The Dynamics of Math Anxiety as it is Transferred through Peer and Teacher Interactions

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Abstract

This research develops a simple dynamical system framework to study the role of social mechanisms on the prevalence of math anxiety in United States education systems. Math anxiety is the self reported discomfort when attempting mathematical problems. This feeling prevents students from pursuing careers in science, technology, engineering, and mathematics in these fields. Female students are disproportionately affected by math anxiety, leading to poor representation. Previous studies have examined how teachers, with and without math anxiety, can "transmit" math anxiety to students. However, to our knowledge no mathematical models have been developed to thoroughly study long term intervention strategies for reducing transmission. In this paper, the effects of female teachers' math anxiety are modeled as a contagion on female students who may become the next generation of teachers. The purpose of this research is to determine intervention strategies to effectively reduce students' math anxiety. From our sensitivity analysis we conclude that, instead of focusing on professional development, math anxiety can be drastically reduced if teachers portray more positive attitudes towards math, and colleges focus on recruiting non-anxious math teachers.

1 Introduction

Throughout the past century, the United States has been a world leader in technology and innovation [19]. Despite being one of the most developed countries in the world, the United States is falling behind other countries in math proficiency scores [10]. Some leaders in academia believe that this phenomenon is caused by the increase of math anxiety in classrooms throughout the United States [3]. Approximately 93 percent of students experience math anxiety with varying degrees of severity [7]. Development of math anxiety in students also carries over to the next generations of students through those students who become teachers with math anxiety [5]. This creates a cycle where math anxiety limits students' perceived capability of pursuing higher level education and careers in Science, Technology, Engineering and Math (STEM) [5, 58]. Because of underlying math anxiety, people avoid critical STEM based careers which are becoming increasingly important for today's scientific and technological advances. Students who would have otherwise been well suited for STEM research choose careers outside of STEM in order to avoid math [4, 30]. Furthermore, females and minorities have valid and important contributions to make to STEM fields. However, because of higher susceptibility to math anxiety and other social factors, females and minorities tend to pursue more socially acceptable fields [5, 11, 58].

1.1 What is Math Anxiety?

Ashcraft defines math anxiety as "a feeling of tension, apprehension, or fear that interferes with math performance [2]." Ashcraft concluded that when solving higher level math problems, feelings of self doubt interfere with a student's ability to successfully complete math problems [2]. As a result of math anxiety, some students develop physiological symptoms such as excessive sweating, nausea, and even heart palpitations [29]. The physical manifestations of math anxiety can cause students who would have otherwise performed well to perform worse than anticipated [31]. Math anxiety can also have long-term psychological effects such as low self-efficacy and even suppression of actual math ability [5]. For our research, we will define math anxiety as the self reported feeling of discomfort and fear when attempting mathematics. One of the ways an individual can self report math anxiety is by taking the Math Anxiety Rating Scale (MARS), a survey that uses Likert scale type questions to measure an individuals feelings of math anxiety [55, 56]. Throughout our study, we considered a person to be math anxious when scoring, on average, 2.5 or above per question on the MARS. (2.5 was chosen because it is one standard deviation above the mean MARS score [55, 56]).

1.2 Contraction of and Recovery from Math Anxiety

Previous studies have shown that the interactions a student has with peers and teachers can have a significant effect on whether or not students and teachers will develop math anxiety [25, 34]. The research identifies several areas in the lives of students and teachers in which math anxiety can be developed or diminished. All of the relationships surrounding a student can affect the student's level of math anxiety, but the relationships with teachers and peers are some of the most significant [5, 21, 25].

One of the most studied areas of math anxiety development is the transmission of anxiety from teachers to students in a classroom. Math anxiety does not necessarily discourage students from pursuing a career as a teacher. Students often aspire to become teachers, particularly at the elementary level, despite their high levels of math anxiety. This inevitably adds to the cycle of math anxiety transmission [31]. Female students are at a greater risk of developing math anxiety than male students, especially from female math anxious teachers. Math anxiety in female teachers plays a major role in shaping female students' beliefs about gender stereotypes regarding math abilities [5]. Even at the collegiate level, some professors perpetuate the idea that females should not take or will not be successful in math courses [25].

In a study by Kelly and Tomhave (1985), the recorded math anxiety levels were identified from the following groups: college freshman who had not yet taken any college level math, college seniors who had not taken college level math, college freshman enrolled in college algebra, participants of a math anxiety workshop, and elementary education majors [26]. The study revealed that elementary education majors tend to score higher on the MARS test than other college students surveyed, outside of the participants in a math anxiety workshop. Eventually, without successful intervention to reduce math anxiety, these students will become teachers who are able to transmit math anxiety to the next generation of young students [18, 26].

Math anxiety can also be transferred between peers through gender stereotypes. Riegle-Crumb et al. (2006) analyzed data from the Adolescent Health and Academic Achievement and the National Longitudinal Study of Adolescent Health in order to see how gender and friendship work together to shape a student's decision making process for choosing to take advanced courses. Their analysis revealed that a student's female friends' success in academia can have substantial effect on whether or not a student will choose to take advanced courses. When considering male dominated subjects, such as physics and calculus, researchers found that early academic success within same sex friendships was a greater indicator of a female students' decision to take higher level courses than for it is male students [34].

As a student trains to become a teacher, a student's level of math anxiety may be influenced by student-teaching and interactions with a student-teaching mentor. A student who had math anxiety prior to student-teaching was more likely to have weaker pedagogy because of the student-teacher's lack of confidence in math [31]. In Perkins' study with eight third and fourth year undergraduate students, who identified as math anxious, students participated in a mentoring program during student-teaching in order to reduce math anxiety. The study showed that participants gained more confidence in their mathematical ability, resulting in lower math anxiety [32].

Attending professional development workshops on teaching math can reduce a teacher's level of math anxiety [7]. Professional development allows teachers to gain a better understanding of the material taught in order to effectively communicate information to students [33]. Workshops help lower a teacher's level of math anxiety by informing the teacher about new education requirements, relevant math research, and instructional methods which build confidence and reduce math anxiety [20].

1.3 Previous Mathematical Models

Similar work in this field includes a mathematical model on the reason behind high school drop out rates by Amdouni et al. (2017). This model describes the factors that move a student from passing, to vulnerable, to failing, to dropping out in terms of high school success rates [1]. Another study analyzed the effects of student-teacher ratio on student and teacher performance in high school. The study determined that lowering class size to 19 or less would optimize both student and teacher performance [14]. Few other mechanistic models have been developed in the field of math education, and no other mechanistic models have been developed to describe the transmission of math anxiety. Previous research in math anxiety has focused on trying to study how math anxiety is developed using statistical analysis to identify its causes and effects. Such studies examine student and teacher populations using data to create statistical models to make generalizations and predictions for development and recovery from math anxiety. For example, Finlayson surveyed pre-service teachers to find causes and possible solutions for math anxiety [18]. Trends in the results were analysed to find statistically significant patterns, but no mechanistic models were developed [18, 25].

1.4 Our Question

In order to mathematically describe how peer and teacher interactions affect a student's level of math anxiety, we decide to view math anxiety as a epidemiological contagion. While math anxiety could develop from other causes such as intimidation by a teacher without math anxiety, studying it as a contagion allows us to better focus on the role of transmission among teachers and students. Our goal is to discover at what point in a female student's progression to becoming a teacher a solution should be implemented to reduce or prevent the development of math anxiety in students and teachers. In our research, we develop a mathematical model to explain how these peer and teacher relationships work together to transmit math anxiety. Because a student can develop math anxiety at any stage of her life, it is important to determine the most effective time of intervention. To our knowledge, we are the first to develop a mechanistic model to describe the dynamics of math anxiety and the interventions that can be used to reduce it among female students and teachers.

In the following sections, we use a system of non-linear ordinary differential equations to model the effects of female teachers' math anxiety on female students who may become the next generation of teachers. In section 2, we describe the basis of our model as well as the assumptions used to create it. Next in section 3, we study the dynamics of our model. Then in section 4, we discuss our results. Finally, we discuss how the results can be applied to the resolution of math anxiety in section 5.

2 Model

2.1 Model Development

We develop a model based on non-linear social interactions to explore how math anxiety spreads in a school district. The time t will be in years. Let $P(t)$ be the number of primary students. Let $S_n(t)$ be the number of female secondary students without math anxiety. Let

$S_a(t)$ be the number of female secondary students with math anxiety. Let $T_n(t)$ be the number of female teachers without math anxiety, and let $T_a(t)$ be the number of female teachers with math anxiety. All of these individuals are in the same school district. Epidemiological assumptions of the dynamics of math anxiety are as follows:
First, all mentions of "anxiety" refer only to math anxiety, and all mentions of "anxious" refer to having math anxiety.

A1: Math anxiety is a contagion that can be spread by anxious students or anxious teachers. We only consider public schools.

A2: There is a constant recruitment of primary students, Λ, in the system, and they graduate at a rate r. Thus, we write the flow of P as

$$\dot{P} = \Lambda - rP \tag{1}$$

A3: Teachers have a probability of spreading math anxiety to the primary students they teach. The proportion of time spent with anxious teachers is

$$\frac{T_a}{T_n + T_a}. \tag{2}$$

Note that the complement proportion, the proportion of time spent with non-anxious teachers, is

$$\frac{T_n}{T_n + T_a}. \tag{3}$$

The probability of a student developing math anxiety given that all of the student's primary school teachers are anxious is Ψ. Thus, the probability of a primary student developing math anxiety by interacting with anxious teachers during primary school is

$$\Psi \frac{T_a}{T_n + T_a}. \tag{4}$$

Thus the flow into the S_a compartment is

$$rP\Psi \frac{T_a}{T_n + T_a}. \tag{5}$$

while the flow into the S_n compartment is

$$rP(1 - \Psi \frac{T_a}{T_n + T_a}). \tag{6}$$

A4: There is a peer to peer effect where anxious students have a probability of affecting non-anxious students and vice versa. The rate at which anxious students recover and become non-anxious students is ϵ_n, while the transmission rate is ϵ_a. Then there is a bi-directional flow of students that can be modelled by the term

$$(\epsilon_n - \epsilon_a)\frac{S_a S_n}{S_a + S_n}. \tag{7}$$

A5: In secondary school, non-anxious teachers can mentor anxious students and help them recover from math anxiety. We define the effective contact rate of anxious students with non-anxious teachers as β_n. Then we have the term

$$\beta_n S_a \frac{T_n}{T_n + T_a}. \tag{8}$$

Similarly, non-anxious students can become anxious students by the influence of anxious teachers. We define the effective contact rate between non-anxious students and anxious teachers as β_a. Then we have the term for becoming anxious through secondary school student and teacher interaction

