

The Role of Transactional Sex in the Spread of HIV/AIDS: A Modeling Perspective

MTBI-02-13M

Titus G.Kassem^{1,2}, Svetlana Roudenko²,
Stephen Tennenbaum³, Carlos Castillo-Chávez²

¹Department of Mathematics, University of Jos, Jos, Nigeria

²Department of Mathematics and Statistics, Arizona State University, Tempe, AZ

³Department of Biometrics, Cornell University, Ithaca NY

Abstract

The sex industry has been implicated in the spread of HIV across the world. In this article we propose a simple theoretical model consisting of two core groups of interacting heterosexual populations. One of the core group consists of male truck drivers and the other group consists of female sex workers. The truck drivers need for entertainment and female companionship make them use the services of the female sex workers in stop-over towns near major transportation routes. The resulting co-mingling of these sexually active, high-risk populations not only explains high prevalence of HIV in truck drivers and female sex workers and the subsequent spread of the disease in general population, but also points out the magnitude of the problem and the urgency of introducing effective controls. Our model assumes (i) a low level of condom use among the trucking population and female sex workers, (ii) high level of HIV in both truck drivers and female sex workers, and (iii) continuous recruitment in both groups when losses due to AIDS or natural factors occur. We give the complete analysis of the disease free and endemic equilibria. With that we also show the effect of reducing HIV cases in both groups by lowering of HIV transmission rates (e.g. by using condoms).

1 Introduction

The sex industry is a major factor in spreading of the human immune deficiency (HIV) across several countries in sub-Saharan Africa and Asia (e.g. Hiesh and Chen, 2003, [HC03]). In Nigeria studies have identified “bridging” populations such as long distance truck drivers, commercial motor cycle riders and the uniformed services who are the primary clients of female sex workers as major contributors to the spread of HIV in the general population (Idoko, 2004, [I04]). The female sex workers are considered as one of the risk groups driving the epidemic because of their high HIV levels and exposure to multiple partners while the truck drivers are at elevated risk because they spend many nights

away from home and their need for entertainment and female companionship makes them use the services of the female sex workers in stop-over towns near major transportation routes. Although sex workers are often subject to great deal of stigma and exploitation, the industry has continued to thrive because of extreme poverty and falling standard of living in Nigeria. Thus, the female sex workers enter the profession out of necessity and only quit when they can. On the other hand, truck driving is a lucrative profession which unskilled males are willing to take, when such opportunities present themselves because of enormous material and monetary benefit associated with it by local standards. Though the truck drivers are engaged in legitimate businesses, they are often seen as a "bridge" population, or the one through which HIV reaches the larger population, particularly people who are considered at lower risk. Most of the truck drivers have wives and other sexual partners in their communities who are always at risk of HIV infection by the truckers. In 1991 a study of truck drivers' sexual cultures was conducted in which the truck drivers reported an average of 6.3 current sexual partners (sex workers), 12 sexual partners during the previous year and 25 partners besides their wife during a lifetime (Orubuloye et al., 1993, [Or93]). A similar study of truck drivers between 1999 and 2001 (Ogboi et al, 2001, [Og01]) found out that the prevalence of HIV infection among truck drivers in Nigerian transit towns was 54 percent as compared to 17 percent in the non-transit towns. A number of studies have documented similar findings across sub-Saharan Africa. Studies of an area along a major highway in Uganda have found an HIV prevalence of 35 percent among truck drivers, and 37 percent of truckers estimated having more than 50 female sexual partners during their lifetimes (Von Reyn, 1990, [VR90]).

The spread of HIV has reached an epidemic proportion in Nigeria and its impact on the transport industry is especially significant when considering that truck drivers are largely transporting goods from the rural areas to major urban centers. In addition, young men and women are being lost to AIDS in their productive years. In 2001, an estimated number of adults and children who died of AIDS was 170,000 (Epidemiological Fact sheets, 2002 update). Thus, most efforts at the understanding the dynamics of the AIDS epidemic within risk groups have been targeted at the two core populations, namely, the truck drivers and female sex workers. For instance, Orubuloye et al in [Or93] considered the role of high-risk occupations in the spread of AIDS among truck drivers and itinerant market women in Nigeria along the Ilorin-Ibadan-Lagos highway and concluded that occupation demands had resulted in a network of multiple partners. Sunmora 2005, [S05] investigated sexual practices and barriers to condom use among truck drivers in Nigeria and concluded that the use of condom among the truck drivers was only 9 percent, though about 70 percent of them knew about condom HIV preventative measure. On the other hand, condom usage is generally acceptable by female sex workers (Orubuloye et al, 1999, [Or99]), but their clients sometimes insist on non usage, thus placing the sex workers as well as their clients at risk of contracting HIV.

In this study, we consider via mathematical modeling the role of transactional sex as it affects the dynamics of HIV/AIDS within these two core groups as a first step on our study on its implication on the general population.

The paper is constructed as follows: in the next section we formulate the main model, after which we analyze the disease-free equilibrium and the endemic equilibrium in Sections 3.1 and 3.2, correspondingly. In Section 4 we study the dependence of the basic reproductive number and the endemic equilibrium on given parameters, in particular, for the latter we include hypothetical simulations of effects from decreasing transmission rates (by using condoms), see Figure 1 and 2 as well as Table 2. Section 5 describes the data we have gathered and parameter estimation we use. The next to last section contains conclusions, discussion, future work, and we conclude with acknowledgments in the last section.

