

# Modeling the Impact of *Batrachochytrium Dendrobatidis* in the Life History Dynamics of *Eleutherodactylus Coqui* Under Seasonality

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August 3, 2010

## Abstract

*Eleutherodactylus Coqui* a native amphibian in Puerto Rico and invasive specie in Hawaii, can be infected by an endemic disease called *Batrachochytrium dendrobatidis*. Puerto Rico and Hawaii have two main seasons, a wet/warm season and a dry/cool season, although not at the same period of time. *E. coqui* reproduces the entire year but tend to breed higher during the wet/warm season. In contrast, it has been found that the disease tend to affect them more in the dry/cool season. The Hawaiian government wants to eradicate *E. coqui* populations, based on the fact that *E. coqui* affects native species and have negative impact in their tourism population. The use of chemical and biological agents, such as *B. dendrobatidis* has been introduced without successful results. We use two stage-structured models to represent the effects of the disease, the juvenile stage and adult stage. Since *E. Coqui* in Puerto Rico gets infected and dies because of the disease or predator-induced mortality, the Puerto Rican model is a susceptible-infected (SI) model. The Hawaiian model is a susceptible-infected-carrier (SIC) model due to the absence of predators in the population and consequently, longer life-span. We look at how the seasonality of the disease impacts these two stages in Puerto Rico and in Hawaii We perform sensitivity analysis on the system equations to study the effect the seasonality function on the transmission of the disease by the environment. Numerical results show that the infected population are higher in Puerto Rico and in Hawaii. The environmental function infer a seasonality atmosphere in Hawaii and that in Hawaii the carriers will higher than the infected population.

## 1 Introduction

The *Eleutherodactylus coqui* is a frog native in Puerto Rico that belongs to the Phylum Chordata, class Amphibia, order Anura, family Leptodactylidae. There reproduction is different from other frogs since they do not need water to reproduce and they do not have a tadpole phase [1]. These amphibians are native of the tropical island of Puerto Rico. This island have a mildly seasonal weather [2].

*Eleutherodactylus Coqui*, is also consider an invader specie in Hawaii; it was accidentally introduce to Hawaii by nursery plants in late 1980's [3]. The Hawaiian government has introduces different strategies with the goal of controlling the *E. Coqui* population. Part of the problem is that there are not enough predators [1]. During the last 10 years *E. coqui* has taken over Hawaii, they loud mating calls and their insectivorous diet has created a havoc in Hawaii [3]. The *E. Coqui* lives mostly near populated areas, the residents and tourists cannot stand their mating calls. Their diets compete with the diets of native species like the Hawaiian hoary bat (*Lasturus cinereus semotus*)[4]. Due to this invasion there has been an increase on research linked to *E. coqui* in Hawaii. There are estimates, growth rates, biological control, identifications of potential predators, descriptions of its life history in Hawaii. The use of *Batrachochytrium dendrobatidis* as a solution of biological control to eradicate the *E. coqui* has been tried[1, 5]. We model the impact of the *B. dendrobatidis* in the different stages of *E. coqui* and study role of environmental seasonality.

## 1.1 Life History of Eleutherodactylus Coqui

*Eleutherodactylus coqui* is commonly found in tropical weather areas with high precipitation [2]. *E. coqui* call his mate in the night to reproduce. The female looks for the loudest or stronger male then they perform the amplexus, embrace of frogs [6]. The fertilization is internal and the development occurs inside the egg [1]. *E. coqui* female clutches after 3 hours of fertilization, then the female deposits the eggs commonly in a leaf litter or rolled leaf and deposits. Approximately 28-41 eggs are in every clutch [7, 1]. After the female has deposited the eggs the male guards them, to keep then away from dryness and predators until they hatch. The male canibal instincts when guarding the eggs; if the eggs are bad or if a predator wrecks them. They eats the male eats 14.3% the eggs on average[2].

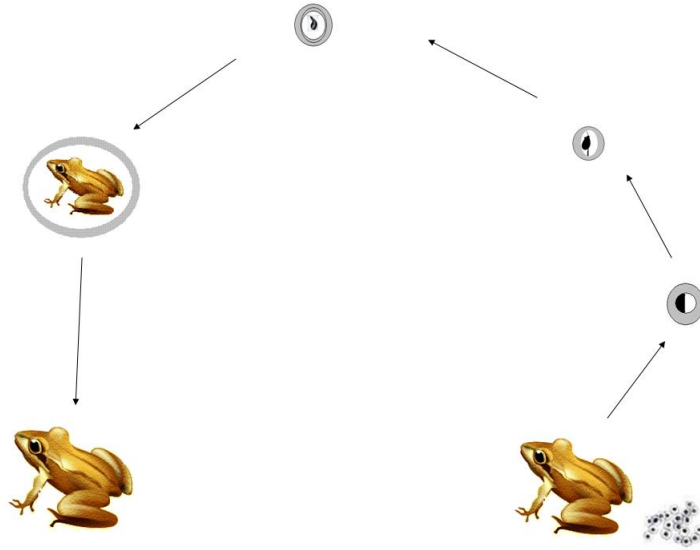


Figure 1: *E. Coqui* Life Cycle: First, the female deposits the eggs, then the egg experience the first division of development, follow by formation of the embryo, then an experience advance embryonic development, then completes development inside the egg and finally hatch This figure was recreated from [6].

*Eleutherodactylus coqui* development is performed inside the egg, see Figure 1. The development carried out over 15 stages of morphological and ethological changes. They experiment stages of tip development, brain tissue, bones and sex characterization [2]. Their growth depends on temperature conditions; that is if the temperature drops their growth is not as effective [7].

After 14-17 days they hatch and become juveniles that is, approximately 5 mm long. Juveniles take 8 to 9 months to develop before they become sexually mature. They reproduce all year, tend to breed more in the wet/warm season [7]. Juveniles die if they are exposed to 5 days of drought conditions [8].

*E. coqui* also have a middle stage phase called subadults. In Puerto Rico they measure approximately 20-22 mm and their prey and predators are different from juveniles and adults [2]. Therefore, they coincide with the adults with the same habitat. Thus, for the purpose the subadult population will be consider part of the adult population.

The *E. Coqui* life expectancy is 4-6 years however in Puerto Rico with 80% of them not surviving the first year [2]. The juveniles to adult mean ratio in Puerto Rico is about 5.3 to 1 [2] while their density is approximately 3-20 adults/100 m<sup>2</sup> [9]. In Hawaii the juvenile to adult mean ratio is about 2.5 to 1 [1] while their density is about 11 to 112 adults per m<sup>2</sup>. Sizes of *E. coqui* vary among regions and according to elevations. Adults measure between, 33.8- 35.5 mm if females and 29.6-32.3 mm if males [9]. Their density varies with elevations, as elevation increases, the density decreases [1]. In Puerto Rico the highest densities are between the months of March to September and the lowest between February and June [2]. *E. coqui* prefers humid environments [1] but habitats vary by

stages. Juveniles live near the ground mainly in leaf litter. Litter gives them rehydration site, predator protection and food [2]. Adults live in higher places, like trees [7] making their density smaller 5.3 to 1.

In Puerto Rico, *E. coqui* has native predators and is an important food web component of the tropical forest. Their mortality in Puerto Rico due to predation is high. However, there are no native predators in Hawaii in the population explosion of *E. coqui*. The Hawaiian government introduced several predators to control the *E. coqui* but they have been ineffective [1].

In Puerto Rico the main predators of the *E. coqui* are birds and invertebrates. Invertebrates prefer juveniles while birds eat more adults. Other of adult predators include owls, hawks, thrasher, thrush and snakes while crab spiders prefer juveniles [2]. Phorid flies, ants and centipedes, snails and spiders prey in of the egg population [2].

