

# OCD Therapeutic Dynamics: Markovian Based Simulations

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## Abstract

Up to 3 percent of the population suffers from a mental disorder known as Obsessive-Compulsive Disorder (OCD) \*. Although full understanding of this disorder still eludes scientists, there is a general consensus on distinct methods of treatment including Selective Serotonin Re-uptake Inhibitors, various types of group therapy, and individual therapy. This paper uses finite Continuous Time Markov Chains (CTMC) to model the treatment dynamics of Obsessive-Compulsive individuals. The models proposed herein include factors such as access to group or individual therapy and health insurance constraints, and are modified to include prescribed treatment regimens. Naturally, added realism moves out of the “neatness” inherent in CTMC models. Realistic extensions in the CTMC models are therefore addressed by numerical means (simulations). To minimize time and cost, we establish necessary conditions between the recruitment rate of individual therapy and the recruitment rate for the group therapy. These conditions depend on the magnitude of the recovery rate for group therapy.

\* <http://www.canmat.org/resources/depression/ocd.html>

## 1 Introduction

Obsessive-Compulsive Disorder (OCD) is a mental condition in which the common symptoms describe an individual who experiences undesired persistent thoughts or obsessions which cause anxiety or distress. The individual responds with a thought or action (compulsion) to find temporary relief from the anxiety. This thought pattern is cyclical, leading from obsessions to compulsions and vice-versa. Approximately 3 percent of the American Population suffers from OCD. Specifically, approximately 2.3 percent of the American population between the ages of 18 and 54 have OCD [1]. There are different forms of treatment including medication, individual therapy and/or group therapy. The standard in medication for OCD is the Selective Serotonin Re-uptake Inhibitors, commonly known as SSRI's.

If individual therapy is carried out by a psychiatrist, treatment includes medication that will in most cases be SSRIs, and some form of therapy, such as Cognitive-Behavior Therapy (CBT), Exposure and Response Prevention, etc. Although individual therapy does not always include medication, people usually achieve greater symptom relief with a combination of medication and therapy [2]. Thus, a general assumption throughout this paper is that all forms of therapy include medication.

We seek to understand the dynamics of how people suffering from Obsessive-Compulsive Disorder choose treatment and how this affects their time and cost until recovery. To study these dynamics, we use a Markov model and numerical simulations of one individual and then use those results to make inferences about the general population of people suffering from OCD. This strategy is justified by the Law of Large Numbers [5], i.e. for a large number of independent individuals with the same transition probabilities, the expected behavior of the population mimics the expected behavior of the individual. That is, most of the time the values  $x_i$  of the random variables  $X_i$  will be such that if there is an arbitrarily small number  $\epsilon$ , for sufficiently large  $n$ :

$$E[X]_1 - \epsilon < s_n/n < E[X]_1 + \epsilon \tag{1}$$

where  $s_n = x_1 + x_2 \dots + x_n$  and where  $x$  is the value of a random variable  $X$ .

We first study a model under the assumption that the individual chooses his preferable therapy, that is, he has unlimited resource or complete insurance coverage. Using Markov Chain theory, we compare analytical results to the numerical simulations to verify that the simulations provide reliable results and conclusions. In reality, the dynamics can transcend probability because choices are limited and influenced by external factors that can go beyond freedom of choice. We then ask how long it takes on average, for an individual to recover if the patient is completely subject to health insurance. Therefore, in a second model, the individual does not choose what he wants but rather what the insurance company will provide. It is also necessary to contemplate the associated cost of therapy and how it plays a role in the dynamics of OCD treatment.

In a third model we combine both situations, freedom of choice and health insurance limitations, to see how the individual will recover, given that in reality preference of therapy and limited insurance coverage both play an important role in OCD treatment. Ultimately we use results of the three models to analyze how the rates can influence rehabilitation and how insurance can find a cost-efficient way of aiding people who suffer from Obsessive-Compulsive Disorder. Although there is no cure for OCD, studies show that patients can reach an emotionally and mentally stable life after a period of consistent (weekly or biweekly) treatment, after which occasional follow-up sessions are required indefinitely. We then assume individuals who do not carry on with the post-treatment follow-up sessions are individuals that quit treatment rather than recover. Thus individuals can quit treatment and eventually start again. The only individuals that “recover” are those that carry out the necessary post-treatments requirements, such as follow-ups and medication.

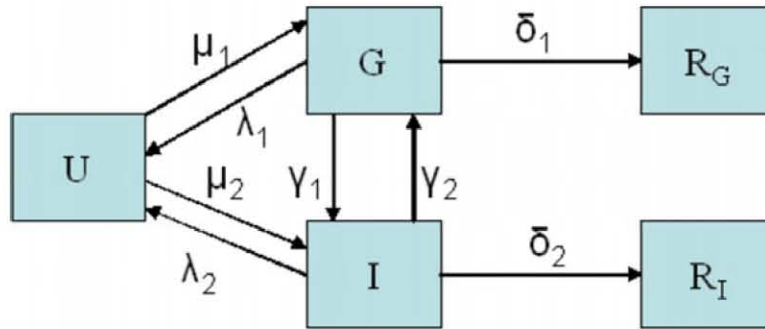


Figure 1: Free Will Model

All models are based on finite Continuous Time Markov Chains which can be simplified by looking at the Embedded Markov Chain (EMC) [8]. Given that an event happens we only need to know the probability of moving from one state to another. With this approach, the events are Poisson processes with the parameter  $\lambda$ , where  $\lambda$  is the sum of the outgoing rates. Therefore the average time between events follows an exponential distribution with parameter  $1/\lambda$ .

The possibilities of this study push far beyond OCD because the model encompasses any disorder with different forms of insurance limited treatment. Thus it follows that models can be adapted to most disorders with the appropriate choice of parameters.

## 2 Free Will Model

The Free Will model (Figure 1) is made of five classes or states which represent being untreated, treated, and recovery. The  $U$  class represents the untreated state. The  $I$  and  $G$  classes represent individual and group therapy respectively. The  $R_I$  class is the state that holds an individual who has recovered and was last in individual therapy, and the  $R_G$  class represents an individual that has recovered and last visited group therapy.