2 Model formulation

For our model we consider a scenario in which the two core groups are experiencing the epidemic (we show that this corresponds to data gathered, refer to Section 5) and act as a reservoir through which the disease spreads in the general population. The model considers the two core groups only and consists of epidemiological processes, namely, the acquisition of infection by the truck drivers and female sex workers, losses in both groups due to AIDS (and natural factors) but an immediate recovery in both groups by the recruitment of new members because of the economic benefits.

2.1 Assumptions

- Sex with clients other than truck drivers by females sex workers is not considered.
- Low or non-existent condom usage by the truck drivers who patronize the services of the female sex workers.
- Random mixing between the two groups.
- Transmission rates are constant over the life of the disease.
- Each of the core groups have losses due to AIDS but the populations size of both core groups remain fixed, since each truck driver and sex worker gets replenished immediately.

Let S_m and S_f denote the number of susceptible truck drivers and female sex workers respectively. Let I_m and I_f be the number of infected male truck drivers and females sex workers, respectively; and A_m and A_f be the number of truck drivers and females sex workers who have developed AIDS, correspondingly. We derive the following system of

differential equations which describes the above dynamics.

$$\begin{aligned}
\dot{S}_m &= \mu_m N_m + \psi_m A_m - \mu_m S_m - \beta_1 S_m \frac{I_f}{K_f} \\
\dot{S}_f &= \mu_f N_f + \psi_f A_f - \mu_f S_f - \beta_2 S_f \frac{I_m}{K_m} \\
\dot{I}_m &= \beta_1 S_m \frac{I_f}{K_f} - (\gamma_m + \mu_m) I_m \\
\dot{I}_f &= \beta_2 S_f \frac{I_m}{K_m} - (\gamma_f + \mu_f) I_f \\
\dot{A}_m &= \gamma_m I_m - (\psi_m + \mu_m) A_m \\
\dot{A}_f &= \gamma_f I_f - (\psi_f + \mu_f) A_f.
\end{aligned} \tag{1}$$

Here, the total populations of truck drivers is $N_m = S_m + I_m + A_m$ and commercial sex workers is $N_f = S_f + I_f + A_f$. We denote the total number of susceptibles and infected by K : $K_m = S_m + I_m$ and $K_f = S_f + I_f$.

The parameters used in the system (1) are indicated in the table:

Symbol	Description
β_1	Rate at which female sex workers infect truck drivers
β_2	Rate at which truck drivers infect female sex workers
μ_m	Natural mortality rate of truck drivers
μ_f	Natural mortality rate of sex workers
ψ_m	Rate at which truck drivers are lost due to AIDS
ψ_f	Rate at which sex workers are lost due to AIDS
γ_m	Rate at which infected truck drivers progress to AIDS
γ_f	Rate at which infected sex workers progress to AIDS

Table 1: Model parameters

Since we assume the constant number of workers in both groups ($N_f = \text{const}$ and $N_m = \text{const}$), the above equations reduce to

$$\begin{aligned}
\dot{S}_m &= \mu_m N_m + \psi_m (N_m - K_m) - \mu_m S_m - \beta_1 S_m \frac{I_f}{K_f} \\
\dot{S}_f &= \mu_f N_f + \psi_f (N_f - K_f) - \mu_f S_f - \beta_2 S_f \frac{I_m}{K_m} \\
\dot{I}_m &= \beta_1 S_m \frac{I_f}{K_f} - (\gamma_m + \mu_m) I_m \\
\dot{I}_f &= \beta_2 S_f \frac{I_m}{K_m} - (\gamma_f + \mu_f) I_f.
\end{aligned} \tag{2}$$

This system is analyzed in the paper.

3 Analysis

We start our analysis with finding biologically meaningful equilibria of system (2), by setting the left-hand side of (2) equal to zero.

$$\begin{aligned}
 0 &= \mu_m N_m + \psi_m(N_m - K_m) - \mu_m S_m - \beta_1 S_m \frac{I_f}{K_f} \\
 0 &= \mu_f N_f + \psi_f(N_f - K_f) - \mu_f S_f - \beta_2 S_f \frac{I_m}{K_m} \\
 0 &= \beta_1 S_m \frac{I_f}{K_f} - (\gamma_m + \mu_m) I_m \\
 0 &= \beta_2 S_f \frac{I_m}{K_m} - (\gamma_f + \mu_f) I_f.
 \end{aligned} \tag{3}$$

3.1 The Disease Free case

First we study the case when the population is free from HIV infections, i.e. $I_m = I_f = 0$. In this case from (3) we obtain the disease free equilibrium $E_0 = (N_m, N_f, 0, 0)$. We compute the basic reproduction number using the next generation operator (e.g. Castillo-Chavez et al, 2002, [CCC02] or Diekmann et al, 1990, [D90]), which is

$$R_0 = \sqrt{\frac{\beta_1 \beta_2}{(\gamma_m + \mu_m)(\gamma_f + \mu_f)}}. \tag{4}$$

The Jacobian matrix at the disease free equilibrium E_0 is

$$J = \begin{pmatrix}
 -(\psi_m + \mu_m) & 0 & -\psi_m & -\beta_1 \frac{N_m}{N_f} \\
 0 & -(\psi_f + \mu_f) & -\beta_2 \frac{N_f}{N_m} & -\psi_f \\
 0 & 0 & -(\gamma_m + \mu_m) & \beta_1 \frac{N_m}{N_f} \\
 0 & 0 & \beta_2 \frac{N_f}{N_m} & -(\gamma_f + \mu_f)
 \end{pmatrix}. \tag{5}$$

The characteristic equation for this case is

$$(\lambda + \psi_m + \mu_m)(\lambda + \psi_f + \mu_f)[(\lambda + \gamma_m + \mu_m)(\lambda + \gamma_f + \mu_f) - \beta_1 \beta_2] = 0. \tag{6}$$

All eigenvalues are negative if and only if $\beta_1 \beta_2 < (\gamma_m + \mu_m)(\gamma_f + \mu_f)$. This implies that the disease free equilibrium is locally stable when $R_0 < 1$ and unstable otherwise.