In Hawaii some of the introduced predators are *Mongoose* and *H. Javanicus* [10] but research on their effectiveness is low. Hawaiian government tried ways eradicating them, with a fungi called *Batrachochytrium dendrobatidis*.

## 1.2 Batrachochytrium dendrobatidis

*Eleutherodactylus* species are endangered in Puerto Rico due to the disease *Batrachochytrium dendrobatidis*. It is a fungi which can rapidly reproduce under any circumstances [11]. *Batrachochytrium* zoospores waterborne can live up to 24 hours. The incubation period in amphibian is about 10-47 days [11]. *B. dendrobatidis* can adapt to cooler temperatures, and since it can survive in water do not susceptible individuals to subsist [11, 12]. This fungi consumes substrates (chitin) from insects cadavers, cellulose and keratin [11].

Chytridiomycosis symptoms on *E. coqui* are discoloration, sloughed skin and lost of movility. The disease make then less active and act moribund toward death [11]. They can have hemorrhage in different parts of the body and pale muscles. Also, the diseases affects their ability to absorb water, leading to a reduce their hydration. [8].

*B. dendrobatidis* can be transmitted by direct contact. This disease can also be transmitted by carriers that is, amphibians who have low or none levels of infection. *E. coqui* serve as a carrier of *B. dendrobatidis*.

*Eleutherodactylus* species seems to be the only amphibians left after a to *B. dendrobatidis* epidemic [13]. In Puerto Rico susceptibility to the disease is higher in juveniles than adults [11]. Also, *E. coqui* susceptibility to *B. dendrobatidis* varies between seasons, they are more susceptible in the dry/cool season [13].

## 1.3 Effects of Seasonality

Puerto Rico presents is characterized by two seasons, by the wet/warm season and the dry/cool season. The wet/warm season takes place between early May through late October. The dry/cool season takes place in late November through late April [13].

The Puerto Rican climate at El Yunque shows the seasonal characteristics discussed before. Temperatures in the dry/cool season is around  $60 - 75F^{\circ}$  in the wet/warm season is around  $80 - 90F^{\circ}$ . Precipitation range in the dry/cool season is less than 300 mm while in the wet warm season is 460-494 mm [8].

In Hawaii the summer season is between May to September and winter is between October to April. Contrary to Puerto Rico the higher precipitation in Hawaii is in win-

ter. Winters are cooler, temperature is about  $75 - 80F^o$  compared to summers that have average temperatures of  $78 - 85C^o$ . Precipitation in the winter is about 400-500 mm while in the summer is about 110-200 mm [14]. Therefore, because *E. coqui* prefer humid enviroments, we are going to propose that the wet/warm season is similar to high precipitation in Hawaii and low precipitation to the dry/cool season.

*E. coqui* is an amphibian characterize by weather conditions. They are more likely to breed in wet/warm conditions. In wet/warm conditions they are more active and more likely to avoid predation [2]. It have been reported that their density decreases if there is a long period of drought [2].

The dry/cool season have been identified as a cause for declines of amphibians due to *B. dendrobatidis* [13]. Moreover, it is found that the prevalence to *B. dendrobatidis* in the dry/cool season is 25% higher than the wet/warm season [13]. Seasonality is cyclic, every year in the dry/cool season *E. coqui* prevalence of *B.dendrobatidis* increases [13].

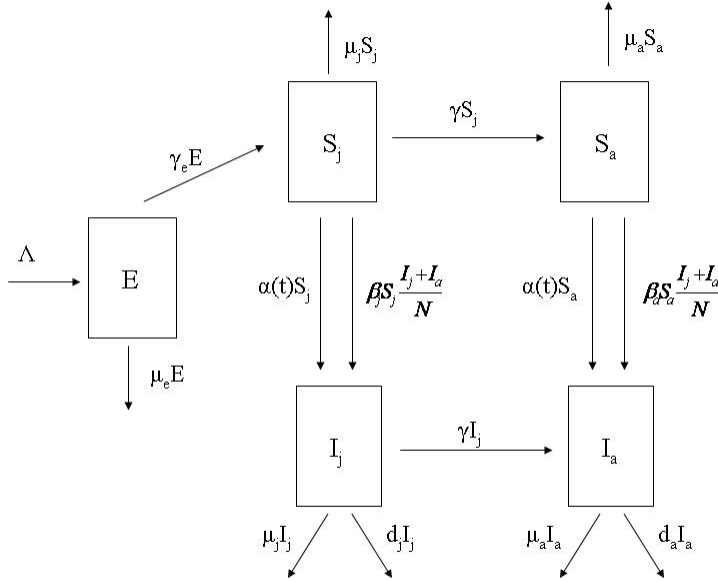


Figure 2: Stage-Structured Compartmental Model for Puerto Rico it is an susceptible-infected (SI).

## 2 Puerto Rican Model

The model is develop to study the dynamics joint of the *B. dendrobatidis* and *Eleutherodactylus Coqui* population. We develop a susceptible-infected stage-structured model. Figure (2) provides a compartmental diagram for our model.

Parameters	Definitions
$\Lambda(N_a)$	egg production rate from adult population
$\beta_j$	transmission rate of infection by direct contact in juveniles
$\beta_a$	transmission rate of infection by direct contact in adults
$\mu_j$	natural mortality rate in juveniles
$\mu_a$	natural mortality rate in adults
$\gamma$	stage progression from juveniles to adults
$d_j$	death rate caused by the disease in juveniles
$d_a$	death rate caused by the disease in adults
$\mu_e$	natural mortality rate in eggs
$\gamma_e$	progression from the egg population to the juvenile population
$\alpha(t)$	transmission rate by the environment
$\delta$	progression rate from the infected class to the carriers class in Hawaiian model

Table 1: Parameters Description for the Two Models

The stage structured model take into account two types of susceptible; the susceptible class for juveniles( $S_j$ ) and the susceptible class for adults $S_a$ . We also consider an eggs population( $E$ ), to account for influx of birth, hatching rates and egg. We take into account infection by adults( $I_a$ ) and juveniles ( $I_j$ ) divided by the total population. Table 2 describes the definition of parameters of our model.

$$\frac{dE}{dt} = \Lambda(N_a) - (\mu_e + \gamma_e)E \quad (1)$$

$$\frac{dS_j}{dt} = \gamma_e E - (\mu_j + \gamma + \alpha(t))S_j - \beta_j \frac{I_j + I_a}{N} S_j \quad (2)$$

$$\frac{dI_j}{dt} = \beta_j \frac{I_j + I_a}{N} S_j + \alpha(t)S_j - (d_j + \mu_j + \gamma)I_j \quad (3)$$

$$\frac{dS_a}{dt} = \gamma S_j - (\alpha(t) + \mu_a)S_a - \beta_a \frac{I_j + I_a}{N} S_a \quad (4)$$

$$\frac{dI_a}{dt} = \gamma I_j + \alpha(t)S_a + \beta_a \frac{I_j + I_a}{N} S_a - (\mu_a + d_a)I_a \quad (5)$$

$$(6)$$

where is the total population of juveniles and adults is

$$N = N_j + N_a = S_j + I_j + S_a + I_a \quad (7)$$

We consider the mortality rate by natural causes (like predation) to be different, because the mortality in juveniles tend to have a higher mortality than adults and eggs mortality is widely different from the other stages. The progression from juveniles to adults,  $\gamma$ , is consider to be linear for simplicity [15]. The mortality disease rate due to the disease is different between stages because the juveniles tend to be more prevalent to the disease. In our model we assume *E. coqui* can get infected through the environment by function  $\alpha(t)$  and by contact with others. The transmission rates between *E. Coqui* are  $\beta_j$  and  $\beta_a$  for juveniles and adults respectively and environment by  $\alpha(t)$  For mathematical

analysis we consider  $\alpha = 0$  and we use it as a function of time for the simulations a way to see the how the environmental transmission affects the system.

