. The behavior before the endemic equilibrium is reached also seems similar in each case. There is a rapid outbreak of the rumor from a small initial source, then the rumor appears to come close to dying out. Afterwards there is another surge and a secondary rumor wave occurs.

The difference between the different cases, that we can see numerically, seems to be the size of the rumor epidemic. The worst epidemic, when considering the absolute size of both spreader classes, seems to be the case of frivolous, because both spreader classes are growing at a large rate, and have very high peaks. The best case is when the rumor is considered to be unbelievable. People are leaving in large numbers straight from the S_1 class to the Z class, and this causes less transmissions of the rumors from both of the spreader classes. However, it is interesting to notice that in all cases the rumor will fade away to almost extinction, but that there seems to always be a large number of stiflers present in the population. This stifler class is also important in controlling the secondary rumor waves that continue to occur with smaller and smaller amplitudes as time progresses.

3.7 R_0 Numerically and Bifurcation

A fact easily noticeable from the numerical simulations of the different cases is that regardless of the values that we choose for q, that the population of people who know the rumor seems to explode. We then calculate the R_0 (Table 3) for each of our cases and find them to be fairly large.

Case	Numerical R_0
Frivolous	29.28
Boring	24.93
Interesting	31.73
Unbelievable	22.27

Table 3: Each Numerical R_0

Considering that $R_0 < 1$ is the condition necessary to insure the that the RFE will be stable this is alarming. The only parameters we seem to be able to change so that the R_0 will be less than one are c_1, k and μ. In order to force $R_0 < 1$ we have to change $\mu > 0.1$, which means that the population is cycling very rapidly. We discount k for modifying because we are asumming that N_2 has a greater activity rate, so k > 1. Lastly, we consider values of c_1 for which $R_0 < 1$ will hold, and we find that the value for c_1 is less than 0.001 (a person transmits a rumor once every thousand days). So we can conclude here that to prevent the outbreak we need either c_1, k, and μ to be unrealistic, or a lot of people leave from the S_1 to the Z class. Note that the parameters for the unbelievable rumor case seem to be somewhat effective in controlling the initial outbreak. The behavior of the endemic equilibrium, as seen in Figure 6 near the value of $R_0 = 1$ is:

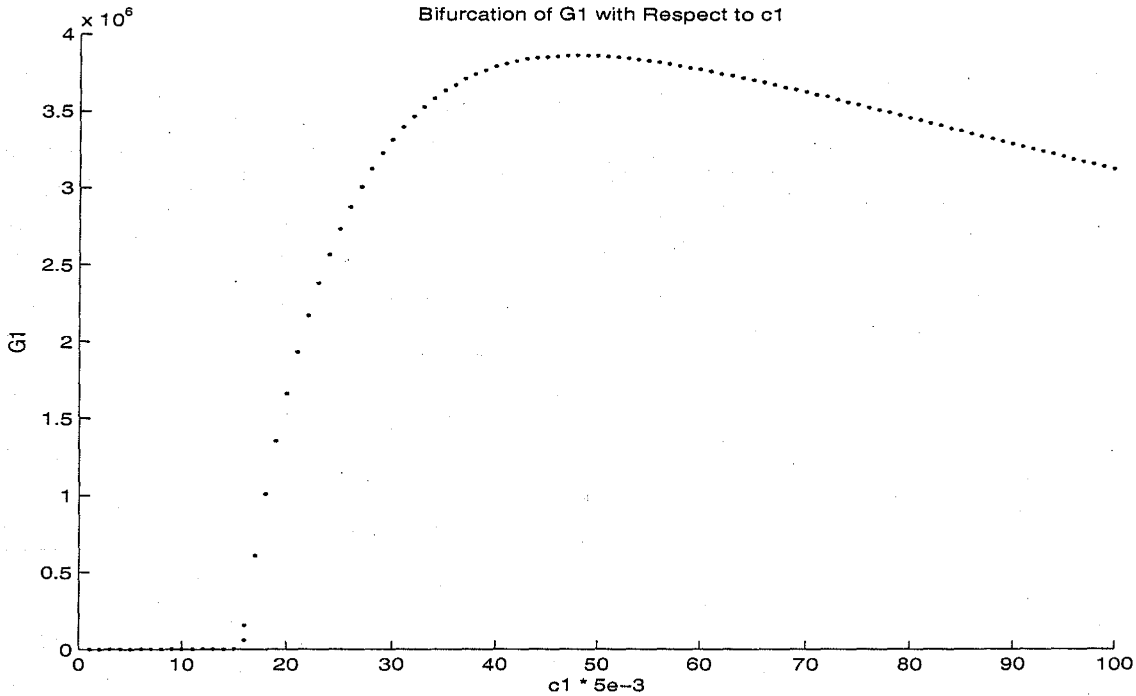

Figure 6: Bifurcation with respect to c_1

What we see is that as R_0 progresses to a very large number, the endemic equilibrium decreases. This has the interesting effect of showing us that the endemic equilibrium decreases with increased activity.