3.2 Endemic equilibrium

In order to determine the endemic equilibrium we substitute $S_m = K_m - I_m$ and $S_f = K_f - I_f$ in the last two equations of system (3), multiply by K_f and K_m , correspondingly, and obtain

$$\beta_1 K_m I_f - \beta_1 I_m I_f - (\gamma_m + \mu_m) K_f I_m = 0,$$

$$\beta_2 K_f I_m - \beta_2 I_f I_m - (\gamma_f + \mu_f) K_m I_f = 0,$$

which, after eliminating $K_m I_f$ term in both equations and canceling I_m (since $I_m \neq 0$), amounts to

$$I_f + \frac{\gamma_m + \mu_m}{\beta_1} K_f = \frac{\beta_2}{\gamma_f + \mu_f} K_f - \frac{\beta_2}{\gamma_f + \mu_f} I_f, \quad (7)$$

Solving for I_f in (7) (and similarly for I_m), we get the following expressions

$$I_f = c_f K_f, \quad (8)$$

$$I_m = c_m K_m, \quad (9)$$

where

$$c_f = \frac{\beta_1 \beta_2 - (\gamma_m + \mu_m)(\gamma_f + \mu_f)}{\beta_1(\gamma_f + \mu_f + \beta_2)}, \quad (10)$$

$$c_m = \frac{\beta_1 \beta_2 - (\gamma_m + \mu_m)(\gamma_f + \mu_f)}{\beta_2(\gamma_m + \mu_m + \beta_1)}. \quad (11)$$

Substituting c_m and c_f for the I_m/K_m and I_f/K_f in the first two equations in (3), we obtain the expression for I_f and I_m at the endemic equilibrium

$$\tilde{I}_f = \frac{(\mu_f + \psi_f)c_f}{(\psi_f + (1 - c_f)(\beta_2 c_m + \mu_f))} N_f, \quad (12)$$

$$\tilde{I}_m = \frac{(\mu_m + \psi_m)c_m}{(\psi_m + (1 - c_m)(\beta_1 c_f + \mu_m)(1 - c_f))} N_m, \quad (13)$$

where $0 \leq c_f, c_m \leq 1$. The values of S_m and S_f are determined by

$$\tilde{S}_f = \frac{1 - c_f}{c_f} \tilde{I}_f,$$

$$\tilde{S}_m = \frac{1 - c_m}{c_m} \tilde{I}_m.$$

Denote this equilibrium by $\tilde{E} = (\tilde{S}_m, \tilde{S}_f, \tilde{I}_m, \tilde{I}_f)$. The linearization of system (2) at the interior equilibrium \tilde{E} , gives the following characteristic equation

$$\begin{vmatrix} \beta_1 c_f + \mu_m + \lambda & 0 & \frac{\psi_m}{c_m} & 0 \\ 0 & \beta_2 c_m + \mu_f + \lambda & 0 & \frac{\psi_f}{c_f} \\ -\beta_1 c_f & 0 & \gamma_m c_f + \mu_m + \lambda & 0 \\ 0 & -\beta_2 c_m & 0 & \gamma_f + \mu_f + \lambda \end{vmatrix} = 0. \quad (14)$$

This is equivalent to

$$\left[(\beta_2 c_m + \mu_f + \lambda)(\gamma_f + \mu_f + \lambda) + \beta_2 \psi_f \frac{c_m}{c_f} \right] \times$$

$$\left[(\beta_1 c_f + \mu_m + \lambda)(\gamma_m + \mu_m + \lambda) + \beta_1 \psi_m \frac{c_f}{c_m} \right] = 0.$$

Equating to zero each factor in the left-hand side of the above equation, we have

$$\lambda^2 + (\beta_1 c_f + 2\mu_m + \gamma_m)\lambda + (\beta_1 c_f + \mu_m)(\gamma_m + \mu_m) + \beta_1 \psi_m \frac{c_f}{c_m} = 0, \quad (15)$$

$$\lambda^2 + (\beta_2 c_m + 2\mu_f + \gamma_f)\lambda + (\beta_2 c_m + \mu_f)(\gamma_f + \mu_f) + \beta_2 \psi_f \frac{c_m}{c_f} = 0. \quad (16)$$

The solutions are

$$\lambda_{1,2} = -\frac{1}{2} \left((\beta_1 c_f + 2\mu_m + \gamma_m) \pm \sqrt{(\beta_1 c_f - \gamma_m)^2 - 4\beta_1 \psi_m \frac{c_f}{c_m}} \right)$$

and

$$\lambda_{3,4} = -\frac{1}{2} \left((\beta_2 c_m + 2\mu_f + \gamma_f) \pm \sqrt{(\beta_2 c_m - \gamma_f)^2 - 4\beta_2 \psi_f \frac{c_m}{c_f}} \right).$$

To establish conditions under which the real part of all λ_i ($i = 1, 2, 3, 4$) is negative, we consider $c_m, c_f \geq 0$. For brevity denote $\lambda_{1,2} = -\frac{1}{2}(b \pm \sqrt{D})$. Then

$$b^2 = (\beta_1 c_f + 2\mu_m + \gamma_m)^2 > (\beta_1 c_m)^2 + \gamma^2 > (\beta_1 c_f)^2 + \gamma_m^2 - 2\beta_1 c_f \gamma_m - 4\beta_1 \psi_m \frac{c_f}{c_m} = D.$$

Therefore, $b^2 > D$, and so $\lambda_{1,2} < 0$. Similarly $\lambda_{3,4} < 0$.