The environmental function is going to be a function will depend on time a it is going to simulate the seasons of low and high infection in Puerto Rico. In order to do this analysis we need to arrange this seasons that represents the seasonality of the transmission by the environment.

## 2.1 Analysis of the Puerto Rican Model for $\alpha = 0$

To perform some analysis and to study the impact of environmental transmission we study the Puerto Rican Model when the function  $\alpha(t) = 0$ . The equilibrium of the system are obtain by setting the equations in System 10 equal to zero. The disease free equilibrium (DFE) is  $DFE_{PR} = (E^*, S_j^*, I_j^*, S_a^*, I_a^*) =$

$$\left( \left( \frac{\Lambda}{\gamma_e + \mu_e} \right), \left( \frac{\Lambda}{\gamma_e + \mu_e} \right) \left( \frac{\gamma_e}{\gamma + \mu_j} \right), 0, \left( \frac{\Lambda}{\gamma_e + \mu_e} \right) \left( \frac{\gamma_e}{\gamma + \mu_j} \right) \left( \frac{\gamma}{\mu_a} \right), 0 \right) \quad (8)$$

Notice that  $S_j$  and  $S_a$  depend of the stage progression factors. Then, for the egg population it can be assume that the average time the spend in the egg population and the mean that enters to the population, for this case the mean birth rate. Now, for the population of susceptible juvenile population, the term for the egg population times the average of juveniles that enters through the progression  $\gamma_e$  times the average time the individuals spend in the susceptible juveniles population. At last, the susceptible adult population depends of the other population and the progression from juveniles to adults,  $\gamma$  times the average time that the individuals spend in the susceptible adult population.

In this case of  $\alpha(t) = 0$  we compute the basic reproductive number that is, average number of infected individuals that a typical infected individual can infect in a population of susceptible. We calculate  $R_0$  using the next generator operator by linearizing the system around the DFE [16, 17]. Let,

$$\mathcal{F} = \begin{pmatrix} \beta_j \frac{I_j + I_a}{N} \\ \beta_a \frac{I_j + I_a}{N} \end{pmatrix}$$

and

$$\mathcal{V} = \begin{pmatrix} (d_j + \mu_j + \gamma)I_j \\ -\gamma I_j + (\mu_a + d_a)I_j \end{pmatrix}$$

Where  $\mathcal{F}$  is the rate of new infection and  $\mathcal{V}$  is the rate of transfer into other compartment of the model [16]. Now,

$$F = \begin{pmatrix} \beta_j \left( \frac{\mu_a}{\gamma + \mu_a} \right) & \beta_j \left( \frac{\mu_a}{\gamma + \mu_a} \right) \\ \beta_a \left( \frac{\gamma}{\gamma + \mu_a} \right) & \beta_a \left( \frac{\gamma}{\gamma + \mu_a} \right) \end{pmatrix}$$

and,

$$V = \begin{pmatrix} d_j + \mu_j + \gamma & 0 \\ -\gamma & \mu_a + d_a \end{pmatrix}$$

Then,

$$FV^{-1} = \begin{pmatrix} \frac{\beta_j \mu_a + \beta_j \mu_a \gamma}{(\mu_a + \gamma)(d_j + \mu_j + \gamma)(\mu_a + d_a)} & \frac{\beta_j \mu_a}{(\mu_a + \gamma)(\mu_a + d_a)} \\ \frac{\beta_a \gamma (\mu_a + d_a) + \beta_a \gamma^2}{(\mu_a + \gamma)(d_j + \mu_j + \gamma)} & \frac{\beta_a \gamma}{(\mu_a + \gamma)(\mu_a + d_a)} \end{pmatrix}$$

The matrix  $FV^{-1} - \lambda I = 0$ , this  $R_0$  is the largest eigenvalue of the matrix. That is,

$$R_0^{PR} = \left( \frac{\beta_j}{\mu_j + d_j + \gamma} \right) \left( \frac{\mu_a}{\gamma + \mu_a} \right) + \left( \frac{\beta_a}{\mu_a + d_a} \right) \left( \frac{\gamma}{\gamma + \mu_a} \right) \\ + \left( \frac{\beta_j}{d_j + \mu_j + \gamma} \right) \left( \frac{\mu_a}{\mu_a + d_a} \right) \left( \frac{\gamma}{\mu_a + d_a} \right)$$

where  $\left( \frac{\beta_j}{\mu_j + d_j + \gamma} \right) \left( \frac{\mu_a}{\gamma + \mu_a} \right)$  is the contribution to  $R_0^{PR}$  by infected juveniles, while  $\left( \frac{\beta_a}{\mu_a + d_a} \right) \left( \frac{\gamma}{\gamma + \mu_a} \right)$  the contribution by infected adults and  $\left( \frac{\beta_j}{d_j + \mu_j + \gamma} \right) \left( \frac{\mu_a}{\mu_a + d_a} \right) \left( \frac{\gamma}{\mu_a + d_a} \right)$  the contribution  $R_0^{PR}$  by juveniles that survive as infection to adults.

## 2.2 Parameters

Parameters are extremely important when understanding epidemic models interpretation of a particular case come to place. Our time units scale will be in months.

*E. coqui* clutches about 4 times a year with about 28 eggs per clutch [7]. With this data we can approximate for this case. Then, the birth rate is  $28 \cdot 4 / 12$  and times the hatching rate 0.44 it gives us the birth rate. Considering the sex ratio is 1:1 [9] the birth rate is estimated to be 3.7.

Researchers have found that the mortality rate of the *E. coqui* population for juveniles 80% and for adults 94% in a year in Puerto Rico. For the calculation of the death rate of each population we use a method found in [18]. Then, because this data is year is required to use data in month for the seasonality analysis, is divided by 12.

Natural mortality of the eggs in Puerto Rico is 16% per week. For the calculation of the eggs mortality is needed to divide by 4 to converted to month, then the value is  $0.64 \text{ month}^{-1}$ . Table 2.2 provide the parameter values including the estimated.

Parameters	Puerto Rico	Reference
$\Lambda$	3.7* individuals per month	[7]
$\beta_j$	0.2+ individuals per month	
$\beta_a$	0.4+ individuals per month	
$\mu_j$	0.067* individuals per month	[7]
$\mu_a$	0.075* individuals per month	[7]
$\gamma$	0.1+ individuals per month	
$d_j$	0.02+ individuals per month	
$d_a$	0.01+ individuals per month	
$\mu_e$	0.64* individuals per month	
$\gamma_e$	0.44* hatch per month	[19]

Table 2: Data assigned to parameters, estimated+ and from literature\*

## 2.3 Numerical Results for the Puerto Rican Model

In this section we study the impact of seasonality or infection by the environmental factors, through the function  $\alpha(t)$ . We let  $\alpha(t) = a(1 + \sin(\frac{\pi}{6}t - c))$ . The  $a$  is the amplitude of



the disease how far this environmental factor can infected the *E. coqui* in the different seasons. Like we mention before wet/warm weather are consider to be favorable for *E. Coqui* reproduction and the dry/cool season is when get affected by the disease [13]. The  $\frac{\pi}{6}$  is the period that we want the seasonality of Puerto Rico because time scale is in months we are going to start in October with medium level of infection and end in October with medium infection and with a peak in January. The c the translation factor is chosen to be  $c=0$ . This seasonality function is represent by the Figure3.