$$\beta_a S_n \frac{T_a}{T_n + T_a}. \tag{9}$$

A6: Secondary school students graduate at a rate σ. The probability that an anxious secondary school graduate becomes a teacher is γ_a. The probability that a college student recovers from math anxiety given a non-anxious student-teaching mentor is q_{an}. The probability of having a non-anxious student-teaching mentor is $\frac{T_n}{T_n+T_a}$. Then we have the term for recovering from anxiety through student teaching

$$\sigma\gamma_a q_{an} \frac{T_n}{T_n + T_a} S_a. \tag{10}$$

Alternatively, anxious students might be assigned to an anxious mentor or to a non-anxious mentor who did not succeed in coaching the student to recover from math anxiety. Then we have the term for retaining anxiety during student teaching

$$\sigma\gamma_a(1 - q_{an} \frac{T_n}{T_n + T_a}) S_a. \tag{11}$$

A non-anxious secondary school student who graduates and interacts with an anxious student-teaching mentor can become an anxious teacher. The probability that a non-anxious secondary school graduate becomes a teacher is γ_n. The probability that a secondary student contracts anxiety with an anxious student-teaching mentor is q_{na}. The probability of having an anxious student-teaching mentor is $\frac{T_a}{T_n+T_a}$. Then we have the term for contracting anxiety during student teaching

$$\sigma\gamma_n q_{na} \frac{T_a}{T_n + T_a} S_n. \tag{12}$$

A non-anxious secondary school student who graduates and interacts with an anxious student-teacher mentor may still become a non-anxious teacher, or the non-anxious student might be assigned a non-anxious mentor. Then we have the term for never having anxiety during student teaching

$$\sigma\gamma_n \left(1 - q_{na} \frac{T_a}{T_n + T_a}\right) S_n. \tag{13}$$

A7: In a district with regular teacher collaboration, such as in curriculum development, there is a teacher-to-teacher effect where anxious teachers have a probability of affecting non-anxious teachers in terms of contracting anxiety or vice versa. The effective contact

Parameter	Units	Description	Best-fit value	Reference
Λ	students/year	Primary students that enter the system each year	346.77	citations[1]
r	1/year	Graduation rate from primary school	0.167	Estimated
σ	1/year	Graduation rate from secondary school	0.143	Estimated
α	1/year	Effective rate of professional development to treat math anxiety	0.0167	[27]
μ	1/year	Retirement rate	0.027	[13, 54]
ω	1/year	Rate of teachers leaving the profession due to math anxiety	0.2	[12]
γ_n	Prob	Probability of becoming a teacher given graduation from high school & non-anxious	0.009	[6, 57]
γ_a	Prob	Probability of becoming a teacher given graduation from high school & anxious	0.037	[6, 57]
q_{an}	Prob	Probability of recovering from anxiety given non-anxious student teacher mentor	0.05	[17, 31]
q_{na}	Prob	Probability of contracting anxiety given anxious student teaching mentor	0.05	[23]
Ψ	Prob	Probability of contracting anxiety given all primary school teachers were anxious	0.32	[22]
ϵ_n	1/time	Rate of students recovering from anxiety given contact with non-anxious students	0.1	[23]
ϵ_a	1/time	Rate of students contracting anxiety given contact with anxious students	0.073	[25]
η_n	1/time	Rate of teachers recovering from anxiety given contact with non-anxious teachers	0.0167	citations[1]
η_a	1/time	Rate of teachers contracting anxiety given contact with anxious teachers	.0324	citations[1]
β_n	1/time	Rate of students recovering from anxiety given contact with non-anxious teachers	0.1	[34]
β_a	1/time	Rate of students contracting anxiety given contact with anxious teachers	0.59	[55]

Table 1: Social meanings, parameter values, and references of parameters in systems (15)-(19)

rate at which the anxious teachers recover from anxiety is η_n, while the rate of teachers contracting anxiety is η_a. Then there is a bi-directional flow that can be modelled by the term

$$(\eta_n - \eta_a)\frac{T_a T_n}{T_n + T_a}. \tag{14}$$

A8: The retirement and natural death rate of teachers is μ. The rate at which anxious teachers leave the profession due to anxiety is ω. The rate at which anxious teachers become non-anxious teachers through professional development is α.

These assumptions result in the following system of non-linear ordinary differential equations. Parameter descriptions can be found in Table 1.

$$\dot{P} = \Lambda - rP \tag{15}$$

$$\dot{S_n} = rP(1 - \Psi\frac{T_a}{T_n + T_a}) + (\epsilon_n - \epsilon_a)\frac{S_a S_n}{S_a + S_n} - \sigma S_n + \beta_n\frac{S_a T_n}{T_n + T_a} - \beta_a\frac{S_n T_a}{T_n + T_a} \tag{16}$$

$$\dot{S_a} = rP\Psi\frac{T_a}{T_n + T_a} - (\epsilon_n - \epsilon_a)\frac{S_a S_n}{S_a + S_n} - \sigma S_a - \beta_n\frac{S_a T_n}{T_n + T_a} + \beta_a\frac{S_n T_a}{T_n + T_a} \tag{17}$$

$$\dot{T_n} = \sigma\gamma_n(1 - q_{na}\frac{T_a}{T_n + T_a})S_n + \sigma\gamma_a q_{an}\frac{T_n}{T_n + T_a}S_a + (\eta_n - \eta_a)\frac{T_a T_n}{T_n + T_a} + \alpha T_a - \mu T_n \tag{18}$$

$$\dot{T_a} = \sigma\gamma_n q_{na}\frac{T_a}{T_n + T_a}S_n + \sigma\gamma_a(1 - q_{an}\frac{T_n}{T_n + T_a})S_a - (\eta_n - \eta_a)\frac{T_a T_n}{T_n + T_a} - (\mu + \omega + \alpha)T_a \tag{19}$$

[1] "citations" references [8, 9, 16, 24, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51]

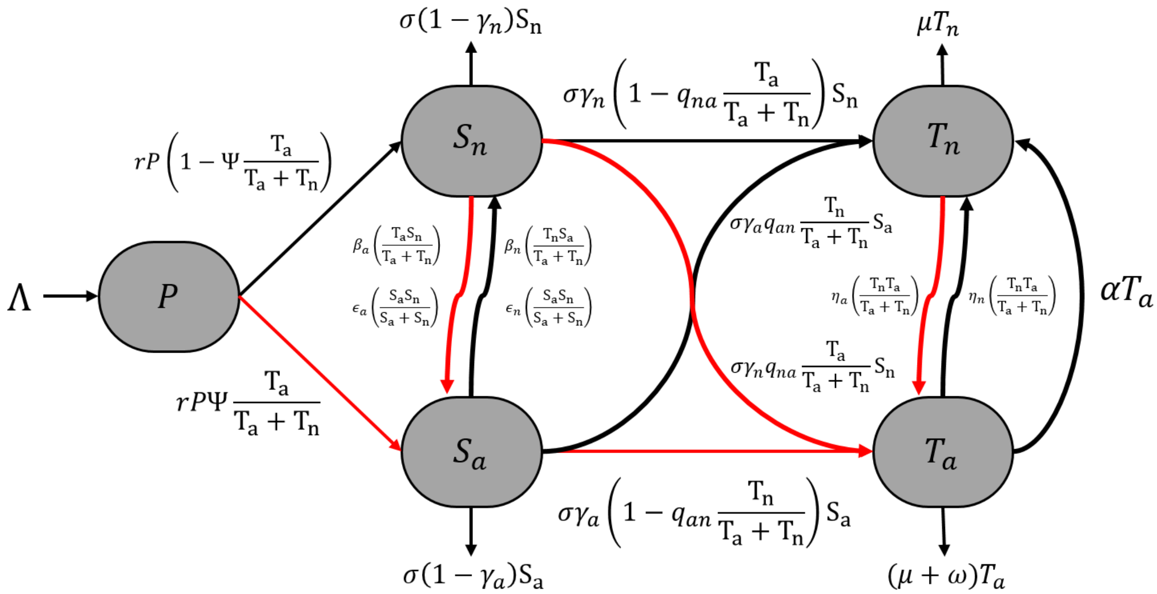

Figure 1: Flow chart describing the interaction between peers and teachers as math anxiety is transmitted

However, we can reduce the dimensionality of our system from five to three if we consider the following

1. We solve $P(t)$ in $\dot{P}$. Thus $P(t) = \frac{\Lambda}{r} - e^{-rt}\left(\frac{\Lambda}{r} - P_0\right)$ where $P(0) = P_0$.
2. We set $S = S_a + S_n$. Then $\dot{S} = \dot{S}_a + \dot{S}_n$. So $\dot{S} = rP - \sigma S$. Solving S, we get $S(t) = e^{-t\sigma}\left(-\frac{\Lambda}{\sigma} + \frac{rP_0 - \Lambda}{r-\sigma} + \frac{(\Lambda - rP_0)e^{t(\sigma-r)}}{r-\sigma} + S_0 + \frac{\Lambda e^{\sigma t}}{\sigma}\right)$ where $S(0) = S_0$.

Then we have the following system of equations

$$\dot{S_a} = rP(t)\Psi\frac{T_a}{T_n+T_a} - (\epsilon_n - \epsilon_a)\frac{S_a(S(t)-S_a)}{S(t)} - \sigma S_a - \beta_n\frac{S_aT_n}{T_n+T_a} + \beta_a\frac{(S(t)-S_a)T_a}{T_n+T_a} \tag{20}$$

$$\dot{T_n} = \sigma\gamma_n(1-q_{na}\frac{T_a}{T_n+T_a})(S(t)-S_a) + \sigma\gamma_a q_{an}\frac{T_n}{T_n+T_a}S_a + (\eta_n - \eta_a)\frac{T_aT_n}{T_n+T_a} + \alpha T_a - \mu T_n \tag{21}$$

$$\dot{T_a} = \sigma\gamma_n q_{na}\frac{T_a}{T_n+T_a}(S(t)-S_a) + \sigma\gamma_a(1-q_{an}\frac{T_n}{T_n+T_a})S_a - (\eta_n - \eta_a)\frac{T_aT_n}{T_n+T_a} - (\mu+\omega+\alpha)T_a \tag{22}$$

2.2 Parameter Estimation

The parameters in the model are categorized into two different groups: well known value estimation and literature estimation. The parameter values of each category were estimated differently. The values r and σ are graduation rates based on the number of years spent in primary and secondary school, respectively. It is assumed that all students graduate within our system. Because the only entry parameter to the system is Λ, then, the equilibrium points of the dynamical system will depend equally on this parameter. Thus, we can assume that any arbitrary value for Λ is correct. To obtain our value of Λ, we set our population size to be a district and estimated the value from one quarter of an average of high school populations across various districts. Using numbers from high school allows us to disregard drop-out rates. ϵ_n, μ, ω, γ_n, γ_a, ϵ_a, Ψ, β_a, α, q_{na}, q_{an}, β_n, η_n, η_a are values estimated from data found in literature relevant to each parameter. Parameter estimates are found in Table 1.