Observe that all entries in \tilde{E} are positive. This is seen from the fact that all parameters involved are positive and $0 \leq c_m, c_f \leq 1$. Therefore, for example, the denominator in (13) is positive (similarly, in (12)) and the whole quantity is non-negative as well.

Here, we remark on the range of c_m and c_f . Positivity of these coefficients is insured when $R_0 \geq 1$. By the definition of R_0 , we have the following inequality

$$\beta_1 \beta_2 \geq (\gamma_m + \mu_m)(\gamma_f + \mu_f). \quad (17)$$

This shows that in our current model the disease-free equilibrium is unstable and HIV spread is growing (monotonously or oscillating) to the endemic equilibrium, existence of which is guaranteed by (17). Note that (17) also implies negativity of eigenvalues for the endemic equilibrium. Therefore, in order to get rid of the disease (i.e. to make \tilde{E} coincide with E_0 or become negative and thus biologically impossible), the condition on the parameters must be

$$\beta_1 \beta_2 < (\gamma_m + \mu_m)(\gamma_f + \mu_f).$$

For further convenience, we rewrite \tilde{E} . Denote by r_m the number of secondary infections an infected male truck driver causes in females in a disease-free population over the length of time a truck driver is both employed and infected, and by r_f the corresponding value for infections caused by females (HIV transmitted from males to females). That is,

$$r_m = \frac{\beta_2}{\gamma_m + \mu_m} \quad \text{and} \quad r_f = \frac{\beta_1}{\gamma_f + \mu_f}.$$

Observe that the basic reproductive number is the geometric mean of the above values $R_0 = \sqrt{r_m \cdot r_f}$. Rewriting the endemic equilibrium value (13) for I_m in terms of r_m and r_f , we obtain

$$\tilde{I}_m = \frac{(\mu_m + \psi_m) \frac{r_m r_f - 1}{r_f (r_m + \beta_1 / \beta_2)}}{\psi_m + \frac{\beta_1 / \beta_2 r_f + 1}{r_f (r_m + \beta_1 / \beta_2)} (\beta_1 \frac{r_m r_f - 1}{r_m (r_f + \beta_2 / \beta_1)} + \mu_m)} N_m, \quad (18)$$

or equivalently, in terms of R_0

$$\tilde{I}_m = \frac{(\mu_m + \psi_m) \frac{R_0 - 1}{R_0 + \beta_2 / (\mu_f + \gamma_f)}}{\psi_m + \frac{\beta_2 / (\mu_f + \gamma_f) + 1}{R_0 + \beta_2 / (\mu_f + \gamma_f)} (\beta_1 \frac{R_0 - 1}{R_0 + \beta_1 / (\mu_m + \gamma_m)} + \mu_m)} N_m.$$

The expression (18) will be used to analyze the dependence of \tilde{I}_m on parameters. A similar expression can be obtained for \tilde{I}_f .

4 Sensitivity Analysis

4.1 Local sensitivity analysis of the Basic Reproductive Number

In this section, we use sensitivity analysis to compute the sensitivity indices of model parameters through local derivatives (for examples see [AH05] or [C04]). This approach gives a local measure as the sensitivity can change when the values change.

Consider the basic reproductive number

$$R_0(\beta_1, \beta_2, \gamma_m, \gamma_f, \mu_m, \mu_f) = \sqrt{\frac{\beta_1 \beta_2}{(\gamma_m + \mu_m)(\gamma_f + \mu_f)}}.$$

Suppose δp is some perturbation to the parameter p where p is any of the six parameters: β_1 , β_2 , γ_m , γ_f , μ_m and μ_f . Let δR_0 be the resulting perturbation to R_0 . Then the normalized forward sensitivity index is defined by

$$Q_p = \frac{\delta R_0}{R_0} / \frac{\delta p}{p},$$

provided $R_0, p \neq 0$ and in the limit is equal to $\frac{p}{R_0} \frac{\partial R_0}{\partial p}$. A linear approximation of the perturbed value R_0 in terms of the sensitivity is

$$R_0(p + \delta p) \sim \left(1 + Q_p \frac{\delta p}{p}\right) R_0,$$

where the sensitivity indices of the six parameters are computed as follows:

$$Q_{\beta_1}(R_0) = \frac{1}{2} \quad \text{and} \quad Q_{\beta_2}(R_0) = \frac{1}{2},$$

$$Q_{\gamma_m}(R_0) = -\frac{1}{2} \frac{\gamma_m}{(\gamma_m + \mu_m)} = -0.3869,$$

$$Q_{\gamma_f}(R_0) = -\frac{1}{2} \frac{\gamma_f}{(\gamma_f + \mu_f)} = -0.3681,$$

$$Q_{\mu_m}(R_0) = -\frac{1}{2} \frac{\mu_m}{(\gamma_m + \mu_m)} = -0.1131,$$

and

$$Q_{\mu_f}(R_0) = -\frac{1}{2} \frac{\mu_f}{(\gamma_f + \mu_f)} = -0.1319.$$

Therefore,

$$R_0(p + \delta p)/R_0 \approx \frac{1}{2} \left(\frac{\delta\beta_1}{\beta_1} + \frac{\delta\beta_2}{\beta_2} \right) - 0.3869 \frac{\delta\gamma_m}{\gamma_m} - 0.3681 \frac{\delta\gamma_f}{\gamma_f} - 0.1131 \frac{\delta\mu_m}{\mu_m} - 0.1319 \frac{\delta\mu_f}{\mu_f}.$$

It is obvious from the above expression that the basic reproductive number is less sensitive to changes in the rate at which the infected are being removed due to AIDS and the natural mortality rate than the rates of HIV transmission (in both classes).