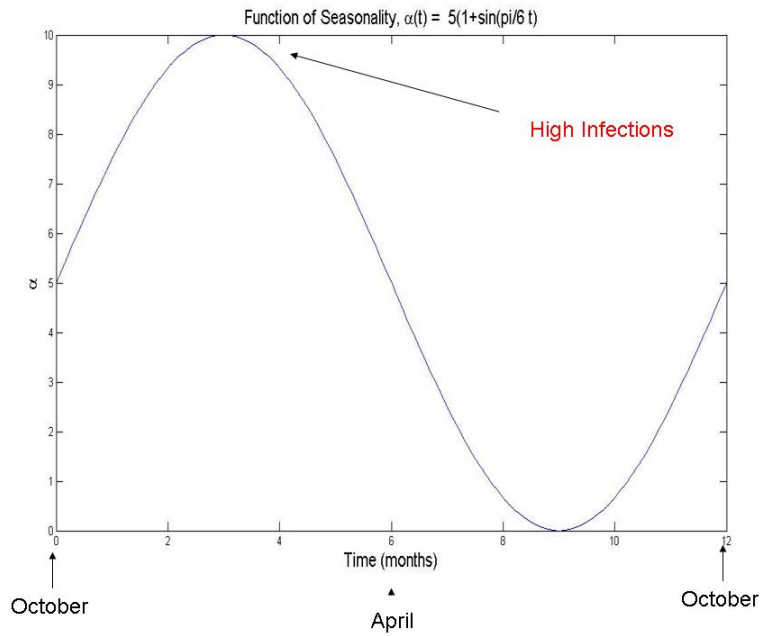


Figure 3: The seasonality function,  $\alpha(t) = 5(1 + \sin(\frac{\pi}{6}t))$  for Puerto Rico

We did simulations to demonstrate how the seasonality can affect the system. However, we use  $\alpha = 0$  first to compare the system without seasonal effects. We use the values of the parameters from the literature and estimated. Figure 4 represent the Puerto Rican model where natural mortality rate is due to predation.

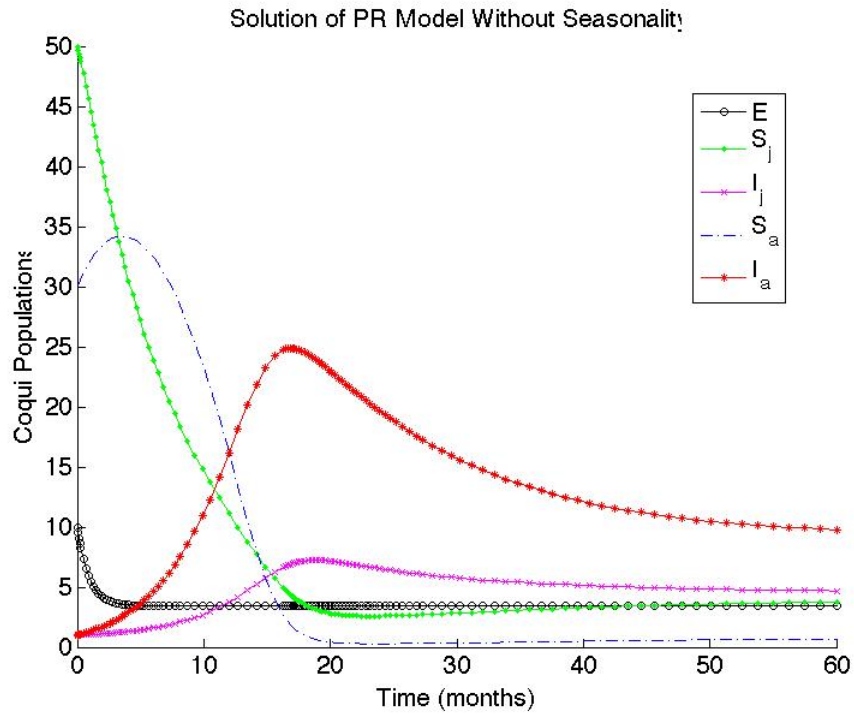


Figure 4: The solution of the Puerto Rican Model without seasonality  $\beta_j = 0.2, \beta_a = 0.4, d_j = 0.02, d_a = 0.01$  and  $R_0^{PR} = 3.66$

Figure 4 shows that the egg population decreases and stay constant as time goes. Susceptible juvenile population decrease due to the transmission of the disease going to infected juveniles populations and from mature to the susceptible adults. For the adult population susceptible increases at first receiving from susceptible juvenile population but decreases because adults are getting infected by direct contact. Finally, the infected adult population increases with a big impulse because it receive population from susceptible adult population and infected adult population but then decreases because of mortality due to the disease and because of the predator population. However, since  $R_0^{PR} = 3.66$  the disease is always endemic in the population.

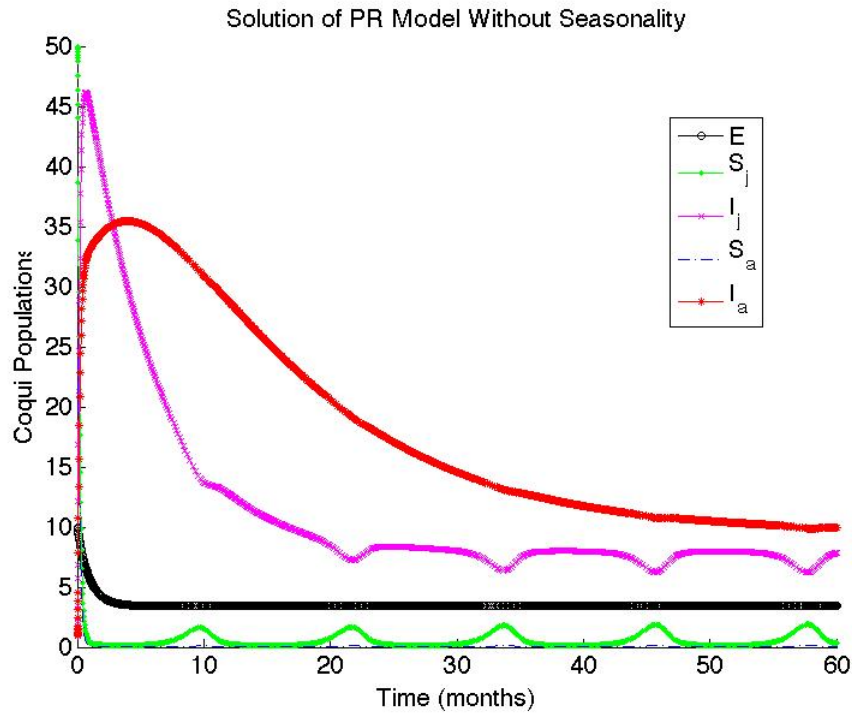


Figure 5: The solution of the Puerto Rican Model with seasonality  $\beta_j = 0.2, \beta_a = 0.4, \gamma = 0.1, d_j = 0.02, d_a = 0.01, \gamma = 0.1$

To study seasonality effects we used the same parameter values as in Figure 4 but with  $\alpha(t) = a(1 + \sin \frac{\pi}{6})$  with  $a = 5$ . First, the egg population stay constant as time progress. The susceptible adult population decreases quickly with the input of the seasonality function that is, environmental transmission and oscillate within. Is interesting to see how the behavior of infected adult population change due to seasonality. It is observe a impulse increase in the first years but drop slowly but with slight little oscillations. Next, looking the behavior of susceptible adult population this the one that do no receive a great amount of population because juveniles get infected or adults get infected. Seasonality is observed but the oscillations are not that significant like the juveniles populations. Finally, it is observe that infected adult population increase at the beginning because of the influx of the the population like susceptible adult population and infected adult population. *E. coqui* populations in Puerto Rico with the high predator population predicts that the *E. coqui* only exist as a infected individual . Now, there is strong evidence that *E. coqui* population can be a carrier, but because of predator population is there in no such population. Therefore, if we consider a a place where is no predators they can stay a carrier. This, describe Hawaii situation where there is no predators

