Social Dynamics of Gang Involvement: A Mathematical Approach

Joshua Austin[1], Emma Smith[2], Sowmya Srinivasan[3], Fabio Sánchez[4]
[1]jaustin1@umbc.edu, [2]emsmith@linfield.edu, [3]ssrinivasa@brynmawr.edu , [4] fabio.sanchez@asu.edu

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Abstract

Gangs have played a significant role in Chicago's social and political history, and continue to impact the city today, as gang violence rates continue to grow despite drops in overall crime[13]. In this paper, we explore the dynamics of gang involvement between at-risk individuals, gang members, and reformed (temporarily removed) gang members. We focus on the effect that reformed gang members have on the at-risk population via a general function, which takes into account cost of gang membership and a threatening factor. We find that the influence of the reformed population is highly sensitive to initial gang member population size, and factors such as cost and recidivism rates play an important role in gang involvement in at-risk environments.

1 Introduction

Street gangs and the violence associated with them have been a persistent problem in Chicago for the past half century. Gang membership is concentrated in neighborhoods of high poverty and consists primarily of Hispanic or African American populations [8]. Furthermore, Chicago's homicide rate has seen little change from 1990 to 2000, and gang violence rates have increased despite overall drops in crime, indicating that gangs affect not only their members, but the community at large [8, 13].

The history of Chicago's gangs is closely related to the flow of minority groups within the city. Many Mexican immigrants and African Americans moved to Chicago in the mid-20th century, drawn by the higher-wage, unionized jobs in steel manufacturing and meatpacking. When the strength of the steel and meatpacking industries diminished in the 1970s, many members of the minority groups were left unemployed. Gentrification of their home neighborhoods has further exacerbated the situation, leading these communities to turn to gangs as a defense against exploitation by more dominant groups [8].

Most gangs in the US are classified as interstitial because they exist over a short period of time; when its members' allegiances shift or its leaders are removed, the gang dissipates. Some cities, such as Chicago, are home to institutionalized gangs, which have persisted over generations. Institutionalized gangs survive leadership changes, have complex organizational structures, and are often involved in community or political activities [8]. In this paper, we focus on members of the established gangs of Chicago, the majority of whom belong to African American or Hispanic groups. Though three of Chicago's top four historically significant gangs have broken into numerous smaller groups, their impact on the city has in no way diminished, since renegade factions of each original gang continue to conflict with each other [8].

Studies have shown that gang membership spans, on average, from ages 13 to 30, indicating that youth are a significant target for gang recruitment [3, 4]. The Office of Juvenile Justice and Delinquency Prevention (OJJDP) cites risk factors that may lead to youth gang involvement, including unstable family life, problems in school, and association with peers involved with gangs [12]. Hill *et al.* (1999) conducted a study that found all of these factors to be significant influences on youth gang involvement [11]. In addition, attractors such as protection, recreation, money, respect, and the desire to belong to a group may contribute to a youth's decision to join a gang [12]. A study conducted by the Department of Justice found that youth who respectfully refused when asked to join a gang rarely faced serious injury or other harm. In contrast, youth who agree to join often endure violent initiation rituals such as beatings or committing a murder, and, once in the gang, face greatly increased chances of being incarcerated, injured, or murdered [21, 22]. Therefore, prevention programs are key to reducing gang involvement because youth must be made aware of the risks associated with joining a gang, as well as be educated about their options should they decide to turn down gang membership.

There have been several notable studies examining the effectiveness of prevention strategies for at-risk youth. Maxson *et al.* (1998) argue that healthy friendship patterns beginning at a young age may reduce the risk of the negative relationships that often lead to gang involvement [15]. In addition, Sheehan *et al.* (1999) found that peer mentoring programs for gang-susceptible youth may help to curb violence in urban areas [19]. Finally, the OJJDP stresses that mentors must build strong relationships with the targeted youth in order to effectively discourage gang involvement [16]. Using this logic, we reason that reformed gang members, who have experienced many of the same challenges, may serve as the most effective mentors for gang-susceptible youth.

In this paper, we model gang involvement using an epidemiological approach [2]. That is, we consider gang membership to be a type of "epidemic" in which those affiliated with gangs impact the rate at which the at-risk population (particularly youth) join gangs. We focus on the institutionalized gangs of Chicago (those that have existed over generations and have a strong organizational structure), and particularly on the impact that former gang members have on at-risk youth through mentoring and other programs.

Various studies have been done modeling other social epidemics in similar ways. For example, the epidemic spread of drug use has been modeled using differential equations [1, 20, 23]. González *et al.* (2003) constructed a model examining the dynamics of peer pressure on college-age bulimia, focusing on the effects of intervention at two stages of the disease [6]. The dynamics of problem drinking has been examined in a similar epidemiological manner, explained further below [18]. In addition, recent research has modeled patterns of gang-related rivalries using an agent-based model [9]. Although we have found studies modeling both gang involvement as well as other social epidemics, little has been done in regard to applying an epidemic model to gang involvement, and in particular, the impact of reformed gang members on this at-risk class. We hope that our model will provide insight into the dynamics of gang involvement and the effect of social influence.

2 Model

We explore the dynamics of gang involvement by representing the interactions between at-risk individuals, gang members, and reformed gang members ages 13 to 30 using a modified version of the classic SIR (susceptible-infected-recovered) model [2]. The population is divided into three different classes: S, G, and T. The at-risk class, S, represents at-risk individuals between the ages of 13 and 30 living in Chicago who have never been involved in a gang. The gang member class, G, is representative of current gang members. The reformed class, T, is representative of gang members who have left the gang class through an intervention program. We assume that there is a constant recruitment rate into the population, μN, and that those leaving the gang class due to incarceration will exit through the rate, μ.

We base our model primarily on the model constructed by Sánchez *et al.* (2006), which models the impact of nonlinear *social influence* on drinking behavior dynamics [18]. They use an SDR model, which contains non- or moderate drinkers, S, problem drinkers, D, and temporarily recovered drinkers, R. When the reformed gang members do not play a role in reducing gang involvement, our model exhibits similar dynamics as between the S and D classes of the drinking model. That is, the susceptible class of each model interacts with a proportion of the problem class, resulting in an increased rate of movement, β, into the problem class. Both models also contain a linear recovery rate γ. Furthermore, both models account for the possibility of relapse (represented by $\rho T \frac{G}{N}$ in our model, where ρ is the recidivism rate), as former gang members often return to gang life as a result of a lack of opportunities or alternative options.

The key difference from the drinking model that we explore is the effect that reformed gang members have on the rate at which the at-risk class enters the gang class. That is, we hypothesize that reformed gang members serving as mentors to at-risk youth have a reducing effect on the rate at which these youth join gangs. We model this reducing factor as a function of T, denoted by $f(\alpha, x)$. This function satisfies the following properties: first, $f(\alpha, x)$ must be a positive, decreasing smooth function. That is,

$$f(\alpha, x) > 0 \text{ and } f'(\alpha, x) \leq 0.$$

The function also includes a "threatening" factor, $\alpha \in [0, 1]$, where values close to 0 indicate that a small proportion of the reformed class is influencing the at-risk class, while values close to 1 indicate a large proportion helping the at-risk class. The threatening factor, α, represents the potential risks to reformed gang members who work to prevent gang involvement. It also reflects the fact that not all reformed gang members are willing or able to interact with the at-risk population.

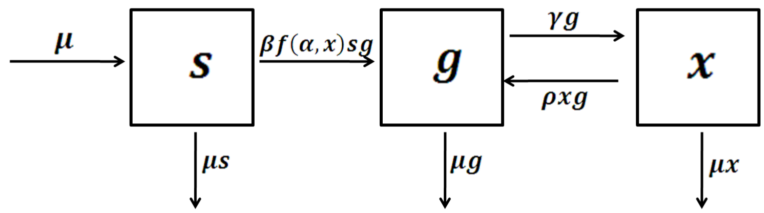

Figure 1: Gang involvement grouped into S (at-risk population), G (gang members), T (reformed gang members).

A system of nonlinear differential equations that describes the previous dynamics is given by:

$$\frac{dS}{dt} = \mu N - \beta f\left(\alpha, \frac{T}{N}\right) S\frac{G}{N} - \mu S, \tag{1}$$

$$\frac{dG}{dt} = \beta f\left(\alpha, \frac{T}{N}\right) S\frac{G}{N} + \rho T\frac{G}{N} - (\gamma + \mu)\, G, \tag{2}$$

$$\frac{dT}{dt} = \gamma G - \rho T\frac{G}{N} - \mu T. \tag{3}$$

The previous model (Equations 1-3) is rescaled to simplify the analysis. That is,

$$s' = \mu - \beta f(\alpha, x)sg - \mu s, \tag{4}$$

$$g' = \beta f(\alpha, x)sg + \rho x g - (\mu + \gamma)g, \tag{5}$$

$$x' = \gamma g - \rho x g - \mu x. \tag{6}$$

where $s = \frac{S}{N}$, $g = \frac{G}{N}$, $x = \frac{T}{N}$, and $s + g + x = 1$. An example of the reducing function $f(\alpha, x)$ that we consider is given by:

$$f\,(\alpha, x) = \frac{\eta}{1 + \alpha x} \tag{7}$$

where $\eta \in (0, 1]$ is an additional reducing factor on β, representing the cost of gang membership. This cost includes violent initiation rituals such as committing criminal activities, including murder and rape, as well as awareness of what gang life is like, such as drug dealing and an increased risk of incarceration[22]. Values of η close to 0 indicate a high cost of gang membership, while values closer to 1 indicate a low cost of gang membership ($\eta = 1$ indicates no cost). Because $x \in [0, 1]$, our example of $f(\alpha, x)$ is bounded by $\left[\frac{1}{2}, 1\right]$.

Table 1 lists all of the model's parameters, their definitions, and values.

Table 1: Parameters

Parameter	Definition	Values
β	recruitment rate into a gang	0.009
μ	departure rate from gang environment	0.00015
η	cost of gang membership	[0,1]
α	threatening factor	[0,1]
ρ	recidivism rate	0.005
γ	intervention rate	0.0027

3 Mathematical Analysis

3.1 Gang-Free Equilibrium and the Basic Reproductive Number

The gang-free equilibrium of the model is

$$(s_0^*, g_0^*, x_0^*) = (1, 0, 0).$$

This is a state in which the gang population is non-existent. Using Equation (5), that is, g', we use the next generation operator method to compute R_0[10]:

$$\mathcal{F} = [\beta f(\alpha, x)sg + \rho xg] \quad \text{and} \quad \mathcal{V} = [(\mu + \gamma)g]$$

where $\mathcal{F}$ contains all terms flowing into g and $\mathcal{V}$ contains all terms flowing out of g.
Now,

$$F = \left[\frac{\partial \mathcal{F}}{\partial g}\right] = [\beta f(\alpha, x)s + \rho x] = [\beta f(0)] \,.$$

Similarly,

$$V = \left[\frac{\partial \mathcal{V}}{\partial g}\right] = [\mu + \gamma] \text{ and } V^{-1} = \frac{1}{\mu + \gamma}.$$

And so, the basic reproductive number is

$$R_0 = FV^{-1} = \frac{\beta f(0)}{\mu + \gamma}$$

where $\frac{1}{\mu+\gamma}$ is the average amount of time spent in the gang class. This, for our choice of $f(\alpha, x)$ (Equation (7)) is given by:

$$R_0 = \frac{\beta \eta}{\mu + \gamma}.$$

The basic reproductive number, R_0, is the average number of secondary gang members recruited by a single gang member. The gang population increases to a stable population size when $R_0 > 1$, and the gang population typically decreases to 0 when $R_0 < 1$. However, past research indicates that in some cases, such as when high relapse rates are a factor in a system, it is still possible to sustain an endemic equilibrium with $R_0 < 1$[7, 18, 24]. This is important because it means that having a value of R_0 less than 1 does not guarantee stability at the gang-free equilibrium.

$R_\rho = \frac{\rho}{\mu+\gamma}$ is analogous to the basic reproductive number for the reformed class. That is, R_ρ is the average number of reformed members that an individual gang member recruits back into gang life. We assume that $\beta\eta < \rho$, and hence, $R_0 < R_\rho$. That is, it is easier for gang members to recruit those who have at one time belonged to a gang.

3.2 Endemic Equilibria

Endemic equilibria exist when $R_0 > 1$ and $R_\rho > 1$, and under certain initial conditions when $R_0 < 1$ and $R_\rho > 1$. In the case when endemic equilibria exist when $R_0 < 1$ and $R_\rho > 1$, the initial gang population and the recidivism rate play a critical role in gang population levels. We study two cases, when $\alpha = 0$ and when $0 < \alpha \leq 1$, using the reducing function $f(\alpha, x) = \frac{\eta}{1+\alpha x}$. This allows us to explore the impact that the reformed class has on gang population dynamics.

3.2.1 $\alpha = 0$ (Absence of Threatening Factor)

Solving for the endemic equilibria of our system when $\alpha = 0$, we obtain the following:

$$\begin{aligned} s_1^* &= \frac{\mu}{\beta\eta g + \mu}, \\ x_1^* &= \frac{\gamma g}{\rho g + \mu}. \end{aligned}$$

Substituting these values into Equation 5 yields:

$$\frac{\beta\eta\mu(\rho g + \mu) + \rho\gamma g(\beta\eta g + \mu)}{(\beta\eta g + \mu)(\rho g + \mu)} = (\mu + \gamma).$$

This leads to the expression $ag^2 + bg + c = 0$ where

$$\begin{aligned} a &= \beta\eta\rho, \\ b &= \beta\eta\mu + \mu\rho + \beta\eta\gamma - \beta\eta\rho, \\ c &= \mu^2 + \mu\gamma - \beta\eta\mu. \end{aligned}$$

Substituting $R_0 = \frac{\beta\eta}{\mu+\gamma}$ and $R_\rho = \frac{\rho}{\mu+\gamma}$ yields the final quadratic $a_2 g^2 + a_1 g + a_0$, where the coefficients are functions of R_0 and R_ρ. That is,

$$\begin{aligned} a_2 &= \rho R_0, \\ a_1 &= (\mu + \gamma)R_0(1 - R_\rho) + \mu R_\rho, \\ a_0 &= \mu(1 - R_0). \end{aligned}$$

When $\alpha = 0$, our system is very similar to the drinking model by Sánchez *et al.* [18]. As shown by Figure 2, the model exhibits a backward bifurcation; that is, it is necessary to have a critical mass of gang members to have a stable endemic equilibrium when $R_0 < 1$ and $R_\rho > 1$[18]. This system exhibits hysteresis, meaning that it is highly sensitive to initial conditions. Unlike typical SIR models, whose behavior is highly predictable given its input, the output of our model can lead to different equilibria under different initial conditions. For example, $R_0 < 1$ does not guarantee a gang-free equilibrium.

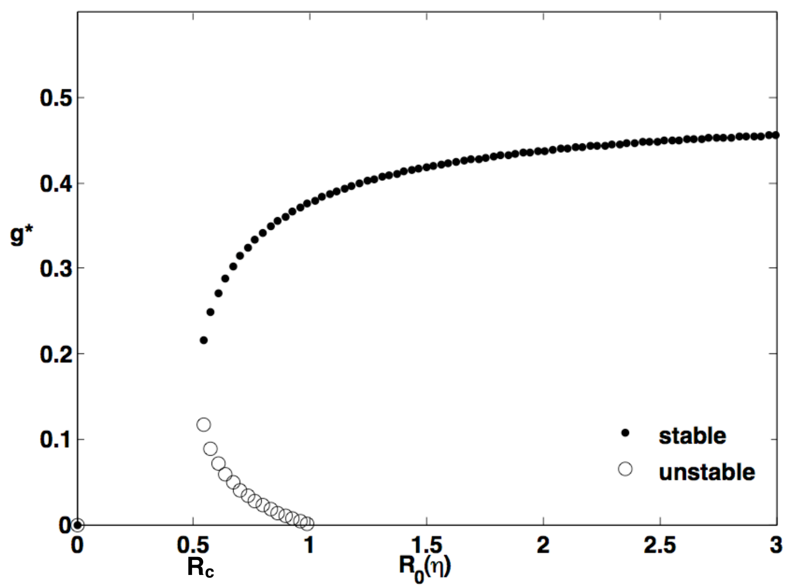

Figure 2: Backward bifurcation with parameters $\mu = 0.00015$, $\beta = 0.009$, $\gamma = 0.0027$, $\alpha = 0$, $\rho = 0.005$, and varied η.

From Figure 2, we can see that when R_0 is below the double root, R_c, there is a gang-free equilibrium. However, given their historical origins, gangs have become well established in the culture of Chicago, so we can deduce that this scenario is unlikely because this critical mass has been surpassed [8]. When $R_0 < R_c < 1$, the gang population tends toward the gang-free equilibrium, and when $R_c < R_0 < 1$, the gang population can tend toward either an endemic or gang-free equilibrium depending on initial gang population size. When $R_0 > 1$, the population tends toward an endemic equilibrium.

Proposition 1. *There exists a double root when the discriminant of the quadratic,* $\Delta = 0$, $g^* = \frac{1}{2}\left(1 - \frac{1}{R_\rho} - \frac{1}{R_0}\right)$, $R_\rho > 1$, *and* $\bar{R}_0 > 1$ *where* $\bar{R}_0 = \frac{\beta\eta}{\mu}$.

Using this value of g^*, we can extrapolate to find the double root, R_c.

3.2.2 $0 < \alpha \leq 1$ (Presence of Threatening Factor)

Solving for the endemic equilibria of our system when $0 < \alpha \leq 1$ leads to the following:

$$\begin{aligned} s_2^* &= \frac{\mu A}{\beta\eta g + \mu A}, \\ x_2^* &= \frac{\gamma g}{\rho g + \mu}. \end{aligned}$$

where $A = 1 + \frac{\alpha\gamma g}{\rho g + \mu}$. Substituting these values into Equation 5 yields:

$$\frac{\beta\eta\mu(\rho g + \mu) + \rho\gamma g(\beta\eta g + \mu A)}{(\beta\eta g + \mu A)(\rho g + \mu)} = (\mu + \gamma).$$

This leads to the cubic expression of g, $ag^3 + bg^2 + cg + d = 0$ where

$$\begin{aligned} a &= \beta\eta\rho^2, \\ b &= 2\beta\eta\rho\mu + \mu\rho^2 + \mu\rho\alpha\gamma + \beta\eta\rho\gamma - \beta\eta\rho^2, \\ c &= \beta\eta\mu^2 + 2\mu^2\rho + \mu^2\alpha\gamma + \beta\eta\mu\gamma + \mu\rho\gamma + \mu\alpha\gamma^2 - 2\beta\eta\mu\rho, \\ d &= \mu^3 + \mu^2\gamma - \beta\eta\mu^2. \end{aligned}$$

Substituting $R_0 = \frac{\beta\eta}{\mu+\gamma}$ and $R_\rho = \frac{\rho}{\mu+\gamma}$ yields the final cubic $a_3g^3 + a_2g^2 + a_1g + a_0$, where the coefficients are functions of R_0 and R_ρ. That is,

$$\begin{aligned} a_3 &= \rho^2 R_0, \\ a_2 &= \rho\left[(\mu+\gamma)R_0\left(1 - R_\rho\right) + \mu\left(R_0 + R_\rho + \frac{\alpha\gamma}{\mu+\gamma}\right)\right], \\ a_1 &= \mu\left[\alpha\gamma + \left(\mu+\gamma\right)\left(R_0 + R_\rho\right) + \mu R_\rho - 2\rho R_0\right], \\ a_0 &= \mu^2\left(1 - R_0\right). \end{aligned}$$

Previous research indicates that the bifurcation of our model exhibits both forward and backward behavior [24]. Figure 3 shows the bifurcation of our model, divided into four regions.

$R_c < 1$ and $R_0^* > 1$ are double roots of the cubic, and are the thresholds that determine the number of endemic equilibria for a given value of R_0. Propositions 2-5 outline the necessary and sufficient conditions for the number of endemic equilibria in each of the four regions.

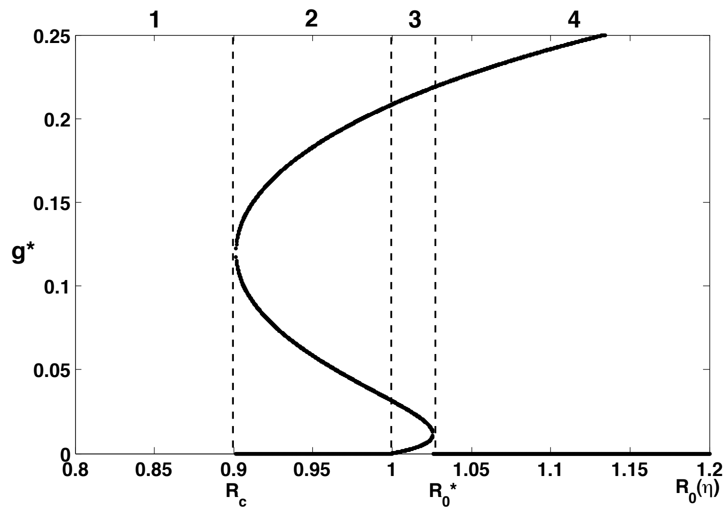

Figure 3: Forward-backward bifurcation with parameters $\mu = 0.0015$, $\gamma = 0.0027$, $\beta = 0.009$, $\alpha = 0.8$, $\rho = 0.0044$ and varied η.

Proposition 2. *A sufficient condition for the gang-free equilibrium is,*

$$0 < R_0 < R_c < 1 \text{ and } R_\rho < 1.$$

In Region 1, when R_0 is less than the double root, R_c, and the recidivism rate, R_ρ, is less than 1, we have a gang-free equilibrium. This means that the per-person recruitment rate is not high enough to sustain a gang population, regardless of the initial gang population size.

Proposition 3. *A necessary condition for two positive equilibria is,*

$$0 < R_c < R_0 < 1 \text{ and } R_\rho > 1.$$

In Region 2, when R_0 is greater than the double root, R_c, and $R_\rho > 1$, two endemic equilibria exist and a backward bifurcation occurs. In this case, initial gang member population size determines if the long-term behavior tends toward an endemic or gang-free equilibrium.

Proposition 4. *A necessary condition for three positive equilibria is,*

$$1 < R_0 < R_0^* \text{ and } R_\rho > 1.$$

In Region 3, three positive equilibria exist, and a forward-backward bifurcation occurs. This region is unique to the cubic function, and occurs as a result of the reducing factor $f(\alpha, x)$. When the initial population of gang members is below the unstable equilibrium, the population tends toward the smaller of the two stable equilibria. That is, with certain initial gang population densities, it is possible that the long-term population will tend toward a smaller endemic equilibrium. However, if this initial gang population is above the unstable equilibrium, the population tends toward a larger endemic equilibrium.

Proposition 5. *A sufficient condition for a unique positive equilibrium is,*

$$1 < R_0^* < R_0 \text{ and } R_\rho > 1.$$

In Region 4, a unique endemic equilibrium exists and the gang population tends toward it, regardless of initial gang population size.

Varying the ρ value of the bifurcation highlights the impact that the recidivism rate has on the gang population dynamics (Figure 4). Figure 4(b) shows that when the recidivism rate is low (but still greater than 1), the bifurcation shifts to the right to where $R_0 < 1$ guarantees a gang-free equilibrium.

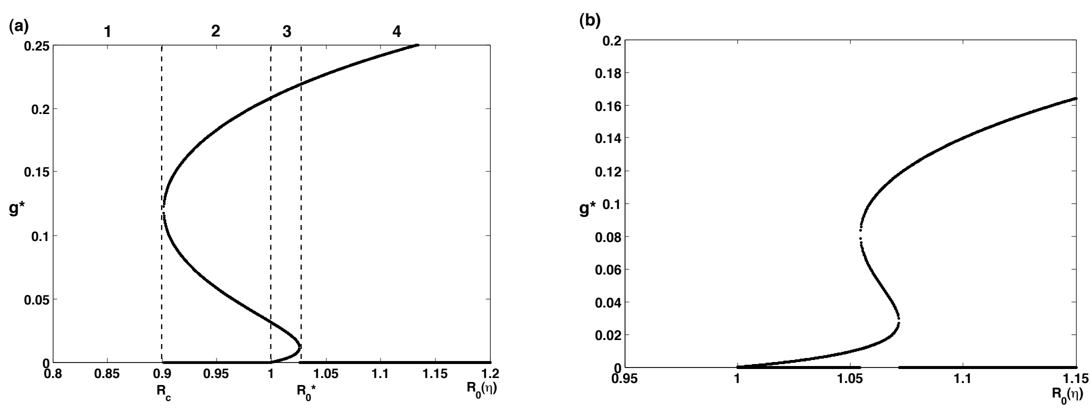

Figure 4: Forward-backward bifurcation with parameters $\mu = 0.00015$, $\gamma = 0.0027$, $\beta = 0.009$, $\alpha = 0.8$, and varied η. (a) $\rho = 0.0044$, $R_\rho = 1.54$; (b) $\rho = 0.004$, $R_\rho = 1.4035$.

With $\rho = 0.004$, reformed gang members have a major influence on gang population dynamics. Figure 5 shows the impact that α has on the behavior of the bifurcation. When α is low, the bifurcation exhibits only a backward bifurcation. When α increases, the bifurcation exhibits the forward-backward behavior, and continues to shift (with the same shape) to the right as α continues to increase to 1.

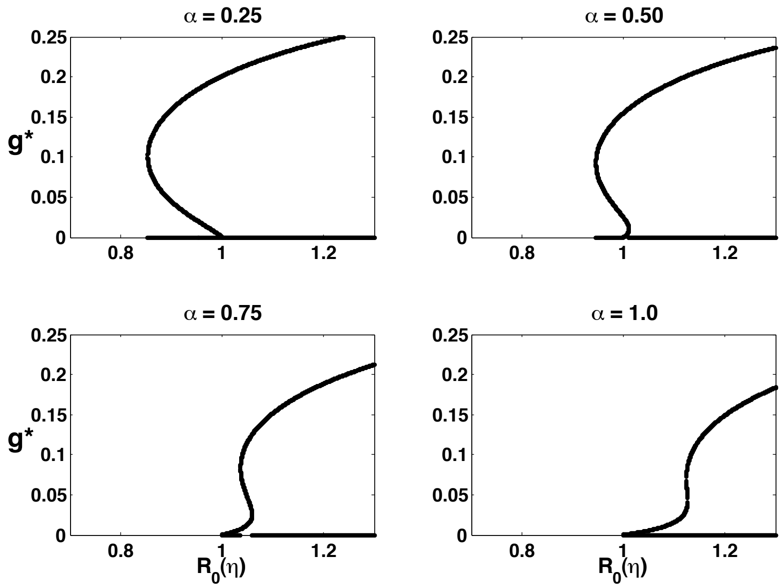

Figure 5: Bifurcation diagrams for varying values of α. Parameters: $\mu = 0.00015$, $\gamma = 0.0027$, $\beta = 0.009$, $\alpha = 0.8$, $\rho = 0.004$, and varied η.

4 Estimation of Parameters

Because of a lack of comprehensive data on Chicago gangs, we use relevant literature to make educated guesses about some of our parameters. Past research indicates that active gang members (those actively involved in recruiting the at-risk population) typically range in age from 13 to 30 years (18 years total), which yields a rate of 0.00015 (in days) for μ[3, 4, 14]. For γ, we assume that gang members leave the gang class at a rate of one per year, or 0.0027 per day. This rate is relatively low due to the assumption that it is often difficult for established members to leave gangs, due to gang hierarchy and lack of available opportunities in mainstream society [14]. Figure 6 shows the range of values for β and η (with fixed values of μ and γ) such that $R_0 < 1$.

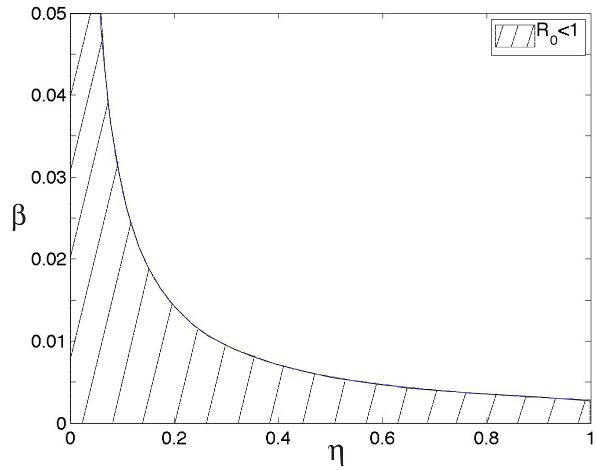

Figure 6: Maximum values for η and β with $\mu = 0.00015$ and $\gamma = 0.0027$ such that $R_0 < 1$.

5 Numerical Analysis

Figure 7 shows numerical simulations of each of the four regions of the forward-backward bifurcation (Figure 3). Figure 7(a) is Region 1 and shows that the gang population tends toward the gang-free equilibrium. Figure 7(b) is Region 2 and shows that, depending on initial gang population size, the long-term gang population can tend toward either an endemic or gang-free equilibrium. It is important to note that for both trajectories shown, $R_0 < 1$. Figure 7(c) is Region 3 and shows that, depending on initial gang population size, the long-term population can tend toward one of two endemic equilibria. Figure 7(d) is Region 4 and shows that the gang population size tends toward an endemic equilibrium.

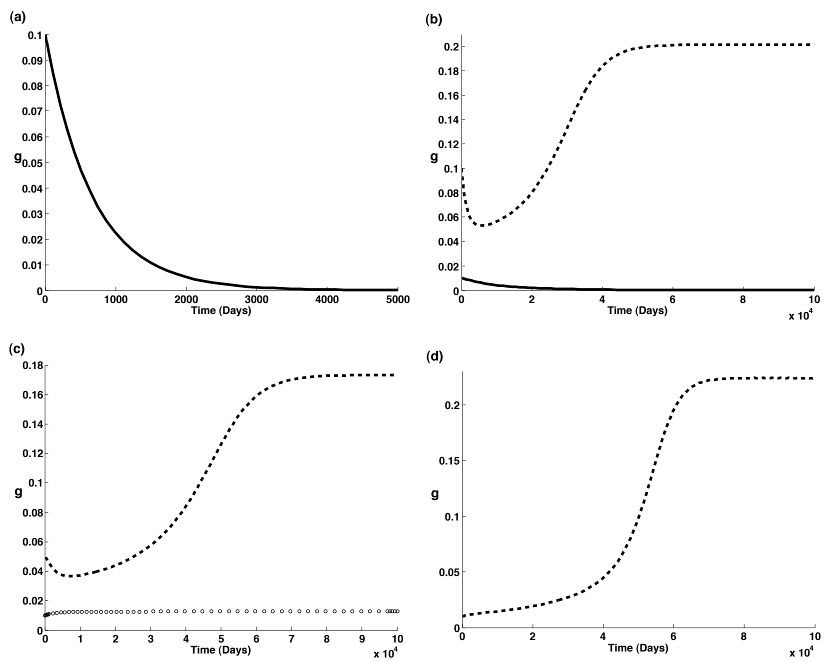

Figure 7: Gang population time series for Regions 1-4 of the forward-backward bifurcation. (a) Region 1 with parameters $\mu = 0.00015$, $\beta = 0.005$, $\gamma = 0.0027$, $\eta = 0.3$, $\alpha = 0.8$, and $\rho = 0.0016$ with initial gang population of 0.1%, $R_0 = 0.5263$, and $R_\rho = 0.5614$; (b) Region 2 with parameters $\mu = 0.00015$, $\beta = 0.0085$, $\gamma = 0.0027$, $\eta = 0.33$, $\alpha = 0.8$, and $\rho = 0.0044$ with initial gang population of 10% (endemic) and 1% (gang-free), $R_0 = 0.9842$, and $R_\rho = 1.5439$; (c) Region 3 with parameters $\mu = 0.00015$, $\beta = 0.009$, $\gamma = 0.0027$, $\eta = 0.33$, $\alpha = 0.8$, and $\rho = 0.0042$ with initial gang population of 5% (large endemic) and 1% (small endemic), $R_0 = 1.0421$, and $R_\rho = 1.4736$; (d) Region 4 with parameters $\mu = 0.00015$, $\beta = 0.009$, $\gamma = 0.0027$, $\eta = 0.33$, $\alpha = 0.8$, and $\rho = 0.0044$ with initial gang population of 1%, $R_0 = 1.0421$, and $R_\rho = 1.5439$.

We use numerical simulations to analyze the long-term gang population dynamics and the effects that threat (α), cost (η), recidivism rate (ρ), and treatment rate (γ) have on these dynamics.

Figure 8 shows the effect that initial gang population size has on long-term gang population involvement. The initial gang member population in Figure 8(a) is 5%, and only the curve corresponding to $\alpha = 1$ tends to the gang-free equilibrium, while the rest tend toward an endemic equilibrium. The initial gang member population in Figure 8(b) is 1%, and only the curve corresponding to $\alpha = 0$ tends toward an endemic equilibrium. This indicates that the system is sensitive to initial conditions and shows that with a smaller initial gang population size, the reformed class has more of an impact on long-term gang population size. Furthermore, these graphs show that it is possible to reach an endemic equilibrium despite $R_0 < 1$ when there are enough gang members in the at-risk environment.

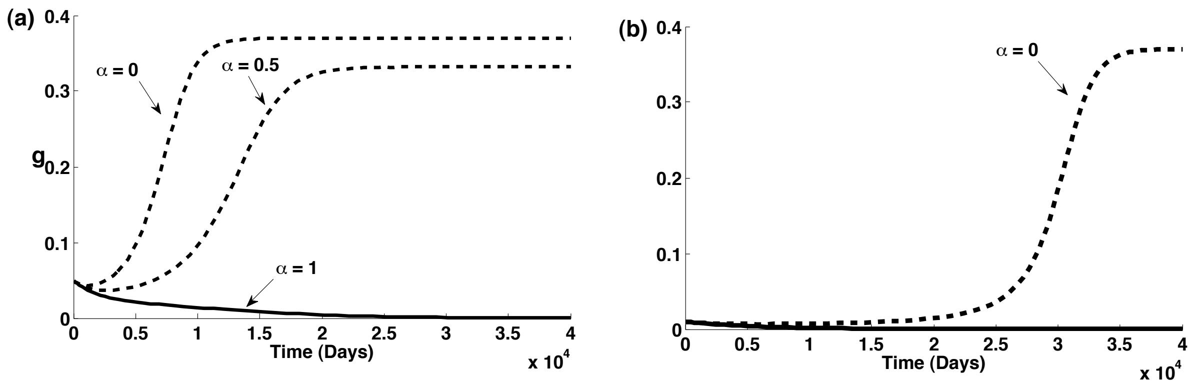

Figure 8: Gang population for different initial gang population sizes. (a) 5% gang members; (b) 1% gang members. Parameters for both are $\mu = 0.00015$, $\beta = 0.009$, $\gamma = 0.0027$, $\eta = 0.3$, $\rho = 0.005$, and $\alpha = [0, 0.5, 1]$ with $R_0 = 0.9474$ and $R_\rho = 1.7544$.

Past research indicates that the recidivism rate plays a major role in the gang population reaching an endemic equilibrium despite $R_0 < 1$ [18]. Figure 9 shows the effect that the recidivism rate, ρ, has on long-term population size. Figure 9(a) has $\rho = 0.005$, and the curves corresponding to both $\alpha = 0.5$ and $\alpha = 0$ tend toward an endemic equilibrium. Figure 9(b) has $\rho = 0.004$, and only the curve corresponding to $\alpha = 0$ tends toward an endemic equilibrium, while the other two tend toward the gang-free equilibrium. This indicates that when recidivism rates are lowered, the reformed class has more of an impact on the long-term gang population size. Furthermore, the endemic equilibrium is lower when $\rho = 0.004$, indicating that keeping recidivism rates low could potentially lower overall gang population size.

Figure 10 shows the effect that the cost of joining a gang, η, has on long-term gang population size. Figure 10(a) has $\eta = 0.3$, and only the curves corresponding to $\alpha =$

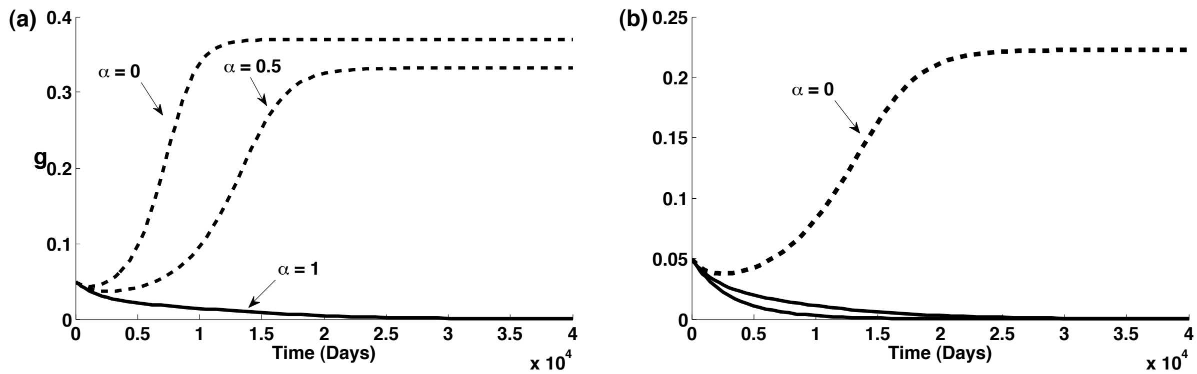

Figure 9: Gang population for different values of ρ. (a) $\rho = 0.005$ with $R_\rho = 1.7544$; (b) $\rho = 0.004$ with $R_\rho = 1.4035$. Parameters for both are $\mu = 0.00015$, $\beta = 0.009$, $\gamma = 0.0027$, $\eta = 0.3$, and $\alpha = [0, 0.5, 1]$ with an initial gang member population of 5% with $R_0 = 0.9474$.

0.5 and $\alpha = 0$ tend toward an endemic equilibrium. Figure 10(b) has $\eta = 0.7$ and all trajectories tend toward an endemic equilibrium. This indicates that if the cost to join a gang is low (i.e. the value of η is high), then reformed gang members do not have a significant impact on gang involvement, regardless of how many of them interact with the at-risk class.

In Figure 11, we examine the relationship between the treatment rate, cost, and R_0. This plot indicates that for higher values of γ, R_0 will be close to 1 despite the value of η. This means that if the treatment rate is high enough, the cost of gang membership is not a factor.

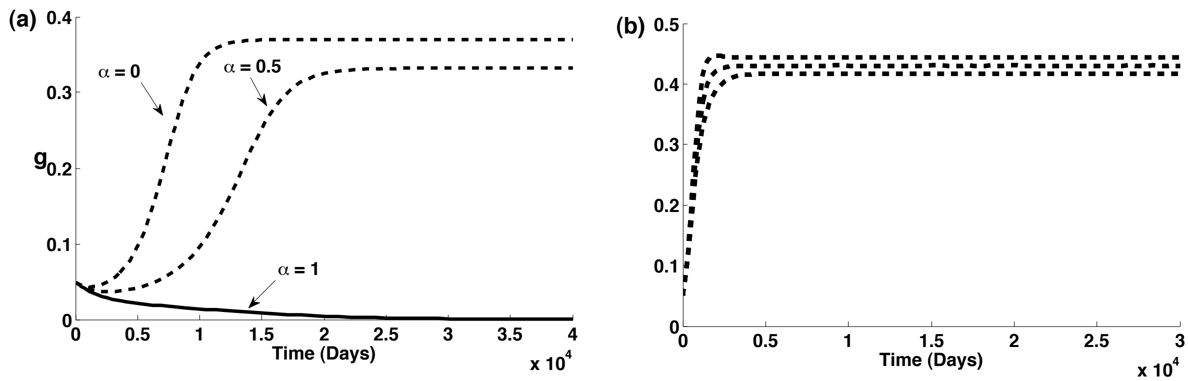

Figure 10: Gang population for different values of η. (a) $\eta = 0.3$ with $R_0 = 0.9474$; (b) $\eta = 0.7$ with $R_0 = 2.2105$. Parameters for both are $\mu = 0.00015$, $\beta = 0.009$, $\gamma = 0.0027$, $\rho = 0.005$, and $\alpha = [0, 0.5, 1]$ with an initial gang member population of 5% and $R_\rho = 1.7544$.

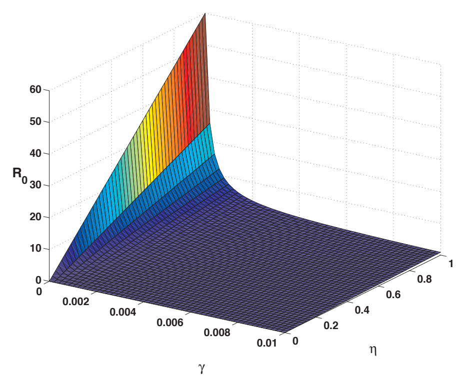

Figure 11: Surface plot of R_0 dependent on γ and η with parameters $\mu = 0.00015$ and $\beta = 0.009$.

6 Discussion

We explored the dynamics of gang involvement, in particular, looking at the *social influence* that reformed gang members have on the at-risk population. Our analysis shows that the impact of the reformed members is highly sensitive to gang population size. For a certain cost (η), a small region exists where $1 < R_0 < R_0^*$ and $R_\rho > 1$ with multiple stable gang populations that are highly dependent on initial gang member population. If initial gang member population in an at-risk environment is large enough, in other words, already established, then the gang member population jumps to the higher endemic equilibrium. Our model shows the influence of established problem communities (in our case, gangs) and it highlights the importance of prevention programs and recidivism rates.

While reformed gang members may have an impact on gang population dynamics, other factors, such as cost and recidivism rate, play a role in the effectiveness of the reformed class. When recidivism rates are low, reformed gang members play a crucial role in gang population dynamics. A high value of α can shift the forward-backward bifurcation to the point where $R_0 < 1$ produces a gang-free equilibrium. This highlights the importance of keeping recidivism rates under control as well as encouraging former gang members to become involved with gang prevention programs. A lack of opportunities could lead reformed gang members to return to gang life; therefore, programs aiming to reintegrate reformed gang members back into society, such as providing tattoo removal services, job placement, and education opportunities, may help reduce the re-involvement of former gang members.

From our model, we also found that the cost of joining a gang has a significant impact on gang population dynamics; if costs are low to join a gang, there is little that reformed gang members can do to decrease gang involvement in at-risk communities. Costs include violent initiation rituals, such being forced to perform illicit activities, including extremes such as armed robberies, murders, and rapes. It also encompasses the knowledge of what gang life is like, such an increased risk of incarceration and drug dealing. Because these factors are difficult for policy makers to control, perhaps an alternative to lowering cost would be to educate at-risk youth about these "costs". This, in turn, may encourage youth to *view* these costs as a deterrent, which could ultimately help lower overall gang involvement. Furthermore, educating youth about other lifestyles and opportunities, such as the pursuit of an education, could help discourage youth from getting involved in gangs. Ultimately, efforts should not only be placed on encouraging reformed gang members to mentor individuals in an at-risk environment, but also on reducing the recidivism rate and helping to educate the at-risk population in order to help contain gang involvement.

7 Future Work

One limitation of our model is that it does not consider the incarcerated population. A gang-affiliated individual who is released from prison has a different rate of entry into the gang class than someone who has never been affiliated with a gang. To account for this, we could extend our model by including a separate compartment for those released from jail or prison. It may also be beneficial to examine other types of prevention and intervention programs for at-risk or current gang members that are independent of the reformed class. Finally, our analysis could benefit by using comprehensive data on gang activity.

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