3 Mathematical Analysis

We provide the basic properties of our system:

3.1 Boundedness and Positivity

Theorem 1. *The system* (15)-(19) *is bounded and positively invariant in* $\mathbb{R}^{+3}$*,*

Proof. As we are dealing with human populations, our solutions must be positive. We prove positive invariance by using Proposition B7 in the text by Smith [53].

Suppose $P = 0$. Then

$$\begin{aligned} \dot{P} &= \Lambda & (23) \\ &\geq 0 & (24) \end{aligned}$$

Suppose $S_n = 0$. Then

$$\begin{aligned} \dot{S}_n &= rP(1 - \Psi\frac{T_a}{T_n + T_a}) + \beta_n S_a \frac{T_n}{T_n + T_a} & (25) \\ &\geq 0 & (26) \end{aligned}$$

Suppose $S_a = 0$. Then

$$\begin{aligned} \dot{S}_a &= rP\Psi\frac{T_a}{T_n + T_a} + \beta_a S_n \frac{T_a}{T_n + T_a} & (27) \\ &\geq 0 & (28) \end{aligned}$$

Suppose $T_n = 0$. Then

$$\begin{aligned} \dot{T}_n &= \sigma\gamma_n(1 - q_{na})S_n + \alpha T_a & (29) \\ &\geq 0 & (30) \end{aligned}$$

Finally, suppose $T_a = 0$. Then

$$\begin{aligned} \dot{T}_a &= \sigma\gamma_a(1 - q_{an}) & (31) \\ &\geq 0 & (32) \end{aligned}$$

□

Proof. We prove boundedness of solutions by using differential inequalities and by looking at limit suprema. Recall that we solved for $P(t)$. Then

$$\limsup_{t\to\infty} P(t) = \frac{\Lambda}{r} \quad (33)$$

and thus the P(t) solutions are always bounded.

Also, recall that we solved for $S(t)$. Then

$$\limsup_{t\to\infty} S(t) = \frac{\Lambda}{\sigma} \tag{34}$$

and thus S(t) is always bounded. Positivity implies that $S_a(t)$ and $S_n(t)$ solutions are also bounded.

Now we prove boundedness of solutions for $T_a(t)$

$$\begin{aligned}\dot{T_a} &= \sigma\gamma_n q_{na}\frac{T_a}{T_n+T_a}(S(t)-S_a)+\sigma\gamma_a(1-q_{an}\frac{T_n}{T_n+T_a})S_a-(\eta_n-\eta_a)\frac{T_aT_n}{T_n+T_a}-(\mu+\omega+\alpha)T_a & (35)\\ &\le \sigma\gamma_n q_{na}(S(t)-S_a)+\sigma\gamma_a S_a+\eta_a T_a-(\mu+\omega+\alpha)T_a. & (36)\end{aligned}$$

This implies that

$$\limsup_{t\to\infty} T_a(t) = \frac{\gamma_n q_{na}(S(t)-S_a)+\sigma\gamma_a S_a}{\alpha-\eta_a+\mu+\omega}. \tag{37}$$

Recall that $S(t)$ and $S_a(t)$ are bounded, thus $T_a(t)$ is bounded. Finally we prove the boundedness of $T_n(t)$ solutions:

$$\begin{aligned}\dot{T_n} &= \sigma\gamma_n(1-q_{na}\frac{T_a}{T_n+T_a})(S(t)-S_a)+\sigma\gamma_a q_{an}\frac{T_n}{T_n+T_a}S_a+(\eta_n-\eta_a)\frac{T_aT_n}{T_n+T_a}+\alpha T_a-\mu T_n & (38)\\ &\le \sigma\gamma_n(S(t)-S_a)+\sigma\gamma_a q_{an}S_a+\eta_n T_n+\alpha T_a-\mu T_n. & (39)\end{aligned}$$

This implies that

$$\limsup_{t\to\infty} T_n(t) = \frac{\sigma(\gamma_n(S(t)-S_a)+\gamma_a q_{an}+\alpha T_a)}{\mu-\eta_n}. \tag{40}$$

□

Recall that T_a is bounded, thus all solutions to the system are bounded above. So we can guarantee that our solutions will be positive and bounded.

Sociological significance: Theorem 1 suggests that our system is well-defined sociologically. This is because we will always have a population that will not grow unbounded over time, nor will it become negative. This makes our model a more accurate representation of students and teachers with and without anxiety.

3.2 Anxiety-Free Equilibrium

In order to study the system, the first step is to solve (15)-(19) for equilibria. We set $S_a^* = 0$ and $T_a^* = 0$ to remove anxiety from the system. We also found $P^* = \lim_{t\to\infty} P(t) = \frac{\Lambda}{r}$ and S_n^* by showing that $S_n^* = S^* = \lim_{t\to\infty} S(t) = \frac{\Lambda}{\sigma}$. We used the previously found equilibrium values to obtain

$$\dot{T_n} = \sigma\gamma_n\frac{\Lambda}{\sigma} - \mu T_n. \tag{41}$$

Then, we solve (41) for T_n^* to give us our Anxiety-Free Equilibrium (AFE):

$$AFE = (P^*, S_n^*, S_a^*, T_n^*, T_a^*) = (\Lambda/r, \Lambda/\sigma, 0, \gamma_n\Lambda/\mu, 0) \tag{42}$$

3.3 Endemic Equilibrium

Let us look at the endemic equilibrium. We set equations (17)-(19) equal to zero:

$$0 = rP^*\Psi\frac{T_a^*}{T_n^*+T_a^*} - (\epsilon_n - \epsilon_a)\frac{S_a^*(S^*-S_a^*)}{S^*} - \sigma S_a^* - \beta_n\frac{S_a^*T_n^*}{T_n^*+T_a^*} + \beta_a\frac{(S^*-S_a^*)T_a^*}{T_n^*+T_a^*} \tag{43}$$

$$0 = \sigma\gamma_n(1 - q_{na}\frac{T_a^*}{T_n^*+T_a^*})(S^*-S_a^*) + \sigma\gamma_a q_{an}\frac{T_n^*}{T_n^*+T_a^*}S_a^* + (\eta_n - \eta_a)\frac{T_a^*T_n^*}{T_n^*+T_a^*} + \alpha T_a^* - \mu T_n \tag{44}$$

$$0 = \sigma\gamma_n q_{na}\frac{T_a}{T_n+T_a}(S(t) - S_a) + \sigma\gamma_a(1 - q_{an}\frac{T_n}{T_n+T_a})S_a - (\eta_n - \eta_a)\frac{T_aT_n}{T_n+T_a} - (\mu+\omega+\alpha)T_a \tag{45}$$

Let the proportion of anxious teachers at the equilibrium be

$$\phi = \frac{T_a^*}{T_n^*+T_a^*}. \tag{46}$$

For simplicity, let $\eta = \eta_n - \eta_a$. We substitute equation (46) into equation (43) and solve for ϕ to get:

$$\phi = \frac{\sigma(S_a^*\beta_n\Lambda - S_a^*\epsilon_a\Lambda + S_a^*\epsilon_n\Lambda + S_a^{*2}\sigma(\epsilon_a - \epsilon_n) + S_a^*\Lambda\sigma)}{\Lambda(\beta_a\Lambda - S_a^*\sigma(\beta_n - \beta_a) + \Lambda\sigma\Psi)}. \tag{47}$$

Solving for T_n in terms of T_a in (46), we obtain

$$T_n^* = T_a^*\left(\frac{1-\phi}{\phi}\right). \tag{48}$$

We substitute equations (47) and (48) into equation (45) to give us the following value

$$T_a^* = \frac{\gamma_a\sigma S_a^*(1 - q_{an}\phi) + \gamma_n q_{na}\sigma\phi(S^* - S_a^*)}{\alpha + \eta(1-\phi) + \mu + \omega}. \tag{49}$$

This allows us to solve for T_n^* in terms of S_a^* by substituting equations (49), (47), and (48) as shown below

$$T_n^* = \left(\frac{\gamma_a\sigma S_a^*(1 - q_{an}\phi) + \gamma_n q_{na}\sigma\phi(S^* - S_a^*)}{\alpha + \eta(1-\phi) + \mu + \omega}\right)\left(\frac{1-\phi}{\phi}\right). \tag{50}$$

With both equations in terms of S_a^*, solving for S_a^* would allow us to find our endemic equilibrium point. Through reduction of the system, we find a fifth degree polynomial, where a solution is $S_a^* = 0$. The polynomial, shown in Appendix A, has an explicit solution for all five equilibria, but not all of them are sociologically viable because not all roots are positive and real.

3.4 Basic reproduction number, R_0

We use the Next-Generation Matrix (NGM) to generate R_0 [15]. Using equations (20) and (22), the equations can be rearranged to get a function of the following form:

$$\dot{f} = \text{“}Entry\text{”} - \text{“}Exit\text{”}$$

Then, we reorder the terms of the function as follows:

$$\dot{S_a} = rP(t)\Psi\frac{T_a}{T_n+T_a} + \beta_a\frac{(S(t)-S_a)T_a}{T_n+T_a} + \epsilon_a\frac{S_a(S(t)-S_a)}{S(t)} - \epsilon_n\frac{S_a(S(t)-S_a)}{S(t)} - \sigma S_a - \beta_n\frac{S_aT_n}{T_n+T_a}$$

and

$$\dot{T_a} = \sigma\gamma_n q_{na}\frac{T_a}{T_n+T_a}(S(t)-S_a) + \sigma\gamma_a(1-q_{an}\frac{T_n}{T_n+T_a})S_a + \eta_a\frac{T_aT_n}{T_n+T_a} - \eta_n\frac{T_aT_n}{T_n+T_a} - (\mu+\omega+\alpha)T_a$$

Thus, we get the following system:

$$f = \begin{bmatrix} rP(t)\Psi\frac{T_a}{T_n+T_a} + \beta_a\frac{(S(t)-S_a)T_a}{T_n+T_a} + \epsilon_a\frac{S_a(S(t)-S_a)}{S(t)} \\ \sigma\gamma_n q_{na}\frac{T_a}{T_n+T_a}(S(t)-S_a) + \sigma\gamma_a(1-q_{an}\frac{T_n}{T_n+T_a})S_a + \eta_a\frac{T_aT_n}{T_n+T_a} \end{bmatrix} \tag{51}$$

And

$$v = \begin{bmatrix} \epsilon_n\frac{S_a(S(t)-S_a)}{S(t)} + \sigma S_a + \beta_n\frac{S_aT_n}{T_n+T_a} \\ \eta_n\frac{T_aT_n}{T_n+T_a} + (\mu+\omega+\alpha)T_a \end{bmatrix} \tag{52}$$

Then, using the following vector X, we create F as the derivative of f in X evaluated at the AFE, and in a similar way, V is created.