### 3 Hawaiian Model

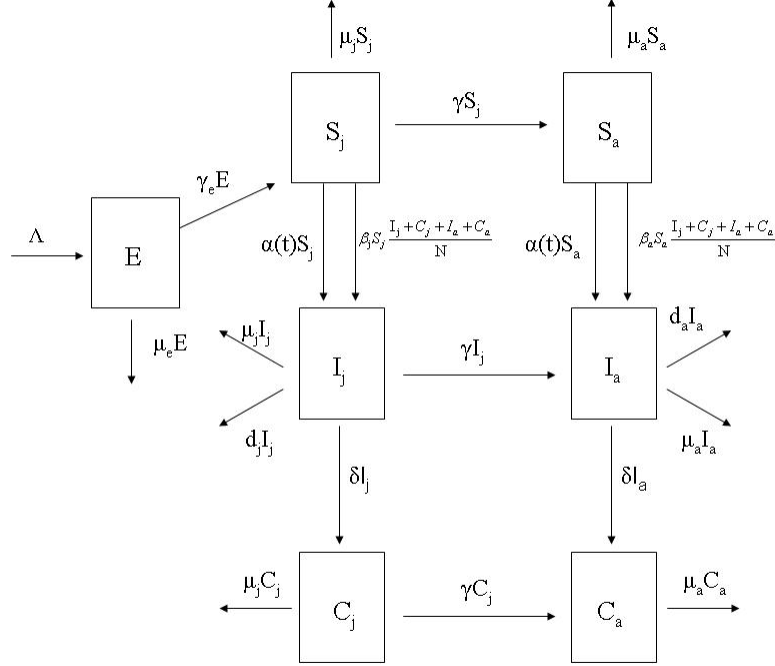


Figure 6: Stage-Structured Compartmental Model: This is the model developed for Hawaii this model is an susceptible-infected-carriers(SIC).

To study the dynamics of the disease in Hawaii we incorporate carriers in the population for both stages. In this model, we introduce the carrier progression of the disease because as we mention before there in low predation in Hawaii different from Puerto Rico that is the main cause of death of *E. Coqui*.

$$\frac{dE}{dt} = \Lambda(N_a) - (\mu_e + \gamma_e)E \quad (9)$$

$$\frac{dS_j}{dt} = \gamma_e E - (\mu_j + \gamma + \alpha(t))S_j - \beta_j S_j \frac{(I_j + I_a + C_j + C_a)}{N} \quad (10)$$

$$\frac{dI_j}{dt} = \beta_j S_j \frac{(I_j + I_a + C_j + C_a)}{N} S_j + \alpha(t)S_j - (d_j + \mu_j + \gamma + \delta)I_j \quad (11)$$

$$\frac{dC_j}{dt} = \delta I_j - (\gamma + \mu_j)C_j \quad (12)$$

$$\frac{dS_a}{dt} = \gamma S_j - (\alpha(t) + \mu_a)S_a - \beta_a S_a \frac{(I_j + I_a + C_a + C_j)}{N} \quad (13)$$

$$\frac{dI_a}{dt} = \gamma I_j + \alpha(t)S_a + \beta_a S_a \frac{(I_j + I_a + C_a + C_j)}{N} S_a - (\mu_a + d_a + \delta)I_a \quad (14)$$

$$\frac{dC_a}{dt} = \delta I_a + \gamma C_j - \mu_a C_a \quad (15)$$

$$\text{where } N = N_j + N_a = S_a + I_a + C_j + S_j + I_j + C_a \quad (16)$$

$$(17)$$

We take into account that the carriers have a progression from juveniles to adults like the others class. The infected class is divide into two: infected and carriers. It is known that the carrier population present low infection levels of the disease and do not experiment symptoms [11]. This type of infections have been found in *E. Coqui* [5]. Individuals acquired the disease and go the infected class but if the do not experiment symptoms they go to the carriers class by the rate of progression,  $\delta$ . Figure 6 provides a compartmental model for this disease.

### 3.1 Parameters from the Literature

We use parameters from the Puerto Rican Model but the mortality rate are consider different. We use the life expectancy of the *E. coqui* is about 6 year and for the the juveniles is 8 months. We assume that the life expectancy is approximately 5 years = 60 months. Thus, the death rate is  $1/60 \text{ months}^{-1}$ , and the same is for juveniles is  $8/12 \text{ year}^{-1}$ . This parameters are use for the Hawaiian Model.

Parameters	Hawaii	Reference
$\Lambda$	3.7* individuals per month	[7]
$\beta_j$	0.2 <sup>+</sup> individuals per month	
$\beta_a$	0.4 <sup>+</sup> individuals per month	
$\mu_j$	0.056* individuals per month	[1]
$\mu_a$	0.017* per month	[1]
$\gamma$	0.1 <sup>+</sup> individuals per month	
$d_j$	0.02 <sup>+</sup> individuals per month	
$d_a$	0.01 <sup>+</sup> individual per month	
$\mu_e$	0.64* individuals per month	
$\gamma_e$	0.44* per hatch per month	[19]
$\delta$	0.014 <sup>+</sup> individuals per month	

Table 3: Data assigned to parameters, estimated<sup>+</sup> and from literature\*

## 4 Analysis for the Hawaiian Model, $\alpha = 0$

In order to calculate the equilibrium where there is no disease we need to assume that there is no environmental transmission that is that  $\alpha = 0$ . For the model describing the dynamics of *B. dendrobatidis* in *E. coqui* population in Hawaii, the  $DFE_H$  is given by  $DFE_H = (E^o, S_j^o, I_j^o, C_j^o, S_a^o, I_a^o, C_a^o) =$

$$\left( \frac{\Lambda}{\gamma_e + \mu_e}, \left( \frac{\Lambda}{\gamma_e + \mu_e} \right) \left( \frac{\gamma_e}{\gamma + \mu_j} \right), 0, 0, \left( \frac{\Lambda}{\gamma_e + \mu_e} \right) \left( \frac{\gamma_e}{\gamma + \mu_j} \right) \left( \frac{\gamma}{\mu_a} \right), 0, 0 \right) \quad (18)$$

Notice that this DFE is similar to the case of Puerto Rican model. Notice that  $S_j$  and  $S_a$  depend of the stage progression factors. Then, for the egg population is the average time a individual can spend in the egg population times the mean that enters to this population, the birth rate. Now, for the population of susceptible juvenile population, the term for the egg population times the average of juveniles that enters through the progression  $\gamma_e$  times the average time the individuals spend in the susceptible juveniles population. At last, the susceptible adult population depends of the other population and the progression from juveniles to adults,  $\gamma$  times the average time that the individuals spend in the susceptible adult population.

By using also the Next Generator Operator, we got the basic reproductive number of the Hawaiian Model. By obtaining the,

$$\mathcal{F} = \begin{pmatrix} \beta_j \frac{I_j + I_a}{N} \\ \beta_a \frac{I_j + I_a}{N} \\ 0 \\ 0 \end{pmatrix}$$

$$\mathcal{V} = \begin{pmatrix} (d_j + \mu_j + \gamma)I_j \\ -\gamma I_j + (\mu_a + d_a)I_j \\ -\delta I_j + (\mu_j + \gamma)C_j \\ -\delta I_a + \mu_a C_a \gamma C_j \end{pmatrix}$$

then

$$F = \begin{pmatrix} \beta_j \left( \frac{\mu_a}{\gamma + \mu_a} \right) & \beta_j \left( \frac{\mu_a}{\gamma + \mu_a} \right) & \beta_j \left( \frac{\mu_a}{\gamma + \mu_a} \right) & \beta_j \left( \frac{\mu_a}{\gamma + \mu_a} \right) \\ \beta_a \left( \frac{\gamma}{\gamma + \mu_a} \right) & \beta_a \left( \frac{\gamma}{\gamma + \mu_a} \right) & \beta_a \left( \frac{\gamma}{\gamma + \mu_a} \right) & \beta_a \left( \frac{\gamma}{\gamma + \mu_a} \right) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Now,  $R_0^H$  is provided by