The distinguishing assumption of this model is that the individual can move freely from state to state, depending solely on fixed initial probabilities. This assumption potentially allows an unlimited number of transitions between the different classes, apparently making the model seem unrealistic. However, with careful choice of the parameters, we reduce the probability of such transitions occurring. One of the major advantages is that the model allows us to trace the history of each individual, see the time spent in each class and how many times each class was visited. Another key point is the ease at which we can compare the numerical simulations with analytic results such as the stationary distribution and the mean time to absorption. This will serve as the basis for the Total and Partial Control models.

## 2.1 Stationary Distribution

The one step transition matrix for a Markov Chain is given by:

$$P = \begin{pmatrix} P_{00} & P_{01} & \dots & \dots & P_{0n} \\ P_{10} & P_{11} & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ P_{n0} & \dots & \dots & \dots & P_{nn} \end{pmatrix} \quad (2)$$

where  $P_{ij}$  is the  $ij$ th element of  $P$ . Each of these elements represents the probability of going from a state  $i$  to state  $j$  in the next step.

In the Free Will model, the transition matrix is given by:

$$P = \begin{pmatrix} P_{UU} & P_{UG} & P_{UI} & P_{UR_g} & P_{UR_i} \\ P_{GU} & P_{GG} & P_{GI} & P_{GR_g} & P_{GR_i} \\ P_{IU} & P_{IG} & P_{II} & P_{IR_g} & P_{IR_i} \\ P_{R_gU} & P_{R_gG} & P_{R_gI} & P_{R_gR_g} & P_{R_gR_i} \\ P_{R_iU} & P_{R_iG} & P_{R_iI} & P_{R_iR_g} & P_{R_iR_i} \end{pmatrix} \quad (3)$$

The one step transition matrix  $P$  of the Markov Chain for the Free Will Model is given by:

$$\begin{pmatrix} 0 & \frac{\mu_1}{\mu_1+\mu_2} & \frac{\mu_2}{\mu_1+\mu_2} & 0 & 0 \\ \frac{\lambda_1}{\lambda_1+\gamma_1+\delta_1} & 0 & \frac{\gamma_1}{\lambda_1+\gamma_1+\delta_1} & \frac{\delta_1}{\lambda_1+\gamma_1+\delta_1} & 0 \\ \frac{\lambda_2}{\lambda_2+\gamma_2+\delta_2} & \frac{\gamma_2}{\lambda_2+\gamma_2+\delta_2} & 0 & 0 & \frac{\delta_2}{\lambda_2+\gamma_2+\delta_2} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Given an initial distribution  $v_0$ , the distribution in the next step is defined as  $v_1 = v_0P$ . Then the distribution after  $n$  steps is found by solving:

$$v_n = v_0P^n \quad (4)$$

The Stationary Distribution is then defined as:

$$v_\infty = v_\infty P^\infty \quad (5)$$

Thus the distribution of states remains unchanged in the next step. Note that each of the simulations determines the movement of one individual until he reaches one of the recovery classes. Hence the stationary distribution serves useful when comparing to the results of a large number of individual simulations. We can use equation 1 to approximate the stationary distribution if we take  $n$  large enough.

For  $\mu_1 = 1/4$ ,  $\mu_2 = 1/6$ ,  $\lambda_1 = 0.0067$ ,  $\lambda_2 = 0.0056$ ,  $\delta_1 = 1/18$ ,  $\delta_2 = 1/15$ ,  $\gamma_1 = 0.0134$ ,  $\gamma_2 = 0.0112$ ,

$$P = \begin{pmatrix} 0 & 0.6 & 0.4 & 0 & 0 \\ 0.0796 & 0 & 0.1777 & 0.7427 & 0 \\ 0.0668 & 0.1337 & 0 & 0 & 0.7995 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

To approximate  $P^\infty$ , we calculate  $P^n$  for a sufficiently large  $n$ .

$$P^\infty \approx \begin{pmatrix} 0 & 0 & 0 & 0.5451 & 0.4549 \\ 0 & 0 & 0 & 0.8118 & 0.1882 \\ 0 & 0 & 0 & 0.1449 & 0.8551 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In all cases our initial condition is one individual at  $U$ , so we use  $v_0 = (1 \ 0 \ 0 \ 0 \ 0)$ . Therefore, the stationary distribution  $v_\infty$  can be approximated to

$$v_\infty \approx (0 \ 0 \ 0 \ 0.5451 \ 0.4549)$$

Hence the probability the individual ends up in  $R_G$  is approximately 0.5451 and the probability that the individual ends up in  $R_I$  is approximately 0.4549. When compared to 10000 simulations for the same parameters, the analytic proportion of  $R_I$  and  $R_G$  was approximately the same as the corresponding proportion in the simulations (Figure 2). In the simulations, the percent of people that went to  $R_i$  or  $R_g$ , was approximately 53 percent and 47 percent respectively.

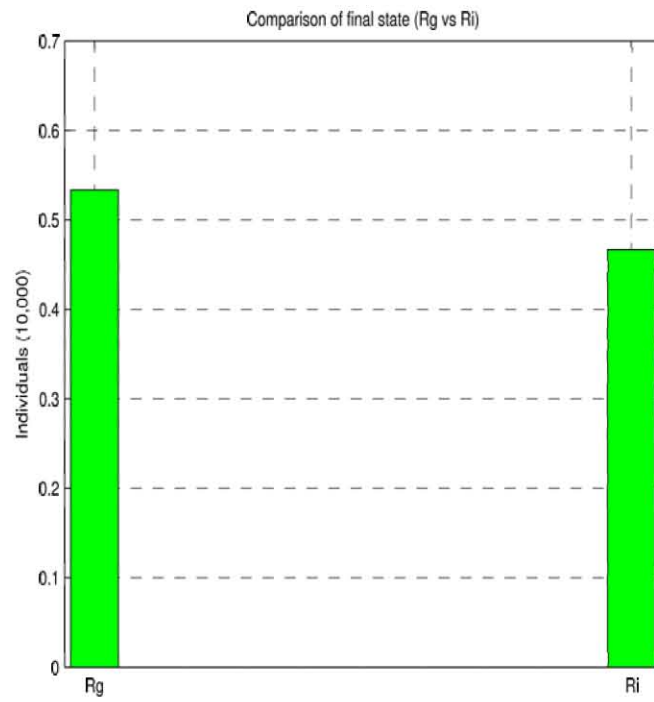


Figure 2:  $R_G$  and  $R_I$  Proportion

## 2.2 Mean Time to absorption

The mean time to absorption is the average time the individual takes to reach one of the absorbing states. To determine the time to absorption for our model, we use the Uniform Discrete time approximation:

$$P = \begin{pmatrix} 1 - \mu_1 \Delta t - \mu_2 \Delta t & \mu_1 \Delta t & \mu_2 \Delta t & 0 & 0 \\ \lambda_1 \Delta t & 1 - \lambda_1 \Delta t - \gamma_1 \Delta t - \delta_1 \Delta t & \gamma_1 \Delta t & \delta_1 \Delta t & 0 \\ \lambda_2 \Delta t & \gamma_2 \Delta t & 1 - \lambda_2 \Delta t - \gamma_2 \Delta t - \delta_2 \Delta t & 0 & \delta_2 \Delta t \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$P$  and its  $ij$ th element are defined as before. The transient state matrix  $P_t$  is given by the transient states or non-absorbing states.

$$P_t = \begin{pmatrix} 1 - \mu_1 \Delta t - \mu_2 \Delta t & \mu_1 \Delta t & \mu_2 \Delta t \\ \lambda_1 \Delta t & 1 - \lambda_1 \Delta t - \gamma_1 \Delta t - \delta_1 \Delta t & \gamma_1 \Delta t \\ \lambda_2 \Delta t & \gamma_2 \Delta t & 1 - \lambda_2 \Delta t - \gamma_2 \Delta t - \delta_2 \Delta t \end{pmatrix}$$

We now find  $A = I - P_t$  is:

$$\Delta t \begin{pmatrix} \mu_1 + \mu_2 & -\mu_1 & -\mu_2 \\ -\lambda_1 & \lambda_1 + \gamma_1 + \delta_1 & -\gamma_1 \\ -\lambda_2 & -\gamma_2 & \lambda_2 + \gamma_2 + \delta_2 \end{pmatrix}$$

Therefore,

$$A^{-1} = \frac{1}{\Delta t} M^{-1} \tag{6}$$

$$\text{where } M = \begin{pmatrix} \mu_1 + \mu_2 & -\mu_1 & -\mu_2 \\ -\lambda_1 & \lambda_1 + \gamma_1 + \delta_1 & -\gamma_1 \\ -\lambda_2 & -\gamma_2 & \lambda_2 + \gamma_2 + \delta_2 \end{pmatrix}$$

Let  $S = (\Delta t)A^{-1}$ .

Solving for S yields:

$$S = \begin{pmatrix} \frac{1}{\mu_1 + \mu_2} \left[ \frac{M_1}{m_1} + (M_1 G_1 + M_2) \frac{l_2 + l_1 G_2}{1 - G_1 G_2} \right] & \frac{1}{\lambda_1 + \gamma_1 + \delta_1} \frac{M_1 + M_2 G_2}{1 - G_1 G_2} & \frac{1}{\lambda_2 + \gamma_2 + \delta_2} \frac{M_2 + M_1 G_1}{1 - G_1 G_2} \\ \frac{1}{\mu_1 + \mu_2} \frac{M_1}{m_1} \frac{l_2 G_1 + l_1}{1 - G_1 G_2} & \frac{1}{\lambda_1 + \gamma_1 + \delta_1} \frac{M_1}{m_1} \frac{1}{1 - G_1 G_2} & \frac{1}{\lambda_2 + \gamma_2 + \delta_2} \frac{M_1}{m_1} \frac{G_1}{1 - G_1 G_2} \\ \frac{1}{\mu_1 + \mu_2} \frac{M_2}{m_2} \frac{l_1 G_2 + l_2}{1 - G_1 G_2} & \frac{1}{\lambda_1 + \gamma_1 + \delta_1} \frac{M_2}{m_2} \frac{G_2}{1 - G_1 G_2} & \frac{1}{\lambda_2 + \gamma_2 + \delta_2} \frac{M_2}{m_2} \frac{1}{1 - G_1 G_2} \end{pmatrix}$$

where

$$M_1 = \frac{m_1}{1 - l_1 m_1}, M_2 = \frac{m_2}{1 - l_2 m_2},$$

$$G_1 = \frac{g_1 + l_1 m_2}{1 - l_2 m_2}, G_2 = \frac{g_2 + l_2 m_1}{1 - l_1 m_1},$$

$$m_i = \frac{\mu_i}{\mu_1 + \mu_2}, l_i = \frac{\lambda_i}{\lambda_i + \gamma_i + \delta_i}, g_i = \frac{\gamma_i}{\lambda_i + \gamma_i + \delta_i}.$$

The  $S_{ij}$ th element represents the time spent in  $j$  after starting in  $i$ . Since our model always starts at  $U$ , we only need  $\sum_{j=1}^3 S_{1j}$ , i.e. the sum of the elements of the first row, in order to know the total time to absorption.

$$S_{1j} = \left( \frac{1}{\mu_1 + \mu_2} \left[ \frac{M_1}{m_1} + (M_1 G_1 + M_2) \frac{l_2 + l_1 G_2}{1 - G_1 G_2} \right] \quad \frac{1}{\lambda_1 + \gamma_1 + \delta_1} \frac{M_1 + M_2 G_2}{1 - G_1 G_2} \quad \frac{1}{\lambda_2 + \gamma_2 + \delta_2} \frac{M_2 + M_1 G_1}{1 - G_1 G_2} \right),$$

This row for the previously used parameters is given by:

$$S_{1j} = ( 2.649276316 \quad 9.769736846 \quad 6.858552633 ).$$

Therefore we see that the time to absorption is approximately 19.28 time units for the selected parameters. In our simulations we record the following average time spent in the respective classes:

$U$ : 2.54 weeks

$G$ : 9.99 weeks

$I$ : 6.77 weeks

In the simulations, the total time to absorption with the same parameters is 19.3 weeks. For the selected parameters there is a 0.1 % error between the simulations and the analysis. Furthermore the time spent in each class correspond as well (Figure 3).