$$\text{Let } X = \begin{bmatrix} S_a \\ T_a \end{bmatrix}$$

$$F = \frac{df}{dX}_{AFE} = \begin{bmatrix} \epsilon_a & \frac{\Psi\mu}{\gamma_n} + \frac{\beta_a\mu}{\sigma\gamma_n} \\ \sigma\gamma_a(1-q_{an}) & q_{na}\mu + \eta_a \end{bmatrix}. \tag{53}$$

$$V = \frac{dv}{dX}_{AFE} = \begin{bmatrix} \sigma+\epsilon_n+\beta_n & 0 \\ 0 & \mu+\eta_n+\omega+\alpha \end{bmatrix} \tag{54}$$

Then the NGM is:

$$FV^{-1} = \begin{bmatrix} \frac{\epsilon_a}{\sigma+\epsilon_n+\beta_n} & \frac{\frac{\Psi\mu}{\gamma_n}+\frac{\beta_a\mu}{\sigma\gamma_n}}{\mu+\eta_n+\omega+\alpha} \\ \frac{\sigma\gamma_a(1-q_{an})}{\sigma+\epsilon_n+\beta_n} & \frac{q_{na}\mu+\eta_a}{\mu+\eta_n+\omega+\alpha} \end{bmatrix} \tag{55}$$

To get the basic reproductive number R_0, we select the maximum eigenvalue of the matrix shown in equation (55). Because all parameters are positive, we identify the following value for R_0

$$R_0 = \frac{1}{2}\Bigg[\frac{\epsilon_a}{\sigma+\epsilon_n+\beta_n} + \frac{q_{na}\mu+\eta_a}{\mu+\eta_n+\omega+\alpha} + \sqrt{\left(\frac{\epsilon_a}{\sigma+\epsilon_n+\beta_n} - \frac{q_{na}\mu+\eta_a}{\mu+\eta_n+\omega+\alpha}\right)^2 + 4\frac{\left(\frac{\Psi\mu}{\gamma_n}+\frac{\beta_a\mu}{\sigma\gamma_n}\right)(\sigma\gamma_a(1-q_{an}))}{(\sigma+\epsilon_n+\beta_n)(\mu+\eta_n+\omega+\alpha)}}\Bigg]. \tag{56}$$

3.5 Sociological Interpretation of R_0

In order to interpret R_0, we can divide it into smaller components:

$$R_S = \frac{\epsilon_a}{\sigma+\epsilon_n+\beta_n}, \tag{57}$$

$$R_T = \frac{q_{na}\mu+\eta_a}{\mu+\eta_n+\omega+\alpha}, \tag{58}$$

$$R_{CTS} = \frac{\sigma\gamma_a(1-q_{an})}{\mu+\eta_n+\omega+\alpha}, \tag{59}$$

$$R_{CST} = \frac{\left(\frac{\Psi\mu}{\gamma_n}+\frac{\beta_a\mu}{\sigma\gamma_n}\right)}{(\sigma+\epsilon_n+\beta_n)}. \tag{60}$$

R_S refers to the transmission of anxiety between secondary school students. Through peer interaction, non-anxious students become anxious students. In a similar way, R_T describes the rate of development of anxious teachers through the influences of other anxious teachers. Anxious teachers are recruited through interaction with other anxious teachers, such as through curriculum coordination meetings. Anxious teachers also mentor non-anxious student-teachers which causes students to become anxious teachers themselves. Both terms, R_S and R_T, are the primary ways that anxiety develops within the system.

The secondary recruitment is given by the terms of the equations R_{CTS} and R_{CST}. R_{CTS} is the promotion from an anxious secondary student to an anxious teacher. This occurs when intervention strategies are ineffective and a student simply ages without any change in anxiety. R_{CST} measures how anxious teachers influence primary students and non-anxious secondary students so that they become anxious secondary students.

So R_0 can be written as shown below:

$$R_0 = \frac{1}{2}\Big(R_S + R_T + \sqrt{(R_S - R_T)^2 + 4R_{CTS}R_{CST}}\Big)$$

To give a sociological explanation of R_0, we first need to examine the R_0 as a mathematical component. We know that if $A, B (\in \mathbb{R}^+)$, then

$$\sqrt{A+B} \leqslant \sqrt{B} + \sqrt{A}$$

and

$$max(A, B) = \frac{A + B + |A - B|}{2}.$$

Since

$$R_0 \leqslant \frac{1}{2}\left(R_S + R_T + \sqrt{(R_S - R_T)^2}\right) + \sqrt{R_{CTS} R_{CST}}$$

and

$$\frac{1}{2}\left(R_S + R_T + \sqrt{(R_S - R_T)^2}\right) \leqslant R_0$$

then,

$$\max(R_S, R_T) \leqslant R_0 \leqslant \max(R_S, R_T) + \sqrt{R_{CTS} R_{CST}}. \tag{61}$$

From the inequality above (61) and the explanation of R_S, R_T, R_{CTS}, and R_{CST}, we can determine the conditions needed to create an epidemic of anxiety and how to avoid it. Since we know that both primary terms, R_S and R_T, are less than R_0, we know that when one of these values is greater than or equal to one, an epidemic will occur. Furthermore, R_0 is also less than the larger of R_S and R_T plus the geometric mean of the secondary recruitment terms, R_{CTS} and R_{CST}. This means that the generational effects can create an epidemic even in scenarios where peer effects by themselves will not support an epidemic.

4 Results

4.1 Simulations

The simulations of our dynamical system were developed using the MATLAB's Ordinary Differential Equation Toolbox. With the help of the function ode45, parameter settings shown in Table 1, and the initial conditions shown below, we get the simulation shown in Figure 2.

$$(P(0), S_n(0), S_a(0), T_n(0), T_a(0)) = (100, 70, 70, 50, 50) \tag{62}$$

For the student population, we found that 21.81% of students are anxious within the total population. Furthermore, 10.91% of teachers are anxious. Also, our simulation shows that there is a female student to female teacher ratio of 22.67.

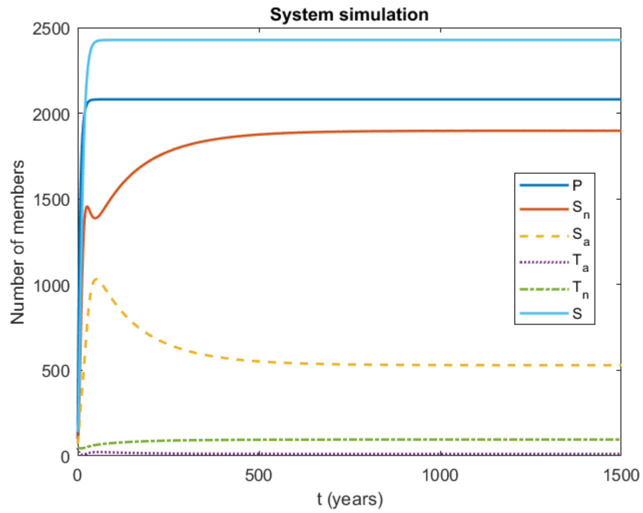

Figure 2: Simulation of the system shown in equations (15) - (19)

4.2 Sensitivity Analysis

In this section, we use sensitivity analysis on R_0 to see which parameters are most influential. Sensitivity can be defined as, "the study of how uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model [35]." In order to find how an individual parameter has an effect on a model we use the following equation:

$$S_e = \frac{p}{f(p)} \cdot \frac{\partial f}{\partial p}, \tag{63}$$

where $p \in \mathcal{P}_f$, with $\mathcal{P}_f$ being the space of parameters of the function f.

To numerically approximate the sensitivity of T_a^* and S_a^*, we use the base parameter values from Table 1. This approximation is calculated using:

$$S_e \approx \frac{\frac{\triangle f}{f}}{\frac{\triangle p}{p}} \tag{64}$$

For each $p \in \mathcal{P}_f$, we ran a simulation changing only one parameter at time, adding 0.0001% to it. Each iteration, we got a change of a value of S_a^*, T_a^* and T_n^*. Thus, applying formula (64), we achieve a numerical approximation of the sensitivity of a desired parameter.

4.2.1 Sensitivity of R_0

By finding the sensitivity index of each parameter, we can determine the percent change that each variable will have on R_0 [35]. The analytic sensitivity analysis of R_0 is shown in Figure 3. The results show that γ_n is the most sensitive parameter of R_0, which is followed by γ_a, β_a, μ, and ω, respectively. In order to decrease R_0, it is recommended to increase γ_n and lower γ_a and β_a, since these are parameters that we can alter, either directly or indirectly.

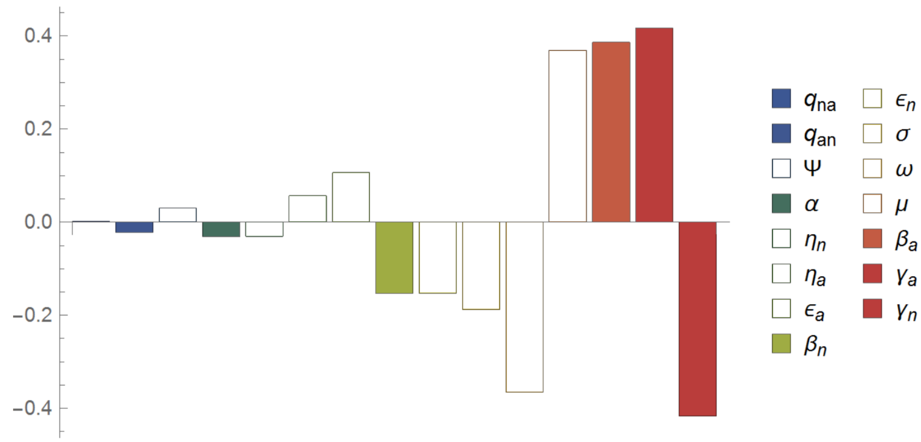

Figure 3: Results of the sensitivity analysis of R_0. Sensitivity indices in white represent parameters that cannot be changed, whereas, colored sensitivity indices represent the parameters that can be changed. Beginning with q_{na} and ending with γ_n, sensitivity to parameters from top to bottom is represented by bars from left to right.

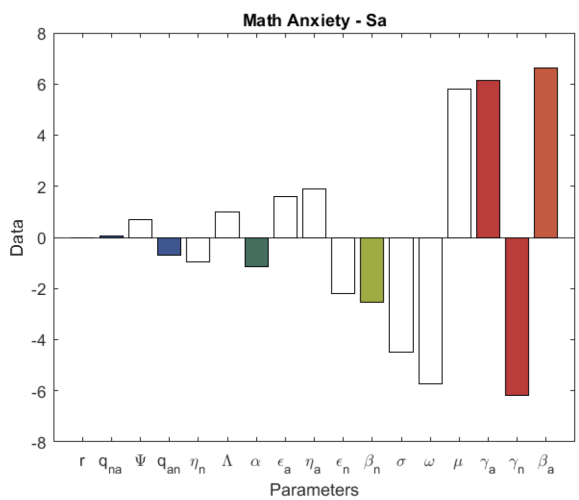

Figure 4: Numeric sensitivity approximation for the S_a^* endemic equilibrium point using base parameter values

4.2.2 Equilibria

Figure 4 shows the sensitivity approximation of S_a^*. The equilibrium value is most sensitive to the following parameters: β_a, γ_n, γ_a, μ, and ω. Next, Figure 5 shows the equilibrium value T_a^* is most sensitive to the following parameters: γ_a, ω, β_a, γ_n and μ. Lastly, Figure 6, shows that γ_n, μ, β_a, γ_a and ω are the parameter values that the equilibrium value T_n^* is most sensitive to.