$$\begin{aligned} R_0^H &= \left( \frac{\beta_j}{\mu_j + d_j + \gamma + \delta} \right) \left( \frac{\mu_a}{\gamma + \mu_a} \right) + \left( \frac{\beta_a}{\mu_a + d_a + \delta} \right) \left( \frac{\gamma}{\gamma + \mu_a} \right) \\ &+ \left( \frac{\beta_j}{\mu_j + d_j + \gamma + \delta} \right) \left( \frac{\delta}{\mu_j + \gamma} \right) \left( \frac{\mu_a}{\gamma + \mu_a} \right) + \left( \frac{\beta_a}{\mu_a + d_a + \delta} \right) \left( \frac{\delta}{\mu_a} \right) \left( \frac{\gamma}{\gamma + \mu_a} \right) \\ &+ \left( \frac{\beta_j}{\mu_j + d_j + \gamma + \delta} \right) \left( \frac{\mu_a}{\gamma + \mu_a} \right) \left( \frac{\gamma}{\mu_a + d_a + \delta} \right) \end{aligned}$$

where, similar to the reproductive number for the Puerto Rican model,  $\left( \frac{\beta_j}{\mu_j + d_j + \gamma + \delta} \right) \left( \frac{\mu_a}{\gamma + \mu_a} \right)$  is the contribution to  $R_0^H$  by infected juveniles, while  $\left( \frac{\beta_a}{\mu_a + d_a + \delta} \right) \left( \frac{\gamma}{\gamma + \mu_a} \right)$  the contribution by infected adults;  $\left( \frac{\beta_j}{\mu_j + d_j + \gamma + \delta} \right) \left( \frac{\delta}{\mu_j + \gamma} \right) \left( \frac{\mu_a}{\gamma + \mu_a} \right)$  is contribution to  $R_0^H$  by carriers juveniles,  $\left( \frac{\beta_a}{\mu_a + d_a + \delta} \right) \left( \frac{\delta}{\mu_a} \right) \left( \frac{\gamma}{\gamma + \mu_a} \right)$  the contribution by carrier adults and  $\left( \frac{\beta_j}{d_j + \mu_j + \gamma} \right) \left( \frac{\mu_a}{\mu_a + d_a} \right) \left( \frac{\gamma}{\mu_a + d_a} \right)$  the contribution  $R_0^H$  by juveniles that survive infection to adults.

This number represent the average number for the Hawaiian population and the difference from the Puerto Rican model is that this include the rate of progression from the infected population to the carriers.

#### 4.1 Numerical Results for the Hawaiian Model

A different seasonality function,  $\alpha(t) = a(1 + \sin(\frac{\pi}{6} - \pi))$  for the case of Hawaii is used to study the effect of the environment factors on this disease. Hawaii present high precipitation in the winter [14]. therefore, we expect more infection is the summer, then the sine function is shifted by  $\pi$ . Starting in October by starting to get low infection then the inflexion point in April and al last going to high infection in the summer.

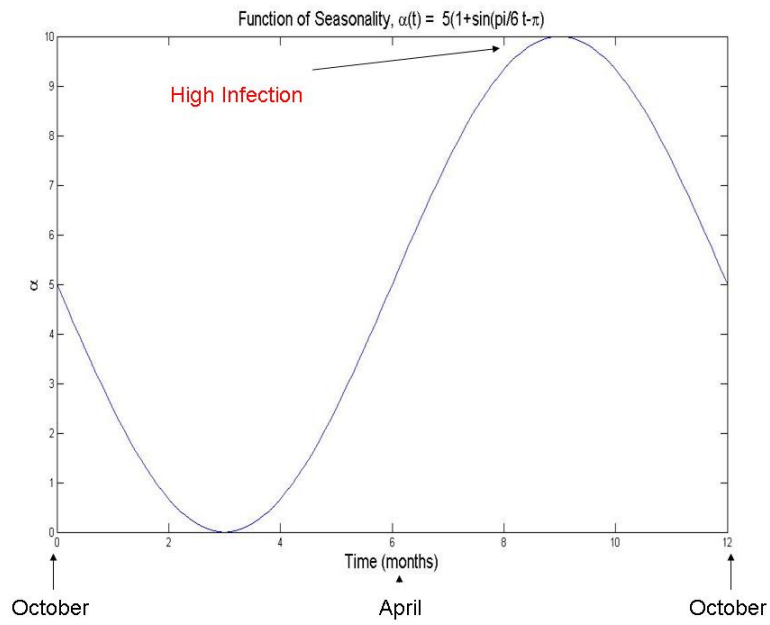


Figure 7: Seasonality function,  $\alpha(t) = 5(1 + \sin(\frac{\pi}{6}t - \pi))$  for Hawaii

First, we are going to observe how the system behaves when there is no environmental transmission that is when  $\alpha = 0$ . In the Figure 8 in show the solution for the system ???. Then, compare the impact of carriers in Hawaii.



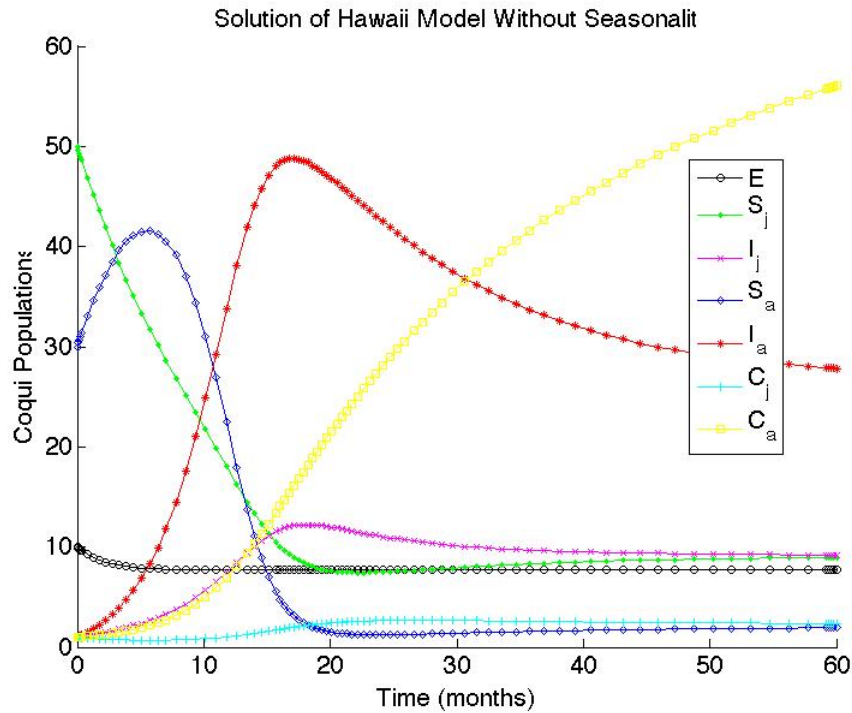


Figure 8: The solution of the Hawaiian Model without seasonality and parameters  $\beta_j = 0.2, \beta_a = 0.4, \gamma = 0.1, \delta = .014, d_j = 0.02, d_a = 0.01$  and  $R_0^H = 6.89$

First, the egg population remains basically constant differing from Figure 4. Next, observing the susceptible juveniles it behave in a similar way and considering that the mortality in Hawaii is lower, it can be infer that the susceptible population is going to other populations. The juveniles infected population increase in time because of the susceptible population becoming infected. The juveniles carriers population increase slowly because of the infected that do not experience symptoms goes to the carrier population. For the susceptible adults population it is observe that this population increases at first but then decreases, going to the infected population, because like it was mention before the mortality is lower and it show the same behavior as the Figure 4. The adult infected population received population from other population like from the susceptible adult population goes to the to the carriers adult population and the infected adult population is observed to be higher as observed in Figure 4. Finally, observing the carriers adult population increases because of the influx coming from the infected adults and carrier juveniles populations. The  $R_0^H = 6.89$  is always endemic and is a high number because of the presence of carrier population