3.8 Sensitivity Analysis

Even though the R_0 for our two attitude model is relatively large, we can determine what incidence rates from the "Mini-Jacobian" have the largest impact on the initial rate of growth. We can do this using the forward normalized sensitivity indices[1] using the process described below.

For a 2x2 "Mini-Jacobian":

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

The Sensitivity Indices are described as the following:

$$S_{a_{ij}} = \frac{a_{ij}\partial\lambda}{\lambda\partial a_{ij}}$$

This will return the change in R_0 relative to the incidence rates in the "Mini-Jacbian" which is the normalized forward sensitivity index with respect to the matrix generating the eigenvalue. Since the largest eigenvalue of the "Mini-Jacobian" is R_0 we can say that $S_{a_{ij}} = \frac{a_{ij}\partial R_0}{R_0\partial a_{ij}}$, where a_{ij} is the given entry we choose. Using the method described in Arriola's preprinted book[2], we can determine the sensitivity for each case:

For Frivolous:
$$\{S_{a_{11}} = 0.0846, S_{a_{12}} = 0.1632, S_{a_{21}} = 0.1632, S_{a_{22}} = 0.5891\}$$
For Interesting:
$$\{S_{a_{11}} = 0.0285, S_{a_{12}} = 0.1398, S_{a_{21}} = 0.1398, S_{a_{22}} = 0.6921\}$$
For Boring:
$$\{S_{a_{11}} = 0.2368, S_{a_{12}} = 0.2498, S_{a_{21}} = 0.2498, S_{a_{22}} = 0.2635\}$$
For Unbelievable:
$$\{S_{a_{11}} = 0.2353, S_{a_{12}} = 0.2498, S_{a_{21}} = 0.2498, S_{a_{22}} = 0.2652\}$$

These indicate that in every case for our q values, apparently it is our incidence rate into G_2 due to contacts with other spreaders in G_2 that are most important when affecting R_0. Similar indices (Table 4) when calculated on R_0 with respect to different parameter values indicates that the activity rates for the different groups are also very sensitive in affecting the outcome of the initial growth rate. The indices for a particular case are:

Parameter	Sensitivity Index	Parameter	Sensitivity Index
c_1	1	q_7	.172
k	.553	q_8	.181
μ	-1	q_9	.172
q_1	.065	q_10	.181
q_2	.069		
q_3	.078		
q_4	.082		

Table 4: Sensitivity Indices

3.9 Levels of Activity

We notice that it is consistent throughout the numerical simulations that the activity rate has a very important affect on the endemic equilibrium. In the bifurcation diagram we see increased activity rates lead to a smaller endemic equilibrium. In the sensitivity analysis, we see that for the frivolous parameters case, the sensitivity to c_1 was highest, followed by k. Hence, the effect of increasing activity in the model was explored. The comparison of G_1 vs time for two different levels of activity, where $k = 5$, is represented in Figure 7.