Not only are we able to keep track of the time history of one individual, but we can construct probability distributions of the number of visits and time spent in each class. Figure 4 shows the probability distributions of the visits to and time spent in each class. It is clear that the time spent in each class follows the exponential distribution. Furthermore, with the selected parameters, most people recover after visiting  $I$  at most once, and of



those people, most people will visit the *I* class once.

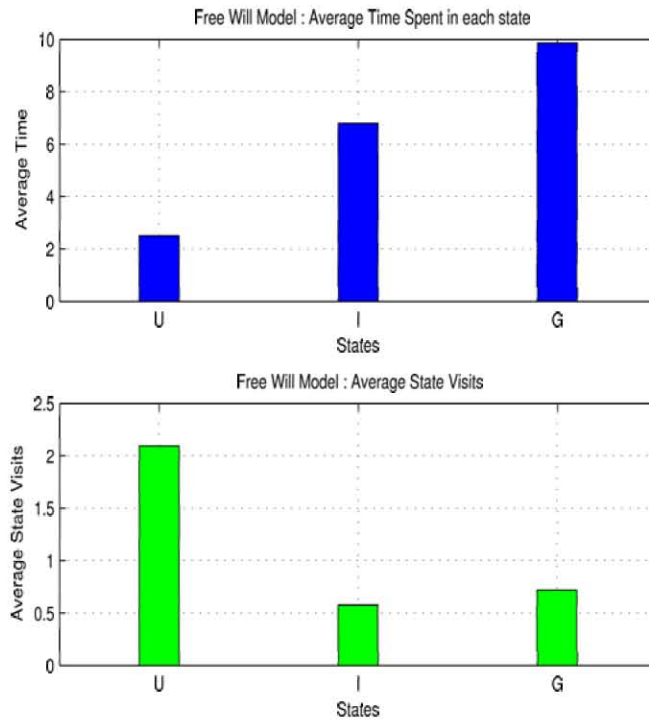


Figure 3: time and number of visits in each class

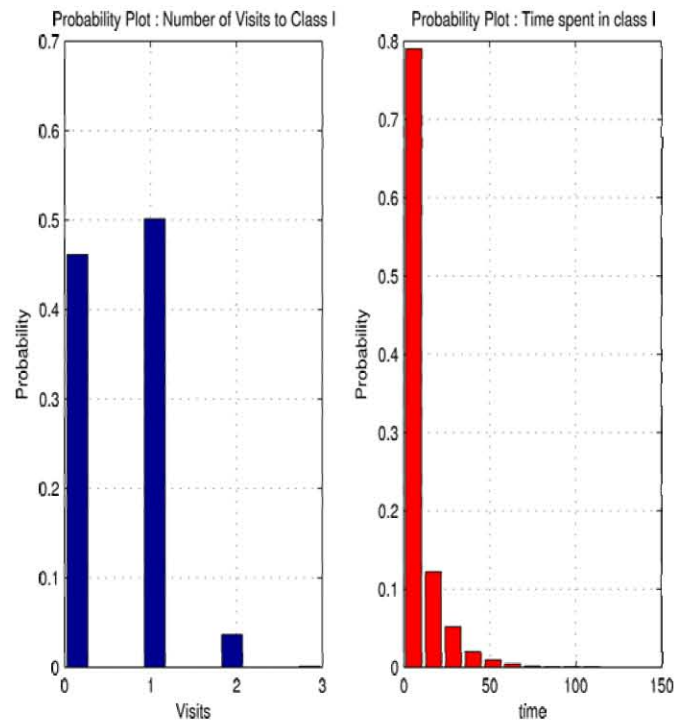


Figure 4: probability plot of time and visits in I

### 3 Total Control Model

Unfortunately in real-life situations, an individual's choice is restricted by other factors such as access to health insurance. For the purpose of imposing insurance health control over the individual, we will base this model (see Figure 5) on a Markov Chain but with an insurance time limit in which the individual is allowed to stay in individual therapy  $I$  until the limit expires. We turn to numerical simulations for the analysis of this model. In this case we assume that the individual wants to stay in  $I$  because it has the fastest recovery rate [3]. The insurance company will also initially cover individual therapy because they want the patient to recover as quickly as possible, but if the patient takes too long to recover, insurance will force the patient into group therapy because it is the less expensive alternative. Once in  $G$ , an individual eventually reaches recovery  $R_G$ . At no point in the model, can the individual return to a previously visited state.

The insurance limit  $\tau$  in  $I$ , represents the time the insurance provides for individual therapy, i.e. an insurance limit.  $T_I$  is the time a certain individual needs to stay in  $I$  before he recovers, without insurance.  $T_I$  is an exponential random variable with parameter  $1/\lambda$ , where  $1/\lambda$  is the expected or average time an individual stays in  $I$ .  $t_G$  is the amount of time an individual needs to stay in  $G$  before he recovers, without insurance.  $t_G$  is an exponential random variable with parameter  $1/\delta$ . The expected or average time an individual stays in  $G$  is  $1/\delta$ . We are assuming that at each point in time, the recovery will be uniform, i.e. the individual has the same improvement in each session. The required amount of time for recovery for a specific individual in  $G$ , with insurance, is  $T_G$ . We are assuming that after  $\tau$  time, there will have been some progress towards recovery, therefore the time until recovery from  $G$ ,  $T_G$ , will be less as  $\tau$  increases.  $T_G = (1 - \tau/T_I)t_G$ . We use time in each form of therapy to estimate cost to the insurance company.