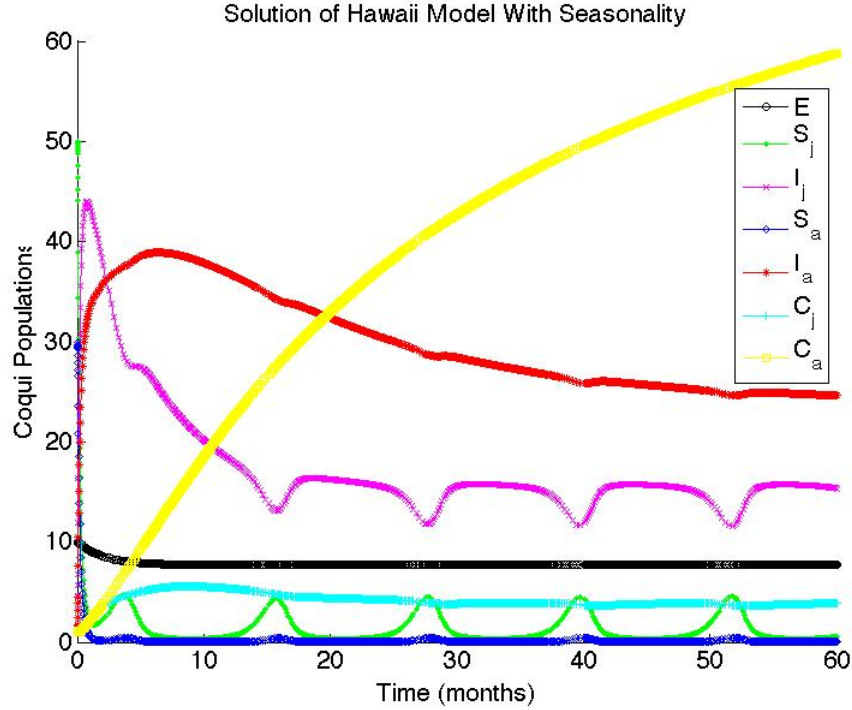


Figure 9: The solution of the Hawaiian Model with seasonality  $\beta_j = 0.2, \beta_a = 0.4, \gamma = 0.1, \delta = .014, d_j = 0.02, d_a = 0.01$  and  $\alpha(t) = 5(1 + \sin(\frac{\pi}{6}t - \pi))$

We consider the  $\alpha(t) > 0$  and the same parameters values as in Figure 8. We are going to compare this simulations with Figures 8, and 5 and look impact of seasonality and carriers. First, egg population stay constant like in Figure 8. The susceptible juveniles population it seems to be affect by the seasonality function like the Figure 5 and when this population decrease the infected juveniles increase. The infected juveniles population increases by the influx of direct transmission coming from susceptible juveniles similar behavior than in Figure 5. The carrier juveniles population increases by the influx from infected juveniles but in a long term stay constant. The adult population is less affected with the seasonality but present some lightly oscillations. Susceptible adults is lower than in Figure 8 and decrease quickly. The infected adult population is not highly affect by the seasonality function but increase with effect different from Figure 8. Finally, the carriers population show the same behavior than in Figure 8 and receive more population from the infected adult and carrier juveniles populations.

## 5 Sensitivity Analysis

Sensitivity Analysis provide the insight on how parameters affect the behavior of the system. Forward Sensitivity Analysis (FSA) is used to study how a perturbation in the parameters can produce a perturbation change to the system [20]. We use FSA is used

effects of  $\alpha(t)$  in the ODE equations of the system 10. Then, to see this change in the system  $\frac{dx}{dt} = f(x, t; p)$ ,  $x(0) = x_o$  the Sensitivity Index (S) is calculated as:

$$S = \lim_{\delta p \rightarrow 0} \frac{\frac{\delta x}{x}}{\frac{\delta p}{p}} = \frac{p}{x} \frac{\partial x}{\partial p}$$

We analyze both system with  $\alpha(t) = a(1 + \sin(\frac{\pi}{6} - c))$ . To calculate the sensitivity index, S, for Puerto Rican model we need to consider the forward problem Equations ?? the system and the (FSA). Now the derivative of the forward system with respect a the amplitude are:

$$\begin{aligned} \frac{d}{dt} \left[ \frac{\partial S_j}{\partial a} \right] &= \gamma_e \frac{\partial E}{\partial a} - \beta_j I_j \frac{\partial S_j}{\partial a} - \beta_j S_j \frac{\partial}{\partial a} \left( \frac{I_j + I_a + C_j + C_a}{N} \right) - \alpha(t) \frac{\partial S_j}{\partial a} \\ &\quad - (1 + \sin(\frac{\pi}{6} - c)) S_j - (\mu_j + \gamma) \frac{\partial S_j}{\partial a} \\ \frac{d}{dt} \left[ \frac{\partial I_j}{\partial a} \right] &= \beta_j I_j \frac{\partial S_j}{\partial a} + \beta_j S_j \frac{\partial}{\partial a} \left( \frac{I_j + I_a + C_j + C_a}{N} \right) + \alpha(t) \frac{\partial S_j}{\partial a} \\ &\quad + S_j (1 + \sin(\frac{\pi}{6} - c)) - (\mu_j + d_j + \gamma) \frac{\partial I_j}{\partial a} \\ \frac{d}{dt} \left[ \frac{\partial S_a}{\partial a} \right] &= \gamma \frac{\partial S_j}{\partial a} - \beta_a I_a \frac{\partial S_a}{\partial a} - \beta_a S_a \frac{\partial}{\partial a} \left( \frac{I_j + I_a + C_j + C_a}{N} \right) - \alpha(t) \frac{\partial S_a}{\partial a} \\ &\quad + S_a (1 + \sin(\frac{\pi}{6} - c)) - \mu_a \frac{\partial S_a}{\partial a} \\ \frac{d}{dt} \left[ \frac{\partial I_a}{\partial a} \right] &= \gamma \frac{\partial I_j}{\partial a} + \beta_a I_a \frac{\partial S_a}{\partial a} + \beta_a S_a \frac{\partial}{\partial a} \left( \frac{I_j + I_a + C_j + C_a}{N} \right) + \alpha(t) \frac{\partial S_a}{\partial a} \\ &\quad + S_a (1 + \sin(\frac{\pi}{6} - c)) - (\mu_a + d_a) \frac{\partial I_a}{\partial a} \\ \frac{\partial}{\partial a} \left( \frac{I_j + I_a + C_j + C_a}{N} \right) &= \frac{N \frac{\partial (I_j + I_a + C_j + C_a)}{\partial a} + (I_j + I_a + C_j + C_a) \frac{\partial N}{\partial a}}{N^2} \end{aligned} \quad (19)$$

We explore the sensitivity of the equations of our system to the seasonality function  $\alpha(t)$  using difference in amplitude. Plotting the Sensitivity index versus time.