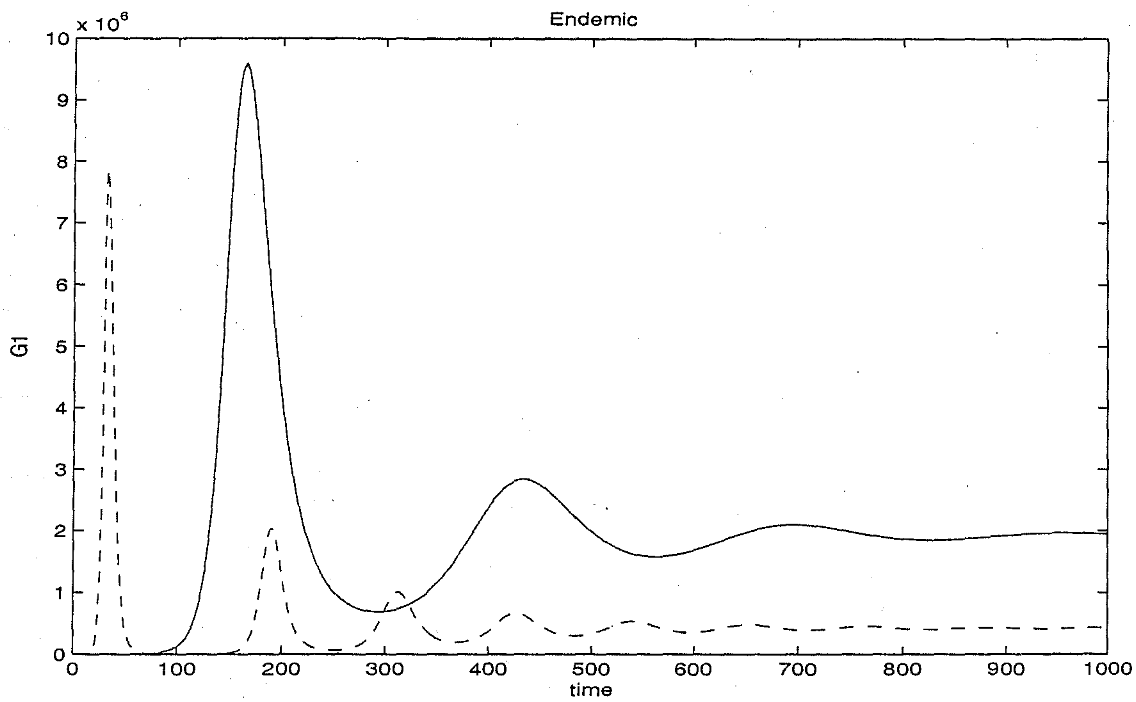

Figure 7: Endemic

The dotted line represents the higher activity rate, and the solid line represents the regular activity rate. We can see that to increase the rate of activity we speed up the time at which the outbreak reaches its peak. This higher activity had the affect of lowering the endemic equilibrium and dampening the secondary rumor waves. With the increased activity we can also see the population of spreaders seems to approach zero to a closer degree than with the regular activity levels. It appears as if the waves reach zero; however, we see a resurgence after the rumor dies out and gets very close to zero. But when carefully examined we see that the rumor waves do not actually reach zero (Figure 8).

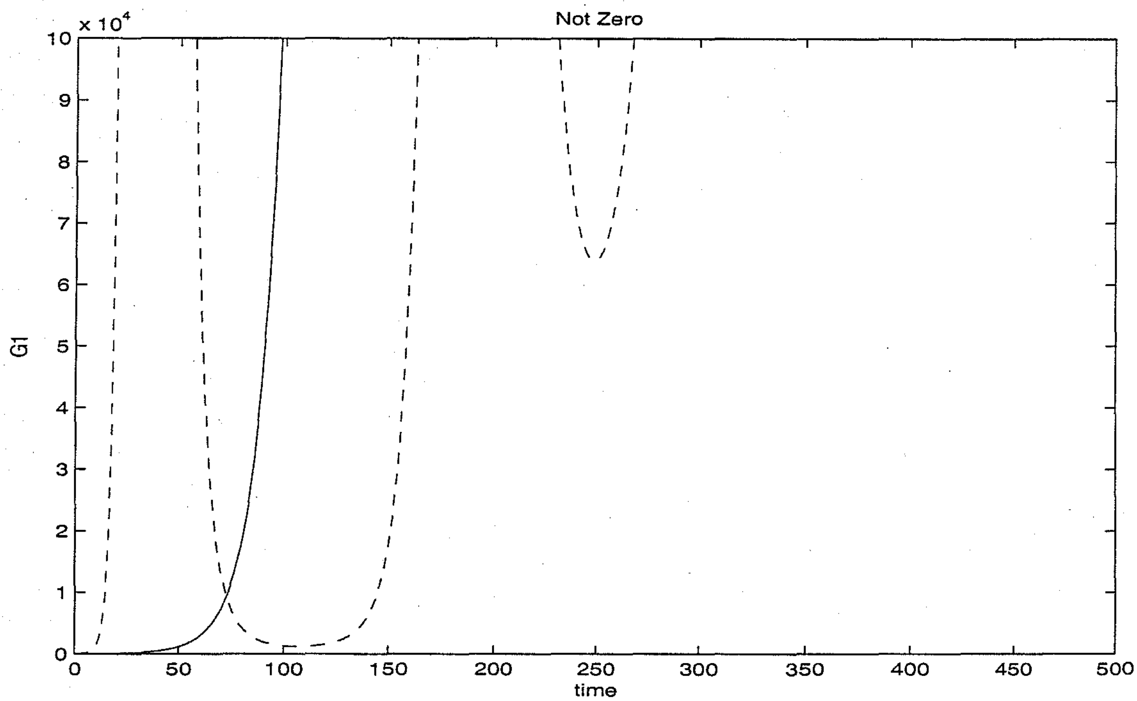

Figure 8: Waves do not reach Zero

4 Model for Internet Networks

4.1 Framework

Chat rooms have been increasing in popularity since the internet became wide spread around 1995. Chat rooms are in many places like individual computer terminals and small networks. Some web-pages begin by putting public chat rooms on their pages for anybody around the world to use. Nowadays, you can go to a public chat room and have a private chat with someone in the current chat room. Microsoft Network (MSN), Yahoo, ICQ, etc. provide you with a private chat room by creating an account on any one of these companies servers. Chat rooms are a useful tool to propagate a rumor very fast. For example, one person from New York can propagate one rumor to China in seconds just by sending an instant message.

Our model of rumors on chat rooms was made to generate numerical simulations of how a rumor propagates and infects people on the internet all over the world. The service that we chose to simulate was MSN instant messenger. This service provides for email, private chat rooms, and in many different ways can provide for public chat rooms and bulletin boards. We also chose this service because we can find a lot of current data for MSN. The number of people using the MSN Hotmail accounts totals 118 million people over the world, with 24,738,000 unique users of MSN instant messenger. The population of MSN instant messenger was divided into N groups. In each group the population is subdivided into Instant Messenger users and non-Instant Messenger users.