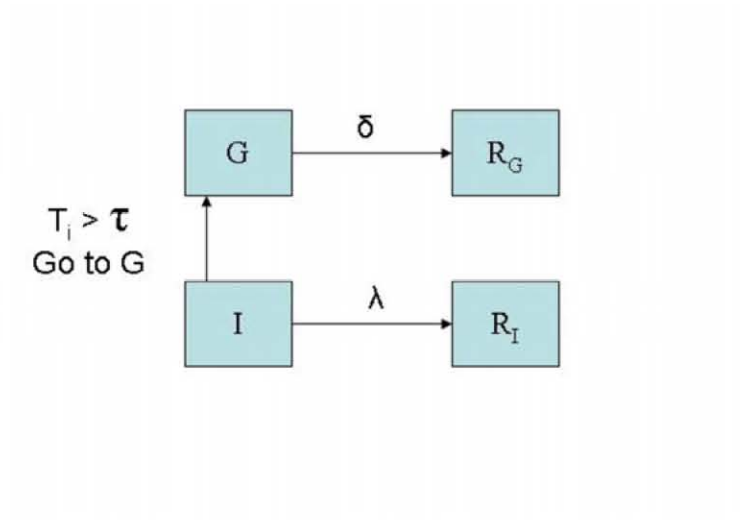


Figure 5: Total Control Model

## 4 Partial Control Model

Realistically, health insurance cannot fully determine the dynamics of recovery, because an individual may have a preference for a type of therapy over another. On the other hand, health insurance may restrict the type of available therapies. Therefore we now study the case in which there is an insurance limit to the  $I$  class, and where the individual is allowed partial freedom of movement. The insurance limit is a time restriction on how much accumulated time you can spend in  $I$ . Therefore we are assuming that  $\tau$  represents the average time that the individual has access to insurance-provided individual treatment. The individual consumes time from his insurance limit only when in  $I$ . Once this time is consumed, the individual is “pushed” out of individual therapy and is no longer allowed to visit  $I$ . The individual is free to move anywhere the patient wants to, except to  $I$ , because insurance will no longer pay for individual therapy. The insurance now only pays for group therapy. The model is the same as the Free Will Model (Figure 1) with the addition of insurance time limit. As usual we assume that the time between each event is an exponentially distributed random variable.

## 5 Parameter Estimation

The following is a list of the rates or parameters used in the models. It is important to understand that these are rates in the sense that they are events per time of movement from one class to another.

parameter	interpretations
$\mu_1$	rate from $U$ to $G$ .
$\mu_2$	rate from $U$ to $I$ .
$\lambda_1$	rate from $G$ to $U$ .
$\lambda_2$	rate from $I$ to $U$ .
$\gamma_1$	rate from $G$ to $I$ .
$\gamma_2$	rate from $I$ to $G$ .
$\delta_1$	rate from $G$ to $R_g$ .
$\delta_2$	rate from $I$ to $R_i$ .

Unfortunately, there is very little or no available information regarding the parameters in our models. This leads to the conclusion that well established or time-proven statistics concerning these matters do not exist. Dr. Sabine Wilhelm of Harvard Medical School said that she unfortunately has not seen any research pertaining to the parameters of our models. Michael A. Jenike, M.D. and professor of psychiatry at Harvard Medical School, stated that he did not know of any studies that contain these rates or parameters. This makes our question much more relevant because understanding these rates and their possible relation to health insurance can help find ways to rehabilitate quickly, and in greater numbers, Obsessive-Compulsive people. In most cases, the parameters we used were estimated by using scarce statistics and/or information. The only useful information found was that people in individual therapy reach rehabilitation after an average of 15 sessions<sup>1</sup>. Since most cases experts recommend seeking therapy weekly [11], we are assuming that it takes an average of 15 weeks for a person in individual therapy to recover. Additional information indicates that individual therapy is as effective but quicker than group therapy [3], and that some patients may have bad results in group therapy [4]. This leads to the assumption that the rate at which people change from group therapy to individual therapy should be greater than the rate of people changing from individual to group therapy. We can also assume that the rate at which individual therapy people quit treatment should be smaller than the rate at which group therapy people quit treatment. The Expert Consensus Panel for Obsessive-Compulsive Disorder suggests that the appropriate number of sessions in group therapy should range from 20-50, which matches other results previously mentioned that state that group therapy is slower than individual [10].

There is a possibility for unrealistic dynamics in the Free Will Model. The Markov scenario allows for many possible state jumps from  $I$  to  $G$  and vice-versa. This seems

very unrealistic because normally an individual might make up his mind on whether to seek individual or group therapy, after visiting each state once. To avoid such unrealistic phenomena in the Markov simulation, we keep  $\gamma_1$  and  $\gamma_2$  considerably low.

<sup>1</sup> Out of 330 patients an average of 83 % were improved after an average of 15 sessions. Taken from Foa, E.B. (1996). The efficacy of behavioral therapy with obsessive-compulsives. *The Clinical Psychologist*, 49, 2, 19-22.

With these conclusions, we set the following parameters:

parameter	value
$\mu_1$	1/4
$\mu_2$	1/5
$\lambda_1$	0.0067
$\lambda_2$	0.0056
$\gamma_1$	0.0134
$\gamma_2$	0.0112
$\delta_1$	1/18
$\delta_2$	1/15

We perform parameter sweeps to see the different dynamics that are possible because of the parameters, and how this can lead to new and better strategies that are cost-effective for the rehabilitation of Obsessive-Compulsive people.

As previously stated, two of the models will have an insurance control or limit. This insurance limit will be called  $\tau$  and represents the average amount of time an individual has access to certain treatment resources that pertain to individual therapy due to his health insurance. When this limit is exceeded, we assume that the insurance company will continue to provide insurance but for group therapy.  $\tau$  will range from 0 to 80, where  $\tau=0$  implies no time spent in individual therapy. 80 is the upper limit of  $\tau$  because there is over 99 % probability of recovery before 80 is reached.