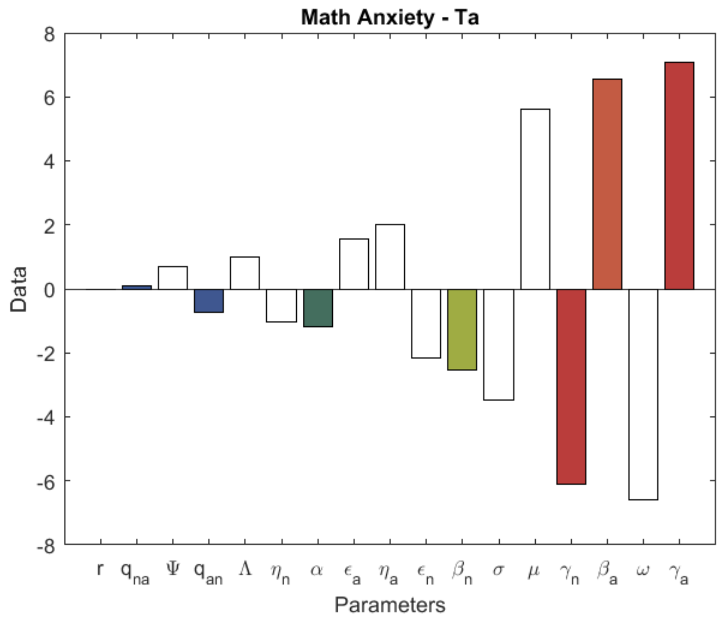

Figure 5: Numeric sensitivity approximation for the T_a^* endemic equilibrium point using base parameter value

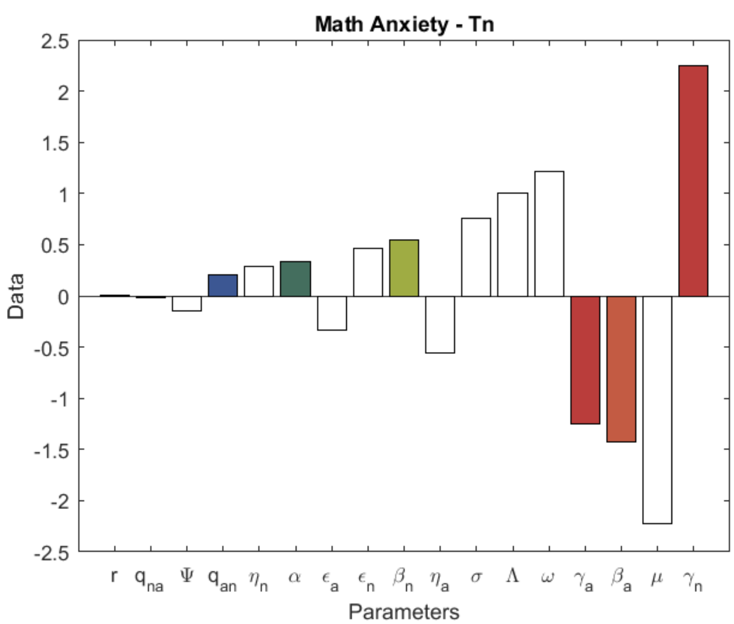

Figure 6: Numeric sensitivity approximation for the T_n^* endemic equilibrium point using base parameter value

4.2.3 Interpretation

Our numeric approximation of the equilibria showed similar results to our sensitivity of R_0. In order to remove anxiety from our system, our analysis suggests that we should focus on the following parameters: μ, ω, β_a, γ_n and γ_a. The rate of teachers leaving the profession because of anxiety, ω, and the retirement rate of teachers, μ, cannot be directly addressed. Thus they cannot be a target for any intervention strategy. Next, β_a represents how teachers interact with secondary school students to cause anxiety within the students. We can indirectly affect β_a by requiring anxious teachers to undergo professional development to improve math instruction. Since γ_n and γ_a have opposite effects, we are able to increase γ_n through programs directed toward increasing recruitment of non-anxious teachers instead of anxious teachers. Ultimately, the results of Figures 4 - 6 suggest that the most effective method to reduce math anxiety in the system is to recruit non-anxious teachers.

4.3 Parameter curves (for select parameters)

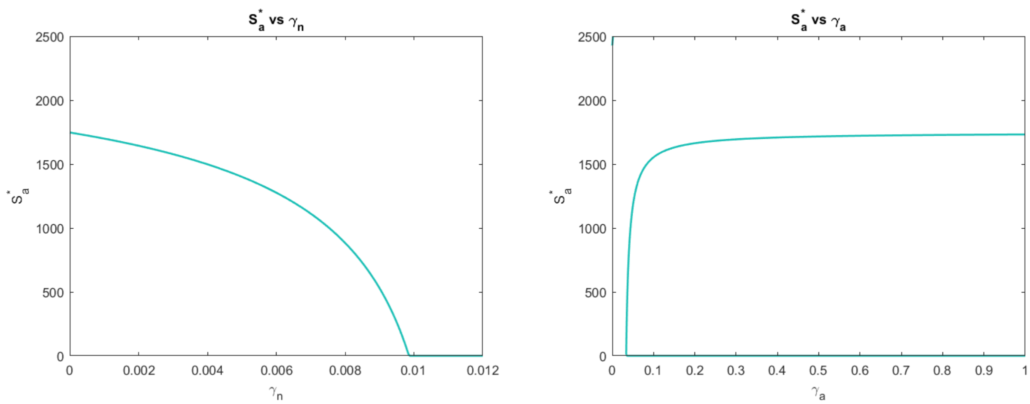

Figure 7: On the left, parameter curve S_a^* vs γ_n and on the right, parameter curve S_a^* vs γ_a

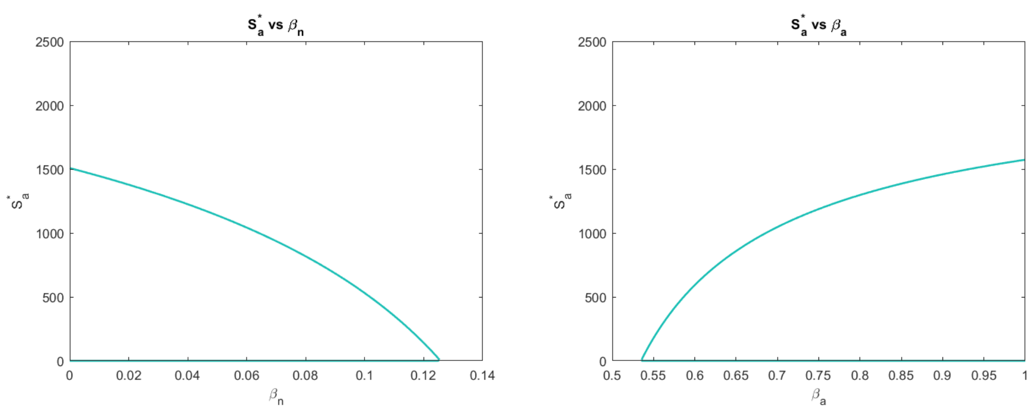

Figure 8: On the left, parameter curve S_a^* vs β_n and on the right, parameter curve S_a^* vs β_a

The curves shown in Figures 7 and 8 are diagrams showing the change in S_a^* over a parameter of the system. The results are calculated from the fifth degree polynomial, S_a^*, using our estimated parameters. When γ_n is decreased, the number of anxious students, will increase as predicted. On the other hand, as γ_a is augmented, S_a^* becomes larger. When β_a is augmented, the contagion through teacher interaction is more effective; more students will become anxious. Analysing β_n, we get a similar result. When β_n decreases, the number of anxious students increases. The parameter curves for α, μ, ω and Ψ are shown in Appendix B in Figures 9 to 12.

4.4 Target Parameter Values

From our simulation, it can be concluded that math anxiety is endemic. In fact, we calculate R_0 to be 1.7145. Therefore, we strive to find values for our parameters in R_0 that will help to reduce R_0. However, not all parameters can be controlled. Thus, we define the intervention parameters as γ_n, γ_a, β_a, and α because these are the parameters that we are able to change in order to reduce math anxiety.

Let $\mathcal{P}$ be a vector of intervention parameters, let $\mathcal{Q}$ be the vector of **all** of the base parameter values, and let P_0 be the vector of base intervention parameter values. Then, let $\mathcal{R} = \mathcal{Q} \setminus P_0$ be the base non-intervention parameters. Then $P_0 \cup \mathcal{R} = \mathcal{Q}$. We were unable to estimate costs, so we use the Euclidean distance as a function to minimize. Because we use Euclidean distances, $\mathcal{P}$ must be a vector of similar parameters (e.g. it cannot be the case that $\mathcal{P} = \{\beta_a, \alpha\}$ but it can be the case that $\mathcal{P} = \{\gamma_n, \gamma_a\}$). We define the Euclidean distance of our parameters as:

$$f(\mathcal{P}) = \sqrt{(\mathcal{P} - P_0)^2}, \tag{65}$$

where P_0 is the base parameter value. We hope to reduce the basic reproductive number from its current value of 1.7145. Thus our constraint is:

$$g(\mathcal{P}) = R_0(\mathcal{P}, \mathcal{R}) - 1. \tag{66}$$

Notice that our constraint is constant. Thus, we are able to use the Lagrangian function. We aim to minimize the Lagrangian function

$$h(\mathcal{P}) = f(\mathcal{P}) - \lambda g(\mathcal{P}) \tag{67}$$

where λ is some real number. We use Lagrange's theorem to find potential extrema by solving the following system of equations

$$\nabla h(\mathcal{P}) = 0 \tag{68}$$

$$g(\mathcal{P}) = 0 \tag{69}$$

From there we choose the points that had all positive components. We are able to get the following values in Table 2.