In this model it is assumed that the users have contacts with other users. Non-users also interact within their own group. Users will have contacts with users from different groups when they logon in the instant messenger. Obviously, non-users will have contact only within their own group. If a significant number of 'infected' people (gossipers) are introduced in the chat room, then the first case of gossiper infecting susceptible will occur in the user population. After the infection, newly infected individuals will then take the rumor back to their own groups generating infections in the non-user and user populations. Within each group, individuals fall into one of 3 classes according to the gossiper model, the classes in this model are X_i, Y_i, Z_i, denoting the numbers of users in the group (i) who are are susceptible, spreaders, and stiflers respectively. S_i, I_i, R_i are used for non-MSN IMU individuals. The total populationa are denoted by $Q_i = S_i + I_i + R_i$ and $T_i = X_i + Y_i + Z_i$ (Refer to Castillo-Chávez & Song [6]).

4.2 The Model

These equations represent people that are Microsoft Network(MSN) Instant Messenger users (IMU).

$$\frac{dX_i}{dt} = p\Lambda_i - A_i - \mu X_i \tag{13}$$

$$\frac{dY_i}{dt} = A_i - B_i - \mu Y_i \tag{14}$$

$$\frac{dZ_i}{dt} = B_i - \mu Z_i \tag{15}$$

These other equations represent people that are not MSN Instant Messenger users (NIMU).

$$\frac{dS_i}{dt} = (1-p)\Lambda_i - C_i - \mu S_i \tag{16}$$

$$\frac{dI_i}{dt} = C_i - D_i - \mu I_i \tag{17}$$

$$\frac{dR_i}{dt} = D_i - \mu R_i \tag{18}$$

Where:

$$A_i = \beta_i b_i X_i[\bar{P}_{a_i} \tfrac{I_i}{T_i\tau_i+Q_i} + \bar{P}_{b_i} \tfrac{Y_i\tau_i}{T_i\tau_i+Q_i} + \sum \bar{P}_{b_j^i} \tfrac{Y_j w_j}{T_i w_j}]$$

$$B_i = \beta_i b_i Y_i[\bar{P}_{b_i} \tfrac{Y_i}{T_i\tau_i+Q_i} + \bar{P}_{a_i} \tfrac{I_i}{T_i\tau_i+Q_i} + \bar{P}_{b_i} \tfrac{Z_i}{T_i\tau_i+Q_i} +$$

$$\bar{P}_{a_i} \tfrac{R_i}{T_i\tau_i+Q_i} + \sum \bar{P}_{b_j^i} \tfrac{Y_j w_j}{T_i w_j} + \sum \bar{P}_{b_j^i} \tfrac{Z_j w_j}{T_i w_j}]$$

$$C_i = \beta_i a_i S_i[\tilde{P}_{a_i} \tfrac{I_i}{T_i\tau_i+Q_i} + \tilde{P}_{b_i} \tfrac{Y_i\tau_i}{T_i\tau_i+Q_i}]$$

$$D_i = \beta_i a_i I_i[\tilde{P}_{a_i} \tfrac{I_i}{T_i\tau_i+Q_i} + \tilde{P}_{a_i} \tfrac{R_i}{T_i\tau_i+Q_i} + \tilde{P}_{b_i} \tfrac{Y_i\tau_i}{T_i\tau_i+Q_i} + \tilde{P}_{b_i} \tfrac{Z_i\tau_i}{T_i\tau_i+Q_i}]$$

$$Q_i = S_i + I_i + R_i$$

$$T_i = X_i + Y_i + Z_i$$

The constants a_i and b_i denote the per-capita contact rates of NIMU and IMU in the neighborhood i. In addition:

$$w_i = \frac{\rho_i}{\sigma_i + \rho_i} \quad \text{and} \quad \tau_i = \frac{\sigma_i}{\sigma_i + \rho_i}$$

represent the fraction of time spend in the chat room, where ρ_i and σ_i denote the rates at which the IMU get on and off the messenger, respectively. The P's are mixing probabilities described as:

1. $P_{a_i a_i} = \tilde{P}_{a_i} = \frac{a_i Q_i}{a_i Q_i + b_i \tau_i T_i}$ is the mixing probability between NIMU from the same neighborhood i.

2. $P_{a_i b_i} = \tilde{P}_{b_i} = \frac{b_i \tau_i T_i}{a_i Q_i + b_i \tau_i T_i}$ is the mixing probability between NIMU and IMU from the same neighborhood i.

3. $P_{b_ia_i} = \bar{P_{a_i}} = \frac{a_iQ_i}{a_iQ_i+b_i\tau_iT_i}\tau_i$ is the mixing probability IMU and NIMU from the same neighborhood i.