## 6 Simulation Results

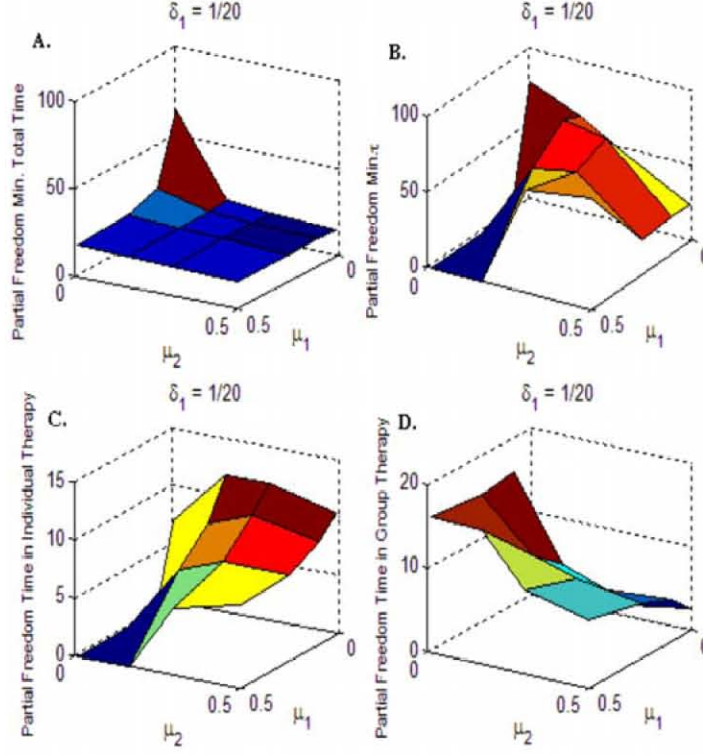


Figure 6: Time Graph for  $\delta_1 = 1/20$

To fully understand our models we need to see how they respond to changes of parameters. The parameters of interest are  $\delta_1$ ,  $\mu_1$ , and  $\mu_2$ .  $\delta_1$  is the rate at which an individual moves from  $G$  to  $R_G$ .  $\mu_1$  is the rate at which an individual moves from  $U$  to  $G$  and  $\mu_2$  is the rate at which an individual moves from  $U$  to  $I$ . The next step is to optimize the Partial Control model because it is a more realistic version of the free will model. Ideally, one would wish to minimize the cost for the insurance company but at the same time, minimize the average time to recovery for a person with OCD. In the Partial Control model, where  $\delta_1=1/20$  and  $\delta_2=1/15$ , we can see that an individual suffering from OCD recovers from individual therapy slightly faster than he does from group therapy. If we look at the graph for minimum average time until recovery (Figure 6), we should make  $\mu_2$  big relative to  $\mu_1$ . Making  $\mu_2$  big with respect to  $\mu_1$  is equivalent to saying that we make an individual move faster from  $U$  to  $I$  than he would from  $U$  to  $G$ . We also have that  $\gamma_1$  is smaller than  $\delta_2$ , meaning that the time a person takes to recover from  $I$  is smaller than the time it takes him to go to  $G$ .

An individual has a higher probability of going from  $U$  to  $I$  than from  $U$  to  $G$ , and a higher probability of recovering than of going to  $G$ . Hence, by letting  $\mu_2$  greater than  $\mu_1$  we are diminishing our time to recovery. The  $\tau$  at which this happens is  $\tau=25$ . Since  $\delta_2 = 1/15$ , this means that a person recovers from  $I$  faster than he is forced to go to  $G$ .

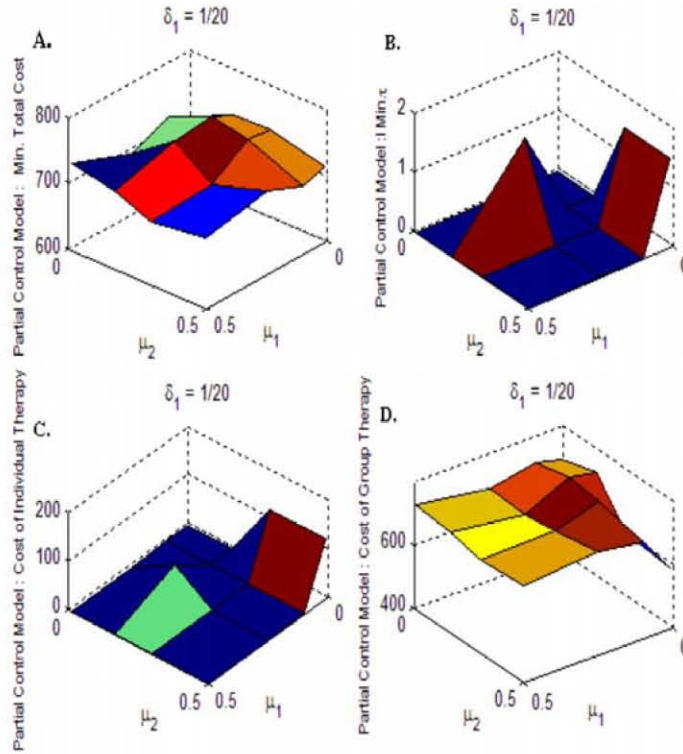


Figure 7: Cost Graph for  $\delta_1 = 1/20$

If we observe the minimum cost values (Figure 7) for  $\mu_1$  and  $\mu_2$  we find they are  $\mu_1=0.5$   $\mu_2=0.3$ . The  $\tau$  at which this happens is zero. This makes sense because  $I$  is more expensive than  $G$ , therefore if we make the insurance time limit zero, we will get our cheapest average total cost by making a person that is in  $U$  class go to group therapy with more probability than going to individual therapy. This means nobody is going to  $I$ .

By taking this value for  $\tau$  we are getting the minimum cost per individual. If we consider the maximum cost per individual we can see that by taking  $\tau$  equal zero we are reducing our cost by approximately 75 dollars per individual.