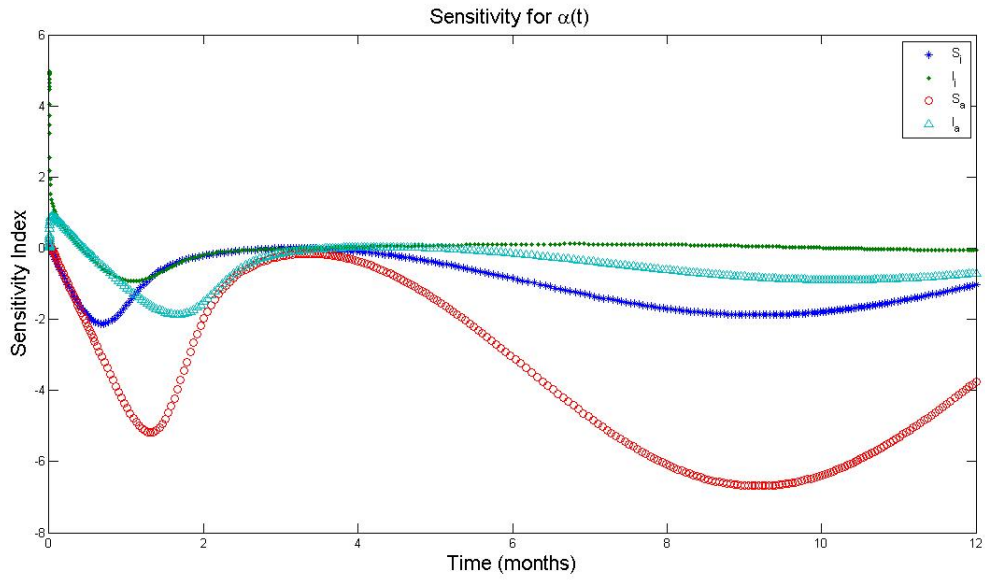


Figure 10: Sensitivity Analysis for the Hawaiian model for  $\alpha(t) = 5(1 + \sin(\frac{\pi}{6}t - \pi))$

When we add a high perturbation to  $a = 5$  the system present high sensitivity indexes. First, the susceptible juveniles population oscillates with respect of the seasonality, this one decrease 2%. The infected juveniles population present a effect that at first decrease but then stay almost constant. The susceptible adults is the one more sensitive to this parameter decreasing a 8%. Finally the infected adult population is affected decreasing at first and then staying almost constant as the time goes.

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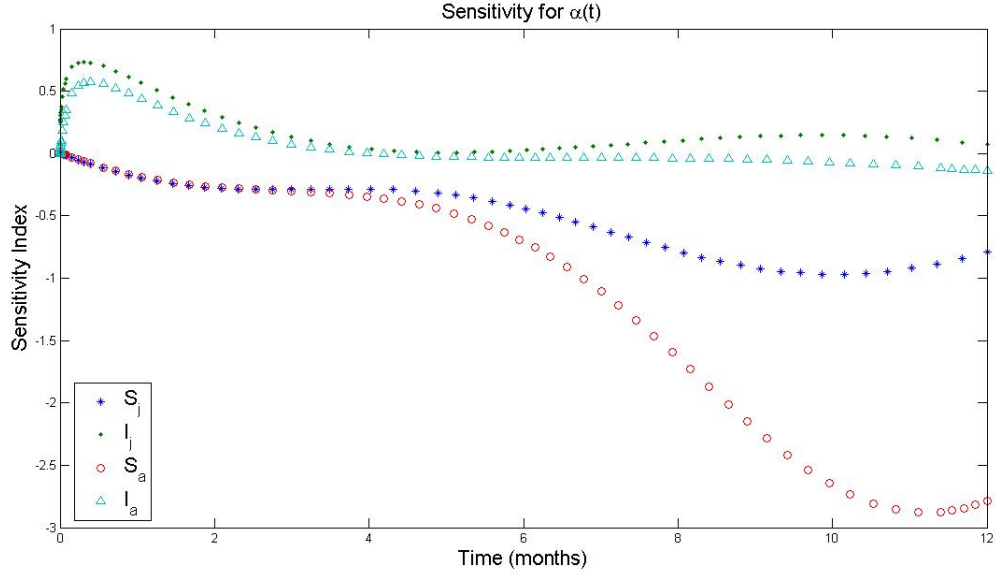


Figure 11: Sensitivity Analysis of the Hawaiian Model for  $\alpha(t) = \frac{1}{4}(1 + \sin(\frac{\pi}{6} - \pi))$

Now we look the behavior and compare to Figure 10 when  $a = 5$ , we observe that for  $a = \frac{1}{4}$  the sensitivity is relative low and that like in Figure 10 the susceptible adult population is the most affect by the parameter  $a$ . First, the infected population increase by the perturbation of the parameter  $a$ . Like the infected juveniles population is affected by a increase of 0.5 %. Then the infected adults looks nearly the same. Next the susceptible adults population decreases a maximum of 3% when there is an increase of  $a$  by 1%. Thus, the sensitivity of the system increases if we increase the amplitude  $a$ .

## 6 Discussion

In Puerto Rico without environmental transmission the models predicts that the susceptible populations decreases due to the influx of the *E. Coqui* going to the infected population. This mean that even they do not have an infection through the environment, the disease is endemic in Puerto Rico. Therefore, the model satisfies the literature found about the behavior of *B. dendrobatidis* is Puerto Rico. When there an implementation of the environmental transmission function, it is a increase of the susceptible population getting infected and when the environmental transmission is chosen to be a sinusoidal function with a high amplitude, may be a decrease of the susceptible population becoming infected. In the model susceptible juveniles where more likely to get infected by the disease but we saw a higher population of adults in the infected population.

It is expected that the carrier population will make that Hawaiian model to behave different from the Puerto Rican Model. We observed when there no is no environmental

transmission the susceptible populations' decrease, like in the Puerto Rican model this means that there an endemic population in Hawaii. The carrier adult populations are the higher population in this model. This implies that the susceptible population of juveniles and adults got a higher rate of infected with direct contact. The disease is endemic in the Hawaiian model but do not seems to affect the *E. coqui* populations because there is present high carrier population that is no affect by the disease. Thus, we expect a long term the same amount of carriers and infected population, because carrier will continue an infecting susceptible individuals. This induce to think that the effect of the natural mortality because of predation in decrease the amount infected individuals. Considering the addition of the environmental transmission function there is an increase in the infected and carrier populations even the thought the low infection seasons decreases the infected populations. Juveniles populations have notable high and low infection seasons than the adult population.

Both models present the oscillation of low and high infection seasons and confirms that the dynamics *E. Coqui* is control by the seasonality of the disease. Thus even that the in Hawaii there no literature in that support that *B. dendrobatidis* has a seasonality behavior, we observe that with low a high infection seasons affects the Hawaiian model. We can infer that in Hawaii can present a seasonality environmental transmission like in Puerto Rico.

Sensitivity Analysis predicts that the environment infection affect the susceptible populations decreasing them and infected population increasing them. The highly affected population was susceptible adult population. Also that the system is highly sensitivity to the model to the transmission environmental function  $\alpha(t)$ . With a increase of amplitude in  $\alpha(t)$  the system can experiment high perturbations affecting populations decreasing or increasing by a percent amount.

We do not suggest *B. dendrobatidis* as biology control mechanism because is observed that *E. coqui* populations stay along as carriers presenting resistant to the disease. These individuals can behave like a susceptible individual and avoid predation. Also, with a low predator population and it will not eliminate the *E. coqui* population in Hawaii.

## 7 Future Work

First, the term of birth rate could be linear depending on the adults population and functions can be used such as the logistic equation to represent it. We can probably. add delay to the hatching rate to explore control measures. The implementation of optimal control to the population in Hawaii, like decreasing the birth rate Finally, it can be added seasonality to the reproduction and study how the seasonality work with the reproduction with the disease.

## 8 Acknowledgements

This project has been supported by grants from the National Science Foundation (NSF - Grant DMPS-0838705), the National Security Agency (NSA - Grant H98230-09-1-0104), the Alfred P. Sloan Foundation; and the President and Provost Offices at Arizona State University. The Mathematical and Theoretical Biology Institute now hosted at the Mathematical, Computational and Modeling Science Center at ASU would like to give thanks

to everybody involve with the program for the past 15 years. We thank Dr. Carlos Castillo-Chavez for giving the opportunity to participate the Mathematical Theoretical Biology Institute. Likewise we thank Dr. Baojun Song, Dr. Leon Arriola, Dr. Jose Flores and Dr. Angela Ortiz for suggestions about the work.

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