4. $P_{b_ib_i} = \bar{P_{b_i}} = \frac{b_i\tau_iT_i}{a_iQ_i+b_i\tau_iT_i}\tau_i$ is the mixing probability between IMU from the same neighborhood i.

5. $P_{b_ib_j} = \bar{P_{b_i^j}} = \frac{b_jw_jT_j}{\sum_{k=1}^{N} b_kw_kT_k}w_i$ is the mixing probability between IMU from neighborhoods i and j.

6. $P_{a_ia_j} = 0$ means NIMU from neighborhood i and j do not have contacts assuming $i \neq j$.

7. $P_{a_ib_j} = 0$ means IMU from neighborhood i and IMU from neighborhood j have no contacts assuming $i \neq j$.

$$\tilde{P_{a_i}} + \tilde{P_{b_i}} = 1, \quad i = 1, 2, ..., N. \tag{19}$$

$$\bar{P_{a_i}} + \bar{P_{b_i}} + \sum_{j=1}^{N} \bar{P_{b_j}} = \tau_i + w_i = 1, \quad i = 1, 2, ..., N. \tag{20}$$

4.3 Estimating Parameters

Our model simulates the propagation of rumor over the internet using the MSN instant messenger. First, we consider two groups, or sub-populations. The first group of people are between the ages of 18 - 34 and the second group is made up of people 55 years and older. 48% of the users are made up of our first group of people and only 11.1% make up the 55+. These people can make contacts depending on how many contacts they have in their contact lists, from that we took the β_1 for the first group to be 8 because this represents the average number of contacts that people within the ages 18-34 can make in one hour. For the second group we choose β_1 to be 2 because the average number of contacts that people of 55 years or over will be less than the younger group.

The older group of poeple are usually only on MSN instant messenger to talk with their family or business contacts. People can log on and off of MSN instant messenger, for this situation we have the parameters $\rho_1, \rho_2, \sigma_1, \sigma_2$. To get these values we looked at a MSN Advantage Marketing survey in which they described that people spend 24 minutes per day on MSN instant messenger. With this data we can find all of the rates at which people get on and off MSN instant messenger with the help of the online internet study. We approximate the contact rates a_i and b_i using the age group sizes and time spent online. The reasoning we use is that the group of 18-34 years will spend more time on the MSN instant messenger than the other group.

Since people spend 24 minutes per day online, we took the first group to have a per-capita contact rate of 2. We assume that the people who logon to MSN instant messenger have 3 times the average number of contacts than people who do not log on. For the second group the per-capita contact rate of users who logon, we approximate to be $\frac{1}{2}$, because we believe that this rate is representative of a lower level of activity. This rate is also three times the rate for people who do not use the MSN instant messenger. The recruitment rate describes the people that are creating new hotmail accounts, and the mortality rates are the people that are no longer users or users whose accounts are deleted by Microsoft for lack of use. The value Λ_1 comes from the number of new Hotmail accounts created. This Λ value in our model is multiplied by a probability, p, that represents the fraction of people who will create accounts and then use the instant messenger. We assume the value to be $\frac{1}{2}$. People that are non-users of MSN messenger have a recruitment rate of $(1-p)\Lambda$. The death rate depends on the values of Λ, the probabilities and the total of user or non-user populations. The size of user and non-user populations are described in Table 5.

N_1	Values	N_2	Values
X_0	$25\text{x}10^6 - 10^3$	X_0	$6\text{x}10^6$
Y_0	10^3	Y_0	0
Z_0	0	Z_0	0
S_0	$31\text{x}10^6 - 10^3$	S_0	$7\text{x}10^6$
I_0	10^3	I_0	0
R_0	0	R_0	0

Table 5: Initial Conditions

4.4 Results

Running a chat room simulation program we wrote, we found some very interesting results for the two groups. In the first group, we found an endemic equilibrium numerically in each class users. In the first group we found people that are 18-34 have an higher endemic equilibrium for users vs. non-users. As time passes, individuals move rapidly from the susceptible to spreader to stifler classes because there are a large number of contacts.

In the second group we found that people who are users seem to gossip more than the people that are non-users, also in all classes we can see that we have an endemic equilibrium. People of this age more care about the rumor, that is why we have fewer stiflers in this group than in the first group. In general we can say from this simulation that users of 18-34 gossip more than users 55 years old or over. We can conclude here that the chat room is a strong tool to spread gossip over the internet. The results are displayed in Figure 9 and Figure 10.