In order to minimize cost we need to make  $\mu_1$  bigger than  $\mu_2$  Therefore, if  $\delta_1 = 1/20$ :

1. In order to minimize the average time to recovery for one individual we need to make  $\mu_2$  bigger than  $\mu_1$ .

2. In order to minimize the cost one individual we need to make  $\mu_1$  bigger than  $\mu_2$ .

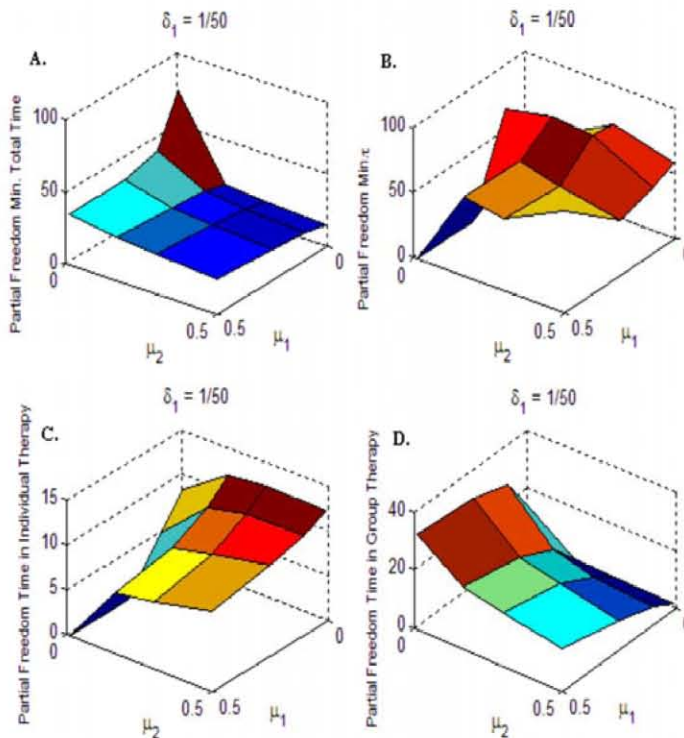


Figure 8: Time Graph for  $\delta_1 = 1/50$

Figure 8 corresponds to the Partial Control Model with respect to time with  $\delta_1 = 1/50$ . We then make the same analysis of parameter variations for  $\mu_1$  and  $\mu_2$  as  $\delta_1 = 1/50$ . In the Figure 8A we measure total time spent before recovery as a function of  $\mu_1$  and  $\mu_2$ . Since we are interested in minimizing the total time it takes for an individual to move to a rehabilitated state. The region colored in dark blue highlights the areas of the graph in which we have relatively low “average total time to recovery” for an individual. Through observation, we may conclude that the optimal total time to recovery occurs when  $\mu_2$  is increased respect to  $\mu_1$ . Given that our  $\delta_1$  is low ( $1/50$ ) and less than half of  $\delta_2$  ( $1/15$ ), we want more individuals to enter individual therapy because it will take them significantly less amount time to recover.

Now we know the sort of behavior we want  $\mu_1$  and  $\mu_2$  to engage in, so we apply them to the next 3 graphs to verify that they correlate with our observations in graph 1. In Figure 8B, when  $\mu_2$  is set so it is big respect to  $\mu_1$  we observe that the insurance limit is very high (approximately 65 weeks). Since the ratio between  $\delta_2$  and  $\delta_1$  is less than  $1/2$  then an individual recovers from  $I$  at less than half the time it would take him to recover from  $G$ . The insurance company allows individuals such a large amount of time in individual therapy because it is quicker and cheaper overall despite being more expensive than group

therapy.

Figure 8C and Figure 8D are also displaying the expected results. In Figure 8C, when we let  $\mu_2$  be bigger than  $\mu_1$  the time that an individual spends in individual therapy increases as it should because the insurance limit Figure 8B increases also. And in Figure 8D when we let  $\mu_2$  be bigger than  $\mu_1$ , the time that an individual spends in group therapy decreases as it should because more people choose individual therapy.

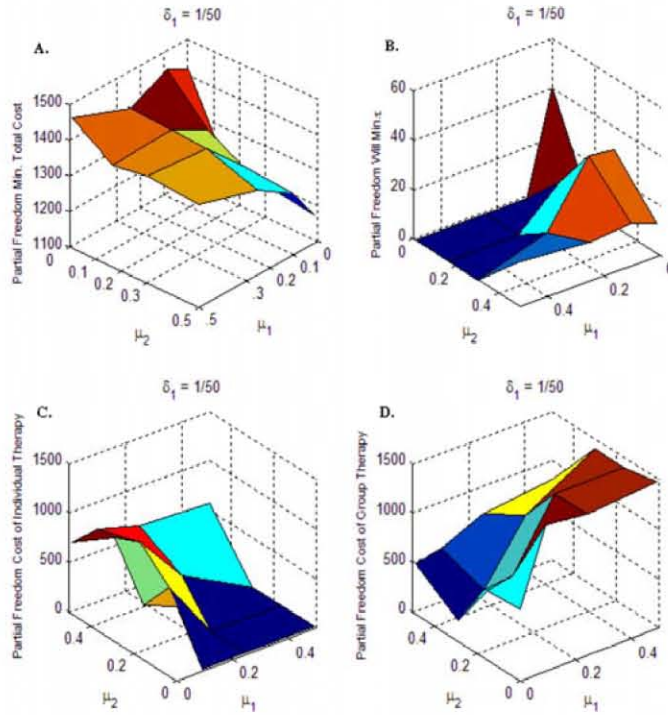


Figure 9: Cost Graph for  $\delta_1 = 1/50$

Figure 9 corresponds to the Partial Control model with respect to cost. Again, the region colored in dark shaded (lower values) highlights the areas of the graph in which we have relatively low “average total cost of treatment” for an individual and the gray shaded region (higher values) denotes relatively high cost. We want the minimum total cost of treatment for an individual prior to their recovery. From Figure 9A we conclude that the optimal total cost of treatment occurs when  $\mu_2$  is bigger than  $\mu_1$ . Since more individuals enter individual therapy, it takes them a significantly shorter amount of time to recover (due to the low rate of recovery from group therapy), this implies it also costs them less