4.2 Sensitivity Analysis of the Endemic Equilibrium

We study dependence of the endemic equilibrium on HIV transmission parameters (β), since in previous section we showed that R_0 is most sensitive to them, and in practice only these parameters could be changed. The question we answer is how the lowering of transmission rate (for example, by truck drivers wearing condoms, assuming that condoms offer complete protection against infection) effects the HIV cases in both groups.

Suppose that $s\%$ of truck drivers wear condoms while interacting with female sex workers. This implies that the HIV transmission rate will decrease by $s\%$, or equivalently, the transmission rates in both directions (from females to males and from males to females) will be reduced as $\beta_i^{new} = k \cdot \beta_i^{old}$, where $k = 1 - s$ and $i = 1, 2$. This implies that the values r_m and r_f will be changed to

$$r_m^{new} = k \cdot r_m \quad \text{and} \quad r_f^{new} = k \cdot r_f.$$

Without loss of generality, consider the truck driver group. Then, modifying (18), the new value \tilde{I}_m^{new} of the infected truck drivers in the endemic equilibrium (assuming that $k\%$ of them will not wear condoms) will be

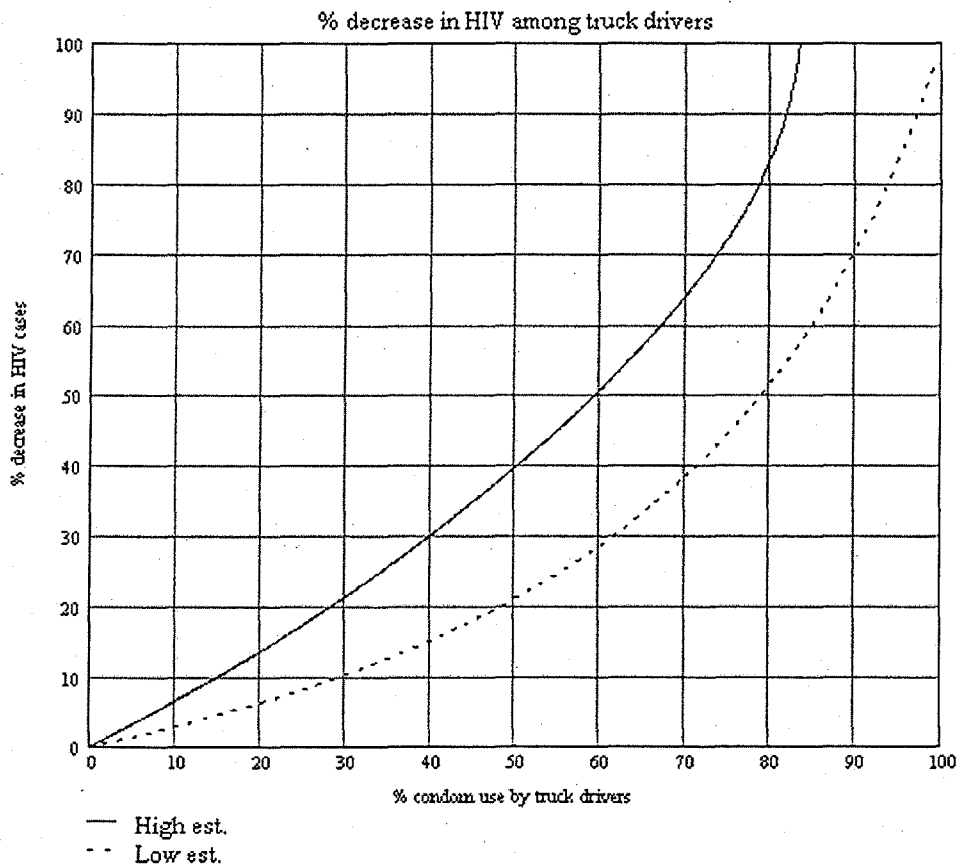
$$\tilde{I}_m^{new} = \frac{(\mu_m + \psi_m) \frac{k^2 r_m r_f - 1}{kr_f(kr_m + \beta_1/\beta_2)}}{\psi_m + \frac{k\beta_1/\beta_2 r_f + 1}{kr_f(kr_m + \beta_1/\beta_2)} (k\beta_1 \frac{k^2 r_m r_f - 1}{kr_m(kr_f + \beta_2/\beta_1)} + \mu_m)} N_m. \quad (19)$$