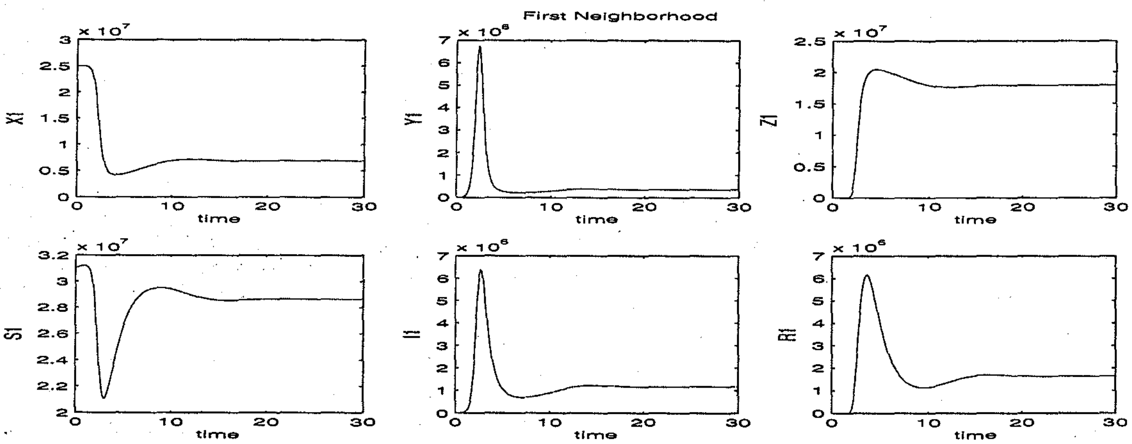

Figure 9: Neighborhood 1 Simulations

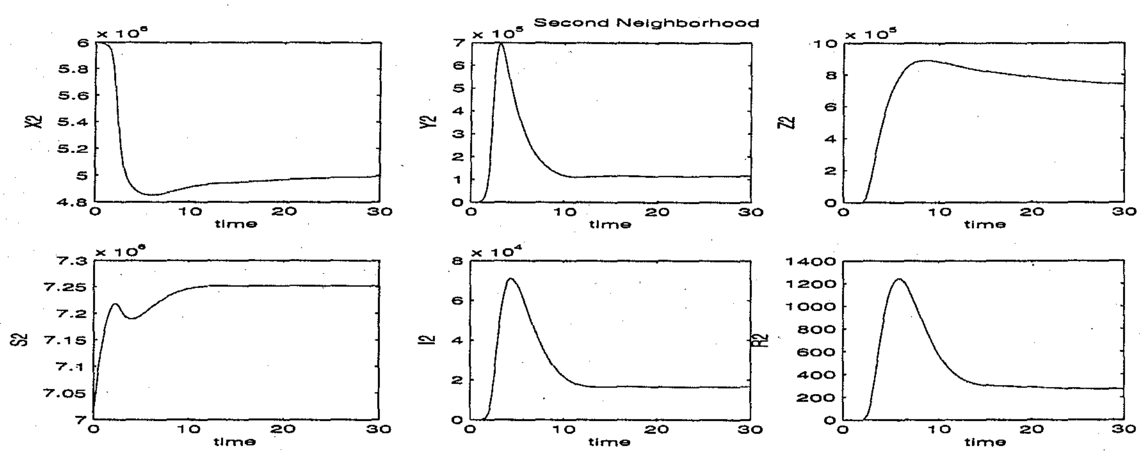

Figure 10: Neighborhood 2 Simulations

5 Concluding Remarks

With our two models for rumor propagation we notice a few similarities. First, when the models are run for a long period of time we find that there exists a small endemic equilibrium that we are unable to solve for analytically. There are also secondary rumor waves that seem to propagate along the solution that eventually dampen and seem to die out. There exists a basic reproductive number for both models, even though we are unable to find this number analytically in the second model, we know that it exists and can be used to control the outbreak of the rumor.

In the two class model there are several interesting aspects. First are the secondary rumor waves that appear every time when the number of stiflers drops followed by a sharp increase in the number of stiflers. These waves have a tendency to dampen and die out. Another is that with increased activity the level of the endemic equilibrium reduces but never quite reaches zero. We were able to determine the sensitivity of R_0 to different parameters, and in the cases that we looked at, we found that the level of activity was the most sensitive parameter. We found that increasing the level of activity actually seemed to help control the size of the outbreak. However, there could be some serious ethical problems in encouraging other people to spread the rumor. It should be noted that even if the outbreak of spreaders is controlled, there will still be a large proportion of individuals who will know the rumor. The safest course of action that we would suggest would be for people to follow the case of the unbelievable rumor, where we were able to show that there was a much smaller outbreak of the rumor. This would mean that people decide they do not want to spread the rumor once they have heard it. Therefore, there are two interesting ways to control the rumor.

In the internet model using the two social groups we were able to see some interesting results. We were able to observe an endemic, and the spread of a rumor from a class without the rumor into both groups. An unnerving but somewhat expected result from this simulation is people with the age between 18-34 are bigger gossipers regardless of whether they are chat users or not. There are still active gossipers in the group of 55 years old or older, but the younger group made more importance of the gossip in a short time than the other group.