money. Figure 8A supports this conclusion. In Figure 9B, when  $\mu_2$  is bigger than  $\mu_1$  we observe the insurance limit is relatively high. However, note the insurance limit will not grow forever as  $\mu_2$  is increased. The insurance company need not allow more than 32 weeks for individual therapy. The insurance company allows individuals a large amount of time in individual therapy because it is a significantly quicker way to recover. Figure 9C and Figure 9D are also displaying the expected results. In Figure 8C, when we let  $\mu_2$  be bigger than  $\mu_1$  the cost of individual therapy increases as it should because the insurance limit Figure 8B increases also. In Figure 8D when we set  $\mu_2$  bigger than  $\mu_1$  the time that an individual spends in group therapy decreases as it should because more people choose individual therapy which allows an adequate insurance limit. The reason setting  $\mu_2$  bigger than  $\mu_1$  lets us optimize cost is because group therapy, while less expensive, takes longer and thus costs more overall.

Therefore, if  $\delta_1 = 1/50$ :

1. In order to minimize the average time to recovery for one individual we need to make  $\mu_2$  bigger than  $\mu_1$ .
2. In order to minimize the cost for one individual we need to make  $\mu_2$  bigger than  $\mu_1$ .

Consequentially, for  $\delta_1 = 1/50$ , to optimize cost and time, we need to make  $\mu_2$  bigger than  $\mu_1$ .

## 6.1 Total Control Model Results

Figure 10 measures the total amount of time spent in therapy, as well as the amount of time spent in group vs. individual therapy. From this we observe the  $\tau$  at which the minimum time to recovery is realized. From observation,  $\tau = 10$  weeks gives us the relative minimum amount of time it takes for an OCD individual to recover, where the minimum is about 12 weeks consisting of a little more individual therapy than group therapy. The minimum time to recovery is an interesting observation, but more importantly we want to optimize (by minimizing) the cost of treatment with respect to some  $\tau$  imposed by the insurance company.  $\tau = 5$  weeks gives us the relative minimum cost of treatment, which is about 750 dollars; most of the cost coming from the cheaper group therapy. The highest amount an insurance company can be charged with for treating an individual with OCD is approximately 1300 dollars. By making  $\tau = 5$ , the insurance company saves 550 dollars, almost half of the amount it spends. So, using our realistic parameters we find that it will be most beneficial to the insurance company to allow an individual 5 weeks of individual therapy before moving them into group therapy to complete their treatment.

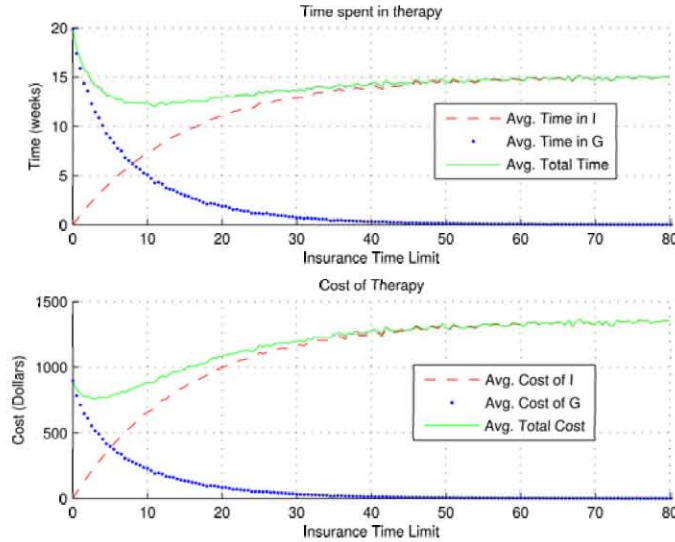


Figure 10: Time and Cost vs Insurance Limit

## 7 Conclusions and Future Work

The Free Will model provides a basic understanding of a Markov chain and allows for comparison between numerical simulations and theoretical analysis. Its simplicity is a foundation which facilitates the construction of the Partial Control and Total Control Model. The Free Will model assumes that the individual has unlimited resources, a basis of comparison to the other models with an insurance time limit. In the partial control model for  $\delta_1=1/20$ , in order to minimize time we need to make  $\mu_2$  bigger than  $\mu_1$ . If we want to minimize cost we let  $\mu_1$  be bigger than  $\mu_2$ . For  $\delta_1=1/50$  we can make  $\mu_2$  bigger than  $\mu_1$  and this will let us minimize time and cost. In the Total Control model, we found that the optimal health insurance time limit for individual therapy is 5 weeks.

In almost all cases  $\tau$  was less than the expected time to recovery from individual therapy without the insurance limit. Therefore an individual is typically forced to spend time in both group and individual therapy. In the partial control model with  $\delta_1=1/50$ , since the recovery rate from individual therapy is significantly greater than the rate of recovery from group therapy, it is more cost efficient to have individuals recover from individual therapy alone.

The main critique of this paper is the lack of data necessary to estimate the parameters of our models. To overcome this we varied our parameters over a wide range to see the overall behavior. This model is easily adapted to other mental disorders since this corresponds to a simple change in parameter values. The importance of these models is therefore enhanced into a more general social-economical understanding of the treatment

of mentally ill people. When the scope moves into this larger scale, the cost to society is increased, thus the urgency of answering questions for these dynamics is increased. Such future studies can provide answers to treating and removing the suffering of the mentally ill population thus improving society into an emotionally healthy future.

Optimizing cost and time to recovery is an ill-posed problem which cannot be solved unless there is an associated cost (to society, and possibly even the insurance companies) of individuals not being in recovery. In the future we would like to explore how adding even minimal cost to the  $U$  class can change the dynamics.

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