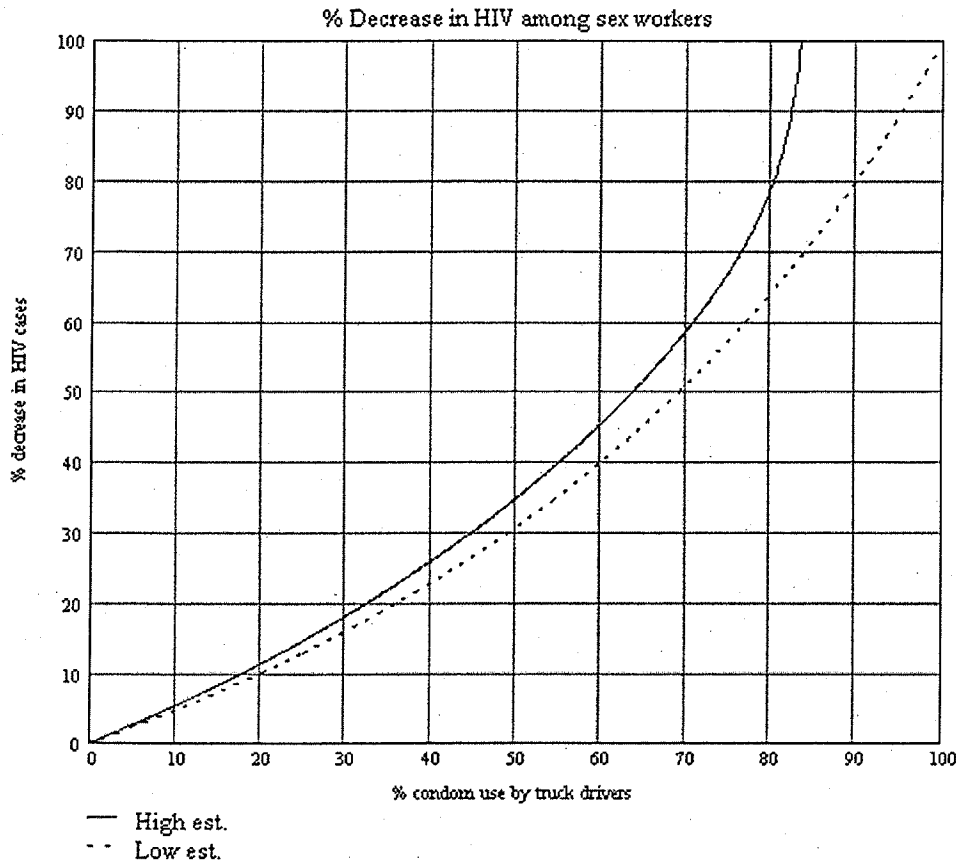
Thus, we have a function $\tilde{I}_m(k)$, where $0 \leq k \leq 1$, and in order to determine the effect from wearing condoms by truck drivers in the endemic population, we study the following quotient

$$J(k) = \frac{\tilde{I}_m(k)}{\tilde{I}_m(1)}$$

The left hand side in the above expression indicates the fraction of the initial endemic number of infected truck drivers. To obtain the percentage *decrease* in endemic HIV cases among truck drivers, we calculate $\mathcal{P}(s) = (1 - J(1 - s)) \cdot 100\%$.

The results for the available data (see Section 5) is given below. First, we show the dependence of \mathcal{P} on s for truck drivers and sex workers by graphs.





In these graphs the two curves show the high and low estimates due to the following ranges in data: (i) the transmission rates β_1 and β_2 have ranges, and (ii) different sources show that on average the number of contacts for a female sex worker can vary between 6 and 30 contacts per week and for truck drivers between 3 and 6 contacts per week (see more details in Section 5).

In the table below we write the numerical values of effects of hypothetical usage of condoms by truck drivers.

% of Truck Drivers wearing condoms	% decrease in HIV cases among Truck Drivers	% decrease in HIV cases among Sex Workers
10%	3-6%	5%
20%	6-14%	10-11%
30%	10-21%	16-18%
40%	15-30%	23-26%
50%	21-40%	31-35%
60%	28-51%	40-45%
70%	38-64%	51-58%
80%	51-83%	64-79%
90%	70-100%	80-100%

Table 2. *Effect of condom use on HIV cases*

Interpretation: If 50% of truck drivers use condoms, HIV cases will be reduced among them by 21-40% and among female sex workers by 31-35%.

Question: What percent of truck drivers would need to wear condoms in order to reduce the prevalence by half (50%)?

Answer: According to data we have gathered (see Section 5) and sensitivity analysis (Section 4), we calculate the value s when $\mathcal{P}(s) = 50\%$, obtaining that 59-79% condom usage by truck drivers will reduce HIV cases among truck drivers by half (50%); similarly, 64-69% condom usage among truck drivers will reduce HIV cases in female sex workers by 50%.

5 Parameters and Data Estimates

In this section we estimate the various parameters used in our model. We describe the model parameters and derivation of them. Due to unavailability of quality data, we try to make the best estimates as possible but in some cases they might be crude.