6 Future Work

We would like to find if there exist some equilibrium point, when we have a distortion of the rumor, as time passes this distortion of the rumor is similar to the original rumor. Also, this research can be expanded to use partial differential equations, with changing time and age. The distortion of the rumor is an important class, for applications to the real life, but also we can make a experiment with more than one rumor in the model having the distortion class and see which rumor spends more time in the system and what group of people allow the continuance of this rumor. The simulation of spreading of rumor, but not only in chat rooms, but over the internet, TV, Radio, cell phones, news paper, magazines, mouth-to-mouth, etc. again using the partial differential equations of the age and time. Having two neighborhoods and two rumors are about of the other neighborhood, what will happen when they encounter each other, using all communication devices, no matter where the neighborhood. We would like to consider a growing population, because the size of the internet is obviously still growing, and it would be interesting to see the dynamics of information exchange over a rapidly growing population. Another interesting goal would be to consider a more complex social model that takes into account small-world networks, the formation of "cliques".

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8 Appendix

Finding R_0

The basic reproductive number (R_0) is defined as the average number of secondary infections produced when one infected individual is introduced into a host population where everyone is susceptible. Thus the basic reproduction number R_0 is often considered as the threshold quantity that determines when an infection can invade and persist in a new host population. Therefore, according to Herbert Hethcote paper the "rumor"-free equilibrium (RFE) is calculated as if disease were never introduced into the system[13]. From this we can see there should be no people in either the infected classes or the recovered class. If we take our system of equations (3)-(7), the RFE corresponds to:

$$\left(S_1 = \frac{\Lambda_1}{\mu}, S_2 = \frac{\Lambda_2}{\mu}, G_1 = 0, G_2 = 0, Z = 0\right)$$

Following finding the RFE we need to linearize the system and create a Jacobian matrix of our system. But because our system of equations is so complex and lengthy we omit this matrix. However, after evaluating the matrix at the RFE the matrix reduces to:

$$\begin{pmatrix} -\mu & 0 & -\frac{c_1^2\Lambda_1(q_1+q_3+q_{13})}{c_1\Lambda_1+c_2\Lambda_2} & -\frac{c_1c_2\Lambda_1(q_2+q_4+q_{14})}{c_1\Lambda_1+c_2\Lambda_2} & 0 \\ 0 & -\mu & -\frac{c_1c_2\Lambda_2(q_7+q_9)}{c_1\Lambda_1+c_2\Lambda_2} & -\frac{c_2^2\Lambda_2(q_8+q_{10})}{c_1\Lambda_1+c_2\Lambda_2} & 0 \\ 0 & 0 & \frac{c_1^2\Lambda_1q_1}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_1c_2\Lambda_2q_9}{c_1\Lambda_1+c_2\Lambda_2} - \mu & \frac{c_1c_2\Lambda_1q_2}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_2^2\Lambda_2q_{10}}{c_1\Lambda_1+c_2\Lambda_2} & 0 \\ 0 & 0 & \frac{c_1c_2\Lambda_2q_7}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_1^2\Lambda_1q_3}{c_1\Lambda_1+c_2\Lambda_2} & \frac{c_2^2\Lambda_2q_8}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_1c_2\Lambda_1q_4}{c_1\Lambda_1+c_2\Lambda_2} - \mu & 0 \\ 0 & 0 & \frac{c_1^2\Lambda_1q_{13}}{c_1\Lambda_1+c_2\Lambda_2} & \frac{c_1c_2\Lambda_2q_{14}}{c_1\Lambda_1+c_2\Lambda_2} & -\mu \end{pmatrix}$$

In order to calculate the eigenvalues of the matrix we need to find the determinant of $(A - \lambda I)$. Using the method found in Braun for calculating eigenvalues[3] we immediately see that $-\mu$ is an eigenvalue with multiplicity three. In finding these three eigenvalues we see that the expansion of the determinant has "removed" the first, second, and fifth columns and rows leaving us with a "Mini-Jacobian":

$$\begin{pmatrix} \frac{c_1^2\Lambda_1q_1}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_1c_2\Lambda_2q_9}{c_1\Lambda_1+c_2\Lambda_2} - \mu & \frac{c_1c_2\Lambda_1q_2}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_2^2\Lambda_2q_{10}}{c_1\Lambda_1+c_2\Lambda_2} \\ \frac{c_1c_2\Lambda_2q_7}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_1^2\Lambda_1q_3}{c_1\Lambda_1+c_2\Lambda_2} & \frac{c_2^2\Lambda_2q_8}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_1c_2\Lambda_1q_4}{c_1\Lambda_1+c_2\Lambda_2} - \mu \end{pmatrix} \tag{21}$$

At this point we want to find the stability of the RFE. We need all the eigenvalues of the matrix to be negative so we can use the Routh-Hurwitz criteria where n=2. The criteria for n=2 requires that the determinant be positive and the trace be negative; however, for our system explicit formulas satisfying both of these conditions is difficult due to the complexity of the entries. Although, after inspecting this matrix we noticed that this is the $M-D$ "Mini-Jacbian" one obtains using the Next Generation Operator (NGO). By using this approach