P	Target Value	ΔP	Sociological Meaning
γ_n	.00986	+0.00086 [+9.5%]	Recruitment on R_0
γ_a	0.0337894	-0.003 [-8.7%]	Weed out Effect on R_0
(γ_n, γ_a)	$(0.00979, 0.0367)$	$(0.00079, -0.0002)$[+8.9%, $-.6$%]	Weed out & Recruitment
β_a	0.534	-0.055[-9.3%]	Better Pedagogy in T_a on R_0
α	0.0382	+.0215[+128.97%]	P.D. on R_0

Table 2: Table of intervention parameters, their target values, the change between target values and base values (along with the percent change), and sociological meaning

5 Discussion

Sensitivity analysis of R_0 reveals similar results to our system of equations. It is important to note that we cannot implement effective intervention strategies for μ, the rate at which teachers retire, and ω, the rate at which teachers quit due to anxiety. The following sections discuss how an increase or decrease in our intervention parameters affect our model to identify potential strategies to reduce anxiety.

5.1 Influence of Recruitment

The most critical point of intervention is before a student chooses to become a teacher because γ_a and γ_n have the most significant sensitivity indices. The most effective change will be intervention in either γ_n or γ_a because any change to these parameters will have a significant impact on anxiety levels in the system as a whole. First, γ_n is the probability that a non-anxious STEM major student will become a teacher. In order to increase γ_n, programs to recruit non-anxious STEM majors should be implemented. Some programs, such as the Noyce scholarship, funded by the National Science Foundation (NSF), aim to recruit STEM majors to pursue careers as high school teachers by offering scholarships to pay tuition and other benefits. The University of Minnesota found that the Noyce program had a significant impact on the amount of qualified STEM teachers in high need areas, which suggests it is effective in adjusting γ_n [28].

However, the probability of an anxious student becoming an anxious teacher, γ_a, remains high [31]. A study by Kelly and Tomhave affirms that many anxious students choose to become teachers, which adds anxiety to the system as a whole [26]. We can only change γ_a by "weed out" policies that would reduce the number of anxious college students who become teachers. Examples include students registering for more math courses, registering for higher level math courses, or colleges setting higher placement test cut-offs. However, the desirability of "weed out" policies has been questioned by researchers. Seymour and Hewitt interviewed undergraduate students who had left STEM majors and switched into non-STEM majors [52]. They found that switching out caused by "weed out" courses was not predicted by a student's grade point average. Instead of narrowing in on the best students, the weed out courses had the opposite effect and also disproportionately steered under-represented groups away from STEM. Therefore, we would not advise targeting γ_a.

As seen in the Figure 7, the behavior of the number of students with anxiety in the equilibrium approximates an asymptote (less than the total amount of students) as the parameter γ_a increases. This effect is important because the system does not allow every student to have anxiety as this parameter continues to increase. So, as said before, when γ_a increases, it increases the anxiety in the system up to a point, but it remains preferable to increase the value of γ_n.

5.2 Influence of Teachers on Secondary Students

The sensitivity analysis also reveals that anxious secondary teachers have a significant impact on whether or not a student will develop anxiety. This is due to β_a, which represents a student interacting with an anxious teacher, interacting with anxious teachers. β_a had the second most sensitive index. Females are especially susceptible to the anxiety of female teachers [5]. When instructors have poor instruction styles or fear math themselves, they are able to spread their anxiety to the students they teach [25]. Although this could be improved through better pedagogy, significant changes would be challenging as teachers are often set in their ways of teaching and change would take time.

5.3 Influence of Peer Interactions

Interactions between teachers can cause the transmission of anxiety from an anxious teacher to a non-anxious teacher which is represented by the parameter η_a. On the contrary, non-anxious teachers can affect anxious teachers to recover from anxiety which is represented by the parameter η_n. Teachers' interactions occur periodically via staff meetings for purposes of planning in schools that use a coordinated curriculum. However, our sensitivity analysis suggests that an intervention strategy to reduce the probability of non-anxious teachers becoming anxious is much weaker than we initially suspected. Time spent reducing anxiety transmission between teachers will not be as effective as other areas of intervention such as recruitment of STEM majors to become teachers.

The classroom consists of anxious and non-anxious students. Students can transmit anxiety or recover from it through peer interactions. The parameter ϵ_n describes the recovery of anxious students from anxiety by interactions with non-anxious students. The parameter ϵ_a describes the transmission rate of anxiety from anxious to non-anxious students by interactions with each other. By school districts implementing policies that promote group work, students that are non-anxious will have a higher probability of interacting with their anxious peers, which could help to reduce math anxiety among anxious students. This concept is shown in a study by Riegle-Crumb et al. which reveals that females' class choice in regards to level of mathematical rigor is affected by the level of math anxiety of their female friends [34]. However, sensitivity analysis showed that intervention in peer interactions would not be as effective as better teacher recruitment.

5.4 Influence of Professional Development

The rate of professional development used to effectively treat anxiety is α. Professional development has shown to be a successful method of reducing anxiety in teachers. For example, Kutaka et al. showed a positive correlation between teachers who went through the process of professional development and their increase of confidence in teaching math. The authors believe that when a teacher has high self-efficacy in mathematics, the teacher's confidence will help them to facilitate a better attitude about math towards students [27].

However, professional development is not as effective as other parameters in the sensitivity analysis. In fact, α has a low sensitivity index. Professional development, although useful for helping individual teachers and their students, does not dramatically affect anxiety in the population. A better allocation of resources would be toward recruitment of non-anxious math teachers to reduce math anxiety in the system as a whole. While professional development programs to reduce math anxiety are not ineffective, it will take more time and effort to alter the levels of math anxiety in the whole population with this method. Professional development helps individual teachers, but not the population as a whole.

5.5 Influence of Student-Teaching Mentors

The parameter q_{an} is the probability that a student-teachers recovers from anxiety given interaction with a non-anxious mentor and q_{na} is the probability of a student-teacher contracting anxiety given an anxious mentor. It is true that a confident mentor can reduce math anxiety in student-teachers, but our results suggest that these mentor effects have little influence on anxiety in the whole population. Resources spent on monitoring mentor to student-teacher interaction may be valuable in other ways, but they do not contribute effectively to an overall reduction in math anxiety in the system as a whole [31].

5.6 Ending the Endemic

Earlier in our analysis, R_0 was calculated to be 1.7145 which reveals that with the current state of our system, math anxiety will continue to exist within a given school district. By using the Lagrange multipliers, we were able to find target intervention parameter values that would help us to reduce R_0 to 1. Our analysis revealed that α was the most changed parameter, which means that intervention strategies for professional development would require extensive implementation. Because our sensitivity analysis revealed that professional development has little impact on reducing math anxiety, school districts should target other intervention strategies. γ_n, γ_a, and β_a were parameters that required very little change (less than 10 percent) to reduce R_0 to less than one. This suggests that interventions targeting teaching methods (β_a) and teacher recruitment (γ_a, γ_n) have the potential to reduce math anxiety dramatically.

6 Conclusions

Due to lack of representation of minorities and females in STEM, math anxiety is an issue that has prevented STEM development from reaching its full potential. Fear of math prevents individuals from pursuing STEM careers that lead to critical scientific and technological advancements, especially in female students. Despite the threat to progress and innovation in STEM that math anxiety presents, little has been done to resolve it.

In order to reduce math anxiety, we examined the transmission of math anxiety between teachers and female students, including the progression of how a student becomes a teacher, to identify the optimal stage in a student or teacher's life to direct intervention efforts. Our objective was to develop a mathematical model to pinpoint an effective strategy for reducing math anxiety. With that, our research question was: "what stage in a female student's or teacher's life should an intervention be implemented to reduce or prevent math anxiety in students and teachers?" Using our model, we were able to conclude that our model is a good representation of how math anxiety is transferred among students and teachers. However, because our research only focuses on a few ways that math anxiety can develop among teachers and students, future research should address other issues like race, parental attitude toward math, parental expectations, and intimidating teachers.

Observing R_0, we were able to determine that intervention strategies should occur at two time periods in the student's progression towards becoming a teacher. The first intervention period should be while the student is in secondary school. The second intervention period should be when the student is in college, before becoming a pre-service teacher. Moreover, because our analysis revealed that professional development was not a significant parameter for reducing math anxiety in the overall population, researchers should develop alternate strategies to help reduce overall math anxiety. Our numeric approximation of sensitivity revealed similar results to our analytic sensitivity analysis of R_0, supporting recruitment of non-anxious teachers and reducing transmission from teachers to secondary school students. Therefore, to reduce math anxiety in the United States, teachers need to portray more positive attitudes towards math, and colleges should focus on recruiting education majors who have positive attitudes toward math.

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A 5th Degree Polynomial, S_a^*

$$0 = C_1 S_a^* + C_2 S_a^{*2} + C_3 S_a^{*3} + C_4 S_a^{*4} + C_5 S_a^{*5} \tag{70}$$

Where

$$C_1 = -\Lambda^4 \sigma \left(\beta_a + \sigma \Psi\right) \left(\mu \gamma_a \left(q_{\text{an}} - 1\right) \left(\beta_a + \sigma \Psi\right) + \gamma_n \left(-\epsilon_a + \beta_n + \epsilon_n + \sigma\right) \left(-\eta_a + \alpha + \mu + \eta_n - \mu q_{\text{na}} + \omega\right)\right)$$

$$\begin{aligned}
C_2 = {} & -\alpha\Lambda^3\Psi\gamma_a\sigma^4 - \Lambda^3\mu\Psi\gamma_a\sigma^4 + \Lambda^3\mu\Psi q_{\text{an}}\gamma_a\sigma^4 - \Lambda^3\Psi\omega q_{\text{an}}\gamma_a\sigma^4 + \alpha\Lambda^3\Psi\gamma_n\sigma^4 + \Lambda^3\mu\Psi\gamma_n\sigma^4 + \Lambda^3\Psi\omega\gamma_n\sigma^4 - \\
& \Lambda^3\mu\Psi q_{\text{na}}\gamma_n\sigma^4 + \Lambda^3\omega q_{\text{na}}\gamma_n\sigma^4 + \Lambda^3\Psi\gamma_a\eta_a\sigma^4 - \Lambda^3\gamma_n\eta_a\sigma^4 - \Lambda^3\Psi\gamma_n\eta_a\sigma^4 - \Lambda^3\Psi\gamma_a\eta_n\sigma^4 + \Lambda^3\gamma_n\eta_n\sigma^4 + \\
& \Lambda^3\Psi\gamma_n\eta_n\sigma^4 - \alpha\Lambda^3\beta_a\gamma_a\sigma^3 - \Lambda^3\mu\beta_a\gamma_a\sigma^3 - 2\Lambda^3\mu\Psi\beta_a\gamma_a\sigma^3 + \Lambda^3\mu q_{\text{an}}\beta_a\gamma_a\sigma^3 + 2\Lambda^3\mu\Psi q_{\text{an}}\beta_a\gamma_a\sigma^3 - \\
& \Lambda^3\omega q_{\text{an}}\beta_a\gamma_a\sigma^3 - \alpha\Lambda^3\Psi\beta_n\gamma_a\sigma^3 + \Lambda^3\mu\Psi\beta_n\gamma_a\sigma^3 - \Lambda^3\mu\Psi q_{\text{an}}\beta_n\gamma_a\sigma^3 - \Lambda^3\Psi\omega q_{\text{an}}\beta_n\gamma_a\sigma^3 + 2\alpha\Lambda^3\beta_a\gamma_n\sigma^3 + \\
& 2\Lambda^3\mu\beta_a\gamma_n\sigma^3 + 2\Lambda^3\omega\beta_a\gamma_n\sigma^3 - 2\Lambda^3\mu q_{\text{na}}\beta_a\gamma_n\sigma^3 - \alpha\Lambda^3\beta_n\gamma_n\sigma^3 - \Lambda^3\mu\beta_n\gamma_n\sigma^3 + \alpha\Lambda^3\Psi\beta_n\gamma_n\sigma^3 + \\
& \Lambda^3\mu\Psi\beta_n\gamma_n\sigma^3 - \Lambda^3\omega\beta_n\gamma_n\sigma^3 + \Lambda^3\Psi\omega\beta_n\gamma_n\sigma^3 + \Lambda^3\mu q_{\text{na}}\beta_n\gamma_n\sigma^3 - \Lambda^3\mu\Psi q_{\text{na}}\beta_n\gamma_n\sigma^3 + 2\Lambda^3\omega q_{\text{na}}\beta_n\gamma_n\sigma^3 + \\
& \alpha\Lambda^3\Psi\gamma_a\epsilon_a\sigma^3 + \Lambda^3\mu\Psi\gamma_a\epsilon_a\sigma^3 - \Lambda^3\mu\Psi q_{\text{an}}\gamma_a\epsilon_a\sigma^3 + \Lambda^3\Psi\omega q_{\text{an}}\gamma_a\epsilon_a\sigma^3 - 2\alpha\Lambda^3\Psi\gamma_n\epsilon_a\sigma^3 - 2\Lambda^3\mu\Psi\gamma_n\epsilon_a\sigma^3 - \\
& 2\Lambda^3\Psi\omega\gamma_n\epsilon_a\sigma^3 + 2\Lambda^3\mu\Psi q_{\text{na}}\gamma_n\epsilon_a\sigma^3 - 2\Lambda^3\omega q_{\text{na}}\gamma_n\epsilon_a\sigma^3 - \alpha\Lambda^3\Psi\gamma_a\epsilon_n\sigma^3 - \Lambda^3\mu\Psi\gamma_a\epsilon_n\sigma^3 \\
& +\Lambda^3\mu\Psi q_{\text{an}}\gamma_a\epsilon_n\sigma^3 - \Lambda^3\Psi\omega q_{\text{an}}\gamma_a\epsilon_n\sigma^3 + 2\alpha\Lambda^3\Psi\gamma_n\epsilon_n\sigma^3 + 2\Lambda^3\mu\Psi\gamma_n\epsilon_n\sigma^3 + 2\Lambda^3\Psi\omega\gamma_n\epsilon_n\sigma^3 \\
& -2\Lambda^3\mu\Psi q_{\text{na}}\gamma_n\epsilon_n\sigma^3 + 2\Lambda^3\omega q_{\text{na}}\gamma_n\epsilon_n\sigma^3 + \Lambda^3\beta_a\gamma_a\eta_a\sigma^3 + \Lambda^3\Psi\beta_n\gamma_a\eta_a\sigma^3 - 2\Lambda^3\beta_a\gamma_n\eta_a\sigma^3 \\
& -\Lambda^3\beta_n\gamma_n\eta_a\sigma^3 - \Lambda^3\Psi\beta_n\gamma_n\eta_a\sigma^3 - \Lambda^3\Psi\gamma_a\epsilon_a\eta_a\sigma^3 + 2\Lambda^3\gamma_n\epsilon_a\eta_a\sigma^3 + 2\Lambda^3\Psi\gamma_n\epsilon_a\eta_a\sigma^3 + \Lambda^3\Psi\gamma_a\epsilon_n\eta_a\sigma^3 \\
& -2\Lambda^3\gamma_n\epsilon_n\eta_a\sigma^3 - 2\Lambda^3\Psi\gamma_n\epsilon_n\eta_a\sigma^3 - \Lambda^3\beta_a\gamma_a\eta_n\sigma^3 - \Lambda^3\Psi\beta_n\gamma_a\eta_n\sigma^3 + 2\Lambda^3\beta_a\gamma_n\eta_n\sigma^3 \\
& +\Lambda^3\beta_n\gamma_n\eta_n\sigma^3 + \Lambda^3\Psi\beta_n\gamma_n\eta_n\sigma^3 + \Lambda^3\Psi\gamma_a\epsilon_a\eta_n\sigma^3 - 2\Lambda^3\gamma_n\epsilon_a\eta_n\sigma^3 - 2\Lambda^3\Psi\gamma_n\epsilon_a\eta_n\sigma^3 - \Lambda^3\Psi\gamma_a\epsilon_n\eta_n\sigma^3 \\
& +2\Lambda^3\gamma_n\epsilon_n\eta_n\sigma^3 + 2\Lambda^3\Psi\gamma_n\epsilon_n\eta_n\sigma^3 + \Lambda^3\omega q_{\text{na}}\gamma_n\epsilon_a^2\sigma^2 + \Lambda^3\omega q_{\text{na}}\gamma_n\epsilon_n^2\sigma^2 - 2\Lambda^3\mu\beta_a^2\gamma_a\sigma^2 + 2\Lambda^3\mu q_{\text{an}}\beta_a^2\gamma_a\sigma^2 \\
& -\alpha\Lambda^3\beta_a\beta_n\gamma_a\sigma^2 + \Lambda^3\mu\beta_a\beta_n\gamma_a\sigma^2 - \Lambda^3\mu q_{\text{an}}\beta_a\beta_n\gamma_a\sigma^2 - \Lambda^3\omega q_{\text{an}}\beta_a\beta_n\gamma_a\sigma^2 - \alpha\Lambda^3\beta_n^2\gamma_n\sigma^2 \\
& -\Lambda^3\mu\beta_n^2\gamma_n\sigma^2 - \Lambda^3\omega\beta_n^2\gamma_n\sigma^2 + \Lambda^3\mu q_{\text{na}}\beta_n^2\gamma_n\sigma^2 + \Lambda^3\omega q_{\text{na}}\beta_n^2\gamma_n\sigma^2 + 2\alpha\Lambda^3\beta_a\beta_n\gamma_n\sigma^2 + 2\Lambda^3\mu\beta_a\beta_n\gamma_n\sigma^2 \\
& +2\Lambda^3\omega\beta_a\beta_n\gamma_n\sigma^2 - 2\Lambda^3\mu q_{\text{na}}\beta_a\beta_n\gamma_n\sigma^2 + \alpha\Lambda^3\beta_a\gamma_a\epsilon_a\sigma^2 + \Lambda^3\mu\beta_a\gamma_a\epsilon_a\sigma^2 - \Lambda^3\mu q_{\text{an}}\beta_a\gamma_a\epsilon_a\sigma^2 \\
& +\Lambda^3\omega q_{\text{an}}\beta_a\gamma_a\epsilon_a\sigma^2 - 3\alpha\Lambda^3\beta_a\gamma_n\epsilon_a\sigma^2 - 3\Lambda^3\mu\beta_a\gamma_n\epsilon_a\sigma^2 - 3\Lambda^3\omega\beta_a\gamma_n\epsilon_a\sigma^2 + 3\Lambda^3\mu q_{\text{na}}\beta_a\gamma_n\epsilon_a\sigma^2 \\
& +\alpha\Lambda^3\beta_n\gamma_n\epsilon_a\sigma^2 + \Lambda^3\mu\beta_n\gamma_n\epsilon_a\sigma^2 + \Lambda^3\omega\beta_n\gamma_n\epsilon_a\sigma^2 - \Lambda^3\mu q_{\text{na}}\beta_n\gamma_n\epsilon_a\sigma^2 - 2\Lambda^3\omega q_{\text{na}}\beta_n\gamma_n\epsilon_a\sigma^2 \\
& -\alpha\Lambda^3\beta_a\gamma_a\epsilon_n\sigma^2 - \Lambda^3\mu\beta_a\gamma_a\epsilon_n\sigma^2 + \Lambda^3\mu q_{\text{an}}\beta_a\gamma_a\epsilon_n\sigma^2 - \Lambda^3\omega q_{\text{an}}\beta_a\gamma_a\epsilon_n\sigma^2 + 3\alpha\Lambda^3\beta_a\gamma_n\epsilon_n\sigma^2 + 3\Lambda^3\mu\beta_a\gamma_n\epsilon_n\sigma^2 \\
& +3\Lambda^3\omega\beta_a\gamma_n\epsilon_n\sigma^2 - 3\Lambda^3\mu q_{\text{na}}\beta_a\gamma_n\epsilon_n\sigma^2 - \alpha\Lambda^3\beta_n\gamma_n\epsilon_n\sigma^2 - \Lambda^3\mu\beta_n\gamma_n\epsilon_n\sigma^2 - \Lambda^3\omega\beta_n\gamma_n\epsilon_n\sigma^2 \\
& +\Lambda^3\mu q_{\text{na}}\beta_n\gamma_n\epsilon_n\sigma^2 + 2\Lambda^3\omega q_{\text{na}}\beta_n\gamma_n\epsilon_n\sigma^2 - 2\Lambda^3\omega q_{\text{na}}\gamma_n\epsilon_a\epsilon_n\sigma^2 - \Lambda^3\gamma_n\epsilon_a^2\eta_a\sigma^2 - \Lambda^3\gamma_n\epsilon_n^2\eta_a\sigma^2 \\
& +\Lambda^3\beta_a\beta_n\gamma_a\eta_a\sigma^2 - 2\Lambda^3\beta_a\beta_n\gamma_n\eta_a\sigma^2 - \Lambda^3\beta_a\gamma_a\epsilon_a\eta_a\sigma^2 + 3\Lambda^3\beta_a\gamma_n\epsilon_a\eta_a\sigma^2 + \Lambda^3\beta_n\gamma_n\epsilon_a\eta_a\sigma^2 \\
& +\Lambda^3\beta_a\gamma_a\epsilon_n\eta_a\sigma^2 - 3\Lambda^3\beta_a\gamma_n\epsilon_n\eta_a\sigma^2 - \Lambda^3\beta_n\gamma_n\epsilon_n\eta_a\sigma^2 + 2\Lambda^3\gamma_n\epsilon_a\epsilon_n\eta_a\sigma^2 + \Lambda^3\gamma_n\epsilon_a^2\eta_n\sigma^2 \\
& +\Lambda^3\gamma_n\epsilon_n^2\eta_n\sigma^2 - \Lambda^3\beta_a\beta_n\gamma_a\eta_n\sigma^2 + 2\Lambda^3\beta_a\beta_n\gamma_n\eta_n\sigma^2 + \Lambda^3\beta_a\gamma_a\epsilon_a\eta_n\sigma^2 \\
& -3\Lambda^3\beta_a\gamma_n\epsilon_a\eta_n\sigma^2 - \Lambda^3\beta_n\gamma_n\epsilon_a\eta_n\sigma^2 - \Lambda^3\beta_a\gamma_a\epsilon_n\eta_n\sigma^2 + 3\Lambda^3\beta_a\gamma_n\epsilon_n\eta_n\sigma^2 + \Lambda^3\beta_n\gamma_n\epsilon_n\eta_n\sigma^2 \\
& -2\Lambda^3\gamma_n\epsilon_a\epsilon_n\eta_n\sigma^2
\end{aligned}$$

$$\begin{aligned}
\frac{C_3}{\Lambda^2\sigma^3} = {} & -\mu\left(q_{\text{an}} - 1\right)\gamma_a\beta_a^2 + \alpha\sigma\gamma_a\beta_a + \alpha\beta_n\gamma_a\beta_a - \alpha\sigma\gamma_n\beta_a - \alpha\beta_n\gamma_n\beta_a - 2\alpha\gamma_a\epsilon_a\beta_a + 3\alpha\gamma_n\epsilon_a\beta_a \\
& +2\alpha\gamma_a\epsilon_n\beta_a - 3\alpha\gamma_n\epsilon_n\beta_a + \gamma_a\left(\epsilon_a - \epsilon_n\right)\left(\mu\left(q_{\text{an}} - 1\right) - \omega q_{\text{an}} + \eta_a - \eta_n\right)\beta_a \\
& +\gamma_a\left(-\sigma + \epsilon_a - \epsilon_n\right)\left(\mu\left(q_{\text{an}} - 1\right) - \omega q_{\text{an}} + \eta_a - \eta_n\right)\beta_a + \sigma\gamma_n\left(-\omega + \mu\left(q_{\text{na}} - 1\right) + \eta_a - \eta_n\right)\beta_a \\
& +\beta_n\gamma_n\left(-\omega + \mu\left(q_{\text{na}} - 1\right) + \eta_a - \eta_n\right)\beta_a + 3\gamma_n\epsilon_n\left(-\omega + \mu\left(q_{\text{na}} - 1\right) + \eta_a - \eta_n\right)\beta_a \\
& +\beta_n\gamma_a\left(\mu\left(q_{\text{an}} - 1\right) + \omega q_{\text{an}} - \eta_a + \eta_n\right)\beta_a + 3\gamma_n\epsilon_a\left(-q_{\text{na}}\mu + \mu + \omega - \eta_a + \eta_n\right)\beta_a - \alpha\beta_n^2\gamma_a \\
& -\alpha\sigma\beta_n\gamma_a + \alpha\beta_n^2\gamma_n - \left(\mu + \omega\right)\left(q_{\text{na}} - 1\right)\beta_n^2\gamma_n + \alpha\sigma\beta_n\gamma_n - \alpha\sigma\Psi\gamma_a\epsilon_a + \alpha\beta_n\gamma_a\epsilon_a + \alpha\sigma\Psi\gamma_n\epsilon_a \\
& -2\alpha\beta_n\gamma_n\epsilon_a + \alpha\sigma\Psi\gamma_a\epsilon_n - \alpha\beta_n\gamma_a\epsilon_n - \alpha\sigma\Psi\gamma_n\epsilon_n + 2\alpha\beta_n\gamma_n\epsilon_n \\
& +\beta_n\left(\gamma_a\left(-\sigma + \epsilon_a - \epsilon_n\right)\cdot\left(-q_{\text{an}}\mu + \mu - \omega q_{\text{an}} + \eta_a - \eta_n\right)\right) \\
& +\beta_n\left(-\gamma_n\left(-\sigma + 2\epsilon_a - 2\epsilon_n\right)\left(-q_{\text{na}}\mu + \mu + \omega - 2\omega q_{\text{na}} + \eta_a - \eta_n\right)\right) \\
& +\gamma_a\left(\left(\omega q_{\text{an}} - \eta_a + \eta_n\right)\sigma^2 + \epsilon_n\left((\Psi+2)\omega q_{\text{an}} - (\Psi+2)\eta_a + 2\eta_n + \Psi\left(-q_{\text{an}}\mu + \mu + \eta_n\right)\right)\sigma\right) \\
& +\gamma_a\left(\epsilon_a^2\left(\omega q_{\text{an}} - \eta_a + \eta_n\right) + \epsilon_n^2\left(\omega q_{\text{an}} - \eta_a + \eta_n\right)\right) \\
& +\gamma_a\left(\epsilon_a\left(2\epsilon_n\left(-\omega q_{\text{an}} + \eta_a - \eta_n\right) - \sigma\left((\Psi+2)\omega q_{\text{an}} - (\Psi+2)\eta_a + 2\eta_n + \Psi\left(-q_{\text{an}}\mu + \mu + \eta_n\right)\right)\right)\right) \\
& +\gamma_n\left(\left(-\omega q_{\text{na}} + \eta_a - \eta_n\right)\sigma^2 - \epsilon_n\left(\omega\left(\Psi + 4q_{\text{na}}\right) - (\Psi+4)\eta_a + 4\eta_n + \Psi\left(-q_{\text{na}}\mu + \mu + \eta_n\right)\right)\sigma\right) \\
& +\gamma_n\left(3\epsilon_a^2\left(-\omega q_{\text{na}} + \eta_a - \eta_n\right) + 3\epsilon_n^2\left(-\omega q_{\text{na}} + \eta_a - \eta_n\right)\right) \\
& \gamma_n\left(\epsilon_a\left(6\epsilon_n\left(\omega q_{\text{na}} - \eta_a + \eta_n\right) + \sigma\left(\omega\left(\Psi + 4q_{\text{na}}\right) - (\Psi+4)\eta_a + 4\eta_n + \Psi\left(-q_{\text{na}}\mu + \mu + \eta_n\right)\right)\right)\right)
\end{aligned}$$

$$\begin{aligned}
\frac{C_4}{\Lambda\sigma^4\left(\epsilon_a-\epsilon_n\right)} \quad = \quad & \mu\gamma_a q_{\mathrm{an}}\beta_n+\omega\gamma_a q_{\mathrm{an}}\left(\beta_a+2\left(-\epsilon_a+\epsilon_n+\sigma\right)+\beta_n\right)-\mu\beta_a\gamma_a q_{\mathrm{an}}-\beta_a\gamma_a\eta_a+\mu\beta_a\gamma_a-2\sigma\gamma_a\eta_a \\
& +2\gamma_a\eta_a\epsilon_a+\alpha\left(\beta_a-\beta_n\right)\left(\gamma_a-\gamma_n\right)-\gamma_a\eta_a\beta_n+\beta_a\gamma_a\eta_n+\gamma_a\beta_n\eta_n+\beta_a\eta_a\gamma_n+\eta_a\beta_n\gamma_n \\
& -\beta_a\gamma_n\eta_n-\mu\gamma_a\beta_n-\mu\beta_a\gamma_n+2\sigma\gamma_a\eta_n+2\sigma\eta_a\gamma_n-2\gamma_a\eta_a\epsilon_n-2\gamma_a\epsilon_a\eta_n+2\gamma_a\eta_n\epsilon_n-3\eta_a\epsilon_a\gamma_n \\
& +3\eta_a\gamma_n\epsilon_n+3\epsilon_a\gamma_n\eta_n+\mu\beta_a\gamma_n q_{\mathrm{na}}-\omega\gamma_n\left(\beta_a-3\epsilon_a q_{\mathrm{na}}-\beta_n+2\beta_n q_{\mathrm{na}}+3\epsilon_n q_{\mathrm{na}}+2\sigma q_{\mathrm{na}}\right) \\
& -\beta_n\gamma_n\eta_n+\mu\beta_n\gamma_n-2\sigma\gamma_n\eta_n-3\gamma_n\eta_n\epsilon_n-\mu\beta_n\gamma_n q_{\mathrm{na}}
\end{aligned}$$

$$C_5=-\sigma^5\left(\epsilon_a-\epsilon_n\right)^2\left(\gamma_a\left(\eta_a-\omega q_{\mathrm{an}}-\eta_n\right)+\gamma_n\left(-\eta_a+\eta_n+\omega q_{\mathrm{na}}\right)\right)$$

B Paramter Curves

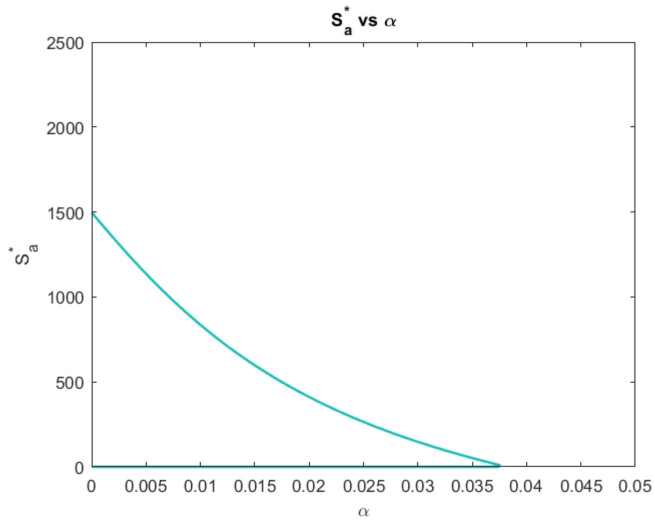

Figure 9: Parameter curve of S_a^* vs α

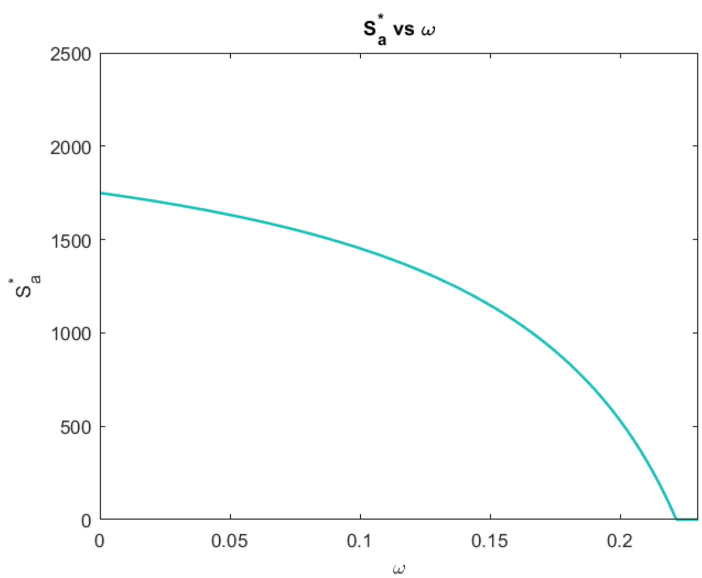

Figure 10: Parameter curve of S_a^* vs ω