CONTACT RATES. The rates of infectious contact, β_1 and β_2 , depend on the total number of contacts per person per year, and the probability of transmission per contact (e.g. [Mo01] and [KZ05]). The risks of infection from sexual transmission are quantified using epidemiological studies called “partners studies” (e.g. [BG94]). Since most truck drivers are clients of sex workers, it is appropriate to consider the rate at which truckers are infecting the sex workers. Combining data from [R97], [M94] and [L91], we obtain a range for the per contact probability of female to male transmission of 0.003 – 0.010, and a range for the per contact probability of male to female transmission of 0.006 – 0.080. From [Or93], [Or99] and [FHI2000], we gather that on average the range of contacts for a truck driver with sex workers is 3-6 times per week and the range of contacts for a female sex worker with truck drivers is 6-30 contacts per week. We calculate the parameters β_1 and β_2 as follows

- $\beta_1 = (\text{probability of transmission from female to male per contact}) \times (\text{number of contacts per week}) \times (52 \text{ weeks per year}) = (0.003-0.01) \times (3-6) \times (52) = 0.468-3.12.$
- $\beta_2 = (\text{probability of transmission from male to female per contact}) \times (\text{number of contacts per week}) \times (52 \text{ weeks per year}) = (0.006-0.8) \times (6-30) \times (52) = 1.872-124.8.$

NATURAL MORTALITY RATE. The non-AIDS related death rate μ_m of truck drivers is related to the life expectancy of a healthy truck driver without AIDS, and the average mortality rate is the reciprocal of the average lifetime of a healthy truck driver following recruitment. According to the World Health Report, 2002 [EPS02], the life expectancy of a healthy Nigerian is 51 years and on average truck drivers joint the profession at 22 years [Or93]. This means that the average life time of a healthy truck driver is 29 years and the natural mortality is $\mu_m = 1/29 = 0.034$. Similarly, we define the natural mortality rate μ_f of female sex workers to be the reciprocal of the average remaining life time of a healthy female sex worker following recruitment. Orubuloye, et al., [Or93] estimates the average age of a female sex worker at 20 years and the average age at the onset of sexual activities is 14 years. We assume that the average age of starting as a sex worker is 16 years and the remaining life time of a healthy female sex worker is 24 years. Therefore, the natural death rate of female sex workers is $\mu_f = 1/24 = 0.0416$.

AIDS-RELATED REMOVAL RATES. The rate of progression to AIDS remains a contentious issue in Nigeria and other parts of Africa. Medical experts says that it takes less time to develop full blown AIDS once an individual is infected because of poor health facilities combined with high level of poverty. However, we are not aware of any research to substantiate the claim. In Hyman (1999), an estimate of 8.6 years was assumed as a mean duration of infection. Since γ_m^{-1} and γ_f^{-1} are AIDS related death for truck drivers and sex workers per year, we assume that since a larger number of HIV infected cases do not get treatment, the same rate of progression to AIDS following infection for both groups which is 0.116 per year.

DISEASE INDUCED DEATH RATES. This is the death rate due to the disease. We compute the death rate due to the disease by considering the reported AIDS related deaths for the country over a period of six years, 1994-1999 [NIMR2000] for both males and females. We then compute the ratio of number of deaths due to AIDS per year to the number of reported AIDS cases that year and take the average over the six year period. The calculated values of $\psi_m=0.1474$ and $\psi_f=0.1998$ with a range of 0.09-0.189 and 0.132-2.08 for males and females respectively.

6 Discussion

Models of the type used in this study are crude at best. We have made a number of simplifying assumptions, among the usual (homogeneous population, random mixing, constancy of rates, etc.) we assume that the core population's interactions with the larger population does not significantly effect the dynamics of the disease, we have assumed that the popula-

tion size is constant over the period of interest and that recruitment exactly matches losses due to retirement and death to name a few. In addition, due to the paucity of data for this part of the world, we sometimes had to resort to little better than “best guess” for some parameter values. Official sources are frequently inconsistent, confusing, or sometimes unreliable from year to year. However, even in light of all this, the results of interest from this model seem rather robust. We find that the more truck drivers would use condoms the more the benefits in terms of reducing disease prevalence. These benefits increase in a non-linear way, i.e. higher usage of condoms would result in even larger reductions in disease than would be expected from projecting the reductions at lower levels. This indicates that a concerted effort made at education and encouragement of truck drivers to use condoms and female sex works to insist on use of condoms would have increasing benefits as such a program progresses. Bringing “traditional health care providers” into such a program by providing financial incentives could have even greater impact. Future work should consist of both collecting better data and refining parameter estimates as well as including non core group components of the population as it would give better understanding the spread of the disease in general population coming from these high risk core groups.

7 Acknowledgments

The research on this project has been partially supported by grants from the National Security Agency, the National Science Foundation, the T Division of Los Alamos National Lab (LANL), the Sloan Foundation, and the Office of the Provost of Arizona State University. All authors are thankful to the Mathematical and Theoretical Biology Institute (MTBI) for summer school 2005 at Los Alamos, NM where most of research for this paper was done. Titus G. Kassem is grateful to the University of Jos Carnegie Partnership Committee for sponsoring his visit to Arizona State University, Tempe, USA and Dr. Carlos Castillo-Chavez for giving him the opportunity and support, as well as Kae Sawyer. We wish to acknowledge with thanks the useful comments of Christopher Kribs-Zaleta and Bajoun Song. The authors are solely responsible for the views and opinions expressed in this research; it does not necessarily reflect the ideas and/or opinions of the funding agencies, Arizona State University, or LANL.

References

- [AH05] ARRIOLA AND HYMAN, Lecture notes on Forward and Adjoint with Applications in Dynamical Systems, Linear Algebra and Optimization. *Mathematical and Theoretical Biology Institute*, Summer 2005.
- [BG94] R. BROOKMEYER AND M.H. GAIL, AIDS Epidemiology: A Quantitative Approach. *Oxford University Press*, 1994.

- [CCC02] C. CASTILLO-CHAVEZ, Z. FENG & W. HUANG, On the computation of R_0 and its role in global stability. in: *Mathematical Approaches for Emerging and Reemerging Infectious Diseases: An Introduction*. C. Castillo-Chavez with S. M. Blower, P van den Driessche, D. Kirschner, A. A. Yakubu (Eds), IMA Vol.125, Springer-Verlag, New York, pp. 229-250.
- [C04] G. CHOWELL, C. CASTILLO-CHAVEZ, P.W. FENIMORE, C.M. KRIBS-ZALETA, L. ARRIOLA AND J.M. HYMAN, Model Parameters and Outbreak Control for SARS. *Emerging Infectious Diseases*, vol.10, No.7, July 2004.
- [D90] O. DIEKMANN, J. A. P. HEESTERBEEK & J. A. J. METZ, On the definition and computation of the basic reproductive ratio in models for infectious diseases in heterogeneous population, *J. Math. Biol.* **28**:365-382.
- [EPS02] Epidemiological Fact Sheets on HIV/AIDS and STI, 2002 update; UN-AIDS/WHO:2002, Nigeria,
www.who.int/emc-hiv/fact_sheets/pdfs/Nigeria_EN.pdf
- [FHI2000] HIV/AIDS Behavioral Surveillance Survey, Nigeria 2000, report and data sheets, Family Health International;
www.fhi.org/en/HIVAIDS/pub/survreports/bssnigeria2000.htm
- [G03] P.B. GILBERT, I.W. MCKEAGUE, G. EISEN, C. MULLINS, A. GUÉYE-NDIAYE, S. MBOUP AND P.J. KANKI, Comparison of HIV-1 and HIV-2 infectivity from a prospective cohort study in Senegal. *Statistics in Medicine*, vol. 22 (2003), p.573-593.
- [HC03] Y. HSIEH. AND C. H. CHEN, Modelling the Social Dynamics of a Sex Industry: Its Implications for Spread of HIV/AIDS. *Bulletin of Mathematical Biology*, vol.66 (2004), p.143-166.
- [HL97] J.M. HYMAN, AND JIA LI, Behaviour Changes in SIS STD Models with selective Mixing. *SIAM J. Appl. Math.* Vol. 57, No.4 (1997), pp 1082-1094.
- [I04] J.A. IDOKO, The plague among us: where is the cure? *University of Jos Inaugural Lecture Series*, 16, 2004.
- [KZ05] C. KRIBS-ZALETA, M. LEE, C. ROMÁN, S. WILEY, C.M. HERNÁNDEZ - SUÁREZ, The effect of the HIV/AIDS epidemic on Africa's truck drivers, preprint, 2004, in press.
- [L91] A. LAZZARIN, A. SARACCO, M. MUSICCO AND A. NICOLOSI, Man-to-woman sexual transmission of the human immunodeficiency virus. Risk factors related to sexual behavior, man's infectiousness, and the woman's susceptibility. *Italian Study Group on HIV Heterosexual Transmission. Archives of Internal Medicine*, Vol. 151 No.12, December 1, 1991.

- [M2000] T. MARCUS, Exposure and experience confounded by structural constraints: Assessing the impact accidents, predation and AIDS on long distance truck drivers. *Report prepared for University of Natal Accident Research Centre (UNIARC)*, 2000.
- [M94] T.D. MASTRO, G.A. SATTEN , T. NOPKESORN, S. SANGKHAROMYA, I.M. LONGINI JR., Probability of female-to-male transmission of HIV-1 in Thailand. *Lancet*, Vol 343 (2004) 07. January 22, 1994.
- [Mo01] L. MORISON, H.A. WEISS, A. BUVE, M. CARAEL, S-C. ABEGA, F. KAONA, L. KANHONON, J. CHEGE AND R. HAYES, for the study group on the heterogeneity of HIV Epidemics in African Cities "Commercial sex and the spread of HIV in four cities in sub-Saharan Africa", *AIDS* (2001), 15(suppl.4), p. S61-S69.
- [NIMR2000] NIGERIAN INSTITUTE OF MEDICAL RESEARCH NIMR HIV-AIDS technical report, Federal Ministry of Science and Technology, Lagos, 2000.
- [Og01] S. J. OGBOI, K. SABITU, P.U. AGU, H.O. OGBOI AND O. AYEGBUSI, Prevalence of HIV Transmission in Nigerian Transit Towns Among Long Distance Drivers. *Laboratory Survey*, 2001; www.aacc.org/ia/transit_towns.pdf.
- [Or93] I.O. ORUBULOYE, P. CALDWELL AND J.C. CALDWELL, The Role of High-Risk Occupations in the Spread of AIDS: Truck Drivers and Itinerant Market Women in Nigeria. *International Family Perspectives*, vol. 19 (1993), n. 2.
- [Or99] I.O. ORUBULOYE, AND F. OGUNTIMEHIN, Intervention for the control of STDs including HIV among commercial sex workers, commercial drivers and students in Nigeria. *The Continuing African HIV/AIDS Epidemic*, ed. I.O. Orubuloye, J.C. Caldwell and J.P.M. Ntozi. Canberra: Australian National University, 1999, pp. 121-129.
- [R97] R.A. ROYCE, A. SENA, W. CATES AND M.S. COHEN, Sexual Transmission of HIV. *New Eng. J. Med.*, 336(15):1072-1078.
- [S05] A.M. SUNMOLA, Sexual Practices, barriers to condom use and its consistent use among long distance truck drivers in Nigeria. *AIDS Care*, February 2005, 17(2):208-221.
- [VR90] C. F. VON RYEN, Epidemiology of HIV infection in Africa. *AIDS and Society*, April-May:3,8, 1990.