R_0 is simply the largest eigenvalue of the final "Mini-Jacboian" matrix. Continuing the NGO process from (10):

$$M = \begin{pmatrix} \frac{c_1^2\Lambda_1 q_1}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_1c_2\Lambda_2 q_9}{c_1\Lambda_1+c_2\Lambda_2} & \frac{c_1c_2\Lambda_1 q_2}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_2^2\Lambda_2 q_{10}}{c_1\Lambda_1+c_2\Lambda_2} \\ \frac{c_1c_2\Lambda_2 q_7}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_1^2\Lambda_1 q_3}{c_1\Lambda_1+c_2\Lambda_2} & \frac{c_2^2\Lambda_2 q_8}{c_1\Lambda_1+c_2\Lambda_2} + \frac{c_1c_2\Lambda_1 q_4}{c_1\Lambda_1+c_2\Lambda_2} \end{pmatrix} \tag{22}$$

$$D = \begin{pmatrix} \mu & 0 \\ 0 & \mu \end{pmatrix} \tag{23}$$

Therefore:

$$D^{-1} = \begin{pmatrix} \frac{1}{\mu} & 0 \\ 0 & \frac{1}{\mu} \end{pmatrix} \tag{24}$$

$$MD^{-1} = \begin{pmatrix} \frac{c_1(c_1\Lambda_1 q_1+c_2\Lambda_2 q_9)}{\mu(c_1\Lambda_1+c_2\Lambda_2)} & \frac{c_2(c_1\Lambda_1 q_2+c_2\Lambda_2 q_{10})}{\mu(c_1\Lambda_1+c_2\Lambda_2)} \\ \frac{c_1(c_1\Lambda_1 q_3+c_2\Lambda_2 q_7)}{\mu(c_1\Lambda_1+c_2\Lambda_2)} & \frac{c_2(c_1\Lambda_1 q_4+c_2\Lambda_2 q_8)}{\mu(c_1\Lambda_1+c_2\Lambda_2)} \end{pmatrix} \tag{25}$$

At this point the eigenvalues are not easier to calculate than before so it is necessary to simplify. We examine any random 2 x 2 matrix:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \text{Where } a \geq 0, b \geq 0, c \geq 0, d \geq 0$$

The characteristic polynomial is:

$$\lambda^2 - (a+d)\lambda + ad - cb = 0$$

From this we calculate the eigenvalues of:

$$\lambda_1 = \frac{(a+d) + \sqrt{(a+d)^2 - 4(ad-cb)}}{2}$$

$$\lambda_2 = \frac{(a+d) - \sqrt{(a+d)^2 - 4(ad-cb)}}{2}$$

$$\text{Where } (a+d)^2 - 4(ad-bc) > 0$$

Since all of our parameters {a,b,c,d} are positive and we assume we meet the condition for real eigenvalues; $\lambda_1 > \lambda_2$. Using NGO it follows that $R_0 = \lambda_1$. For stability of the DFE we need $R_0 < 1$ we need do some more simplification so we can better analyze the meaning of R_0.

$$\lambda_1 = \frac{(a+d) + \sqrt{(a+d)^2 - 4(ad-cb)}}{2} < 1$$

$$\sqrt{(a+d)^2 - 4(ad-cb)} < 2 - (a+d)$$

$$(a+d)^2 - 4(ad-cb) < 4 - 4(a+d) + (a+d)^2$$

$$-4(ad-cb) < 4 - 4(a+d)$$

$$-4(ad-cb) + 4(a+d) < 4$$

$$-(ad-cb) + (a+d) < 1$$

$$(a+d) - (ad-cb) < 1$$

This says that the trace minus the determinant must be less than one in order for the DFE to be stable. We can now simply substitute for the parameters that we obtain from the entries of the (21).

$$R_0 = \left(\frac{c_1(c_1\Lambda_1 q_1 + c_2\Lambda_2 q_9)}{\mu(c_1\Lambda_1 + c_2\Lambda_2)} + \frac{c_2(c_1\Lambda_1 q_4 + c_2\Lambda_2 q_8)}{\mu(c_1\Lambda_1 + c_2\Lambda_2)}\right)$$

$$-\left(\frac{c_1(c_1\Lambda_1 q_1 + c_2\Lambda_2 q_9)}{\mu(c_1\Lambda_1 + c_2\Lambda_2)}\frac{c_2(c_1\Lambda_1 q_4 + c_2\Lambda_2 q_8)}{\mu(c_1\Lambda_1 + c_2\Lambda_2)} - \frac{c_2(c_1\Lambda_1 q_2 + c_2\Lambda_2 q_{10})}{\mu(c_1\Lambda_1 + c_2\Lambda_2)}\frac{c_1(c_1\Lambda_1 q_3 + c_2\Lambda_2 q_7)}{\mu(c_1\Lambda_1 + c_2\Lambda_2)}\right) < 1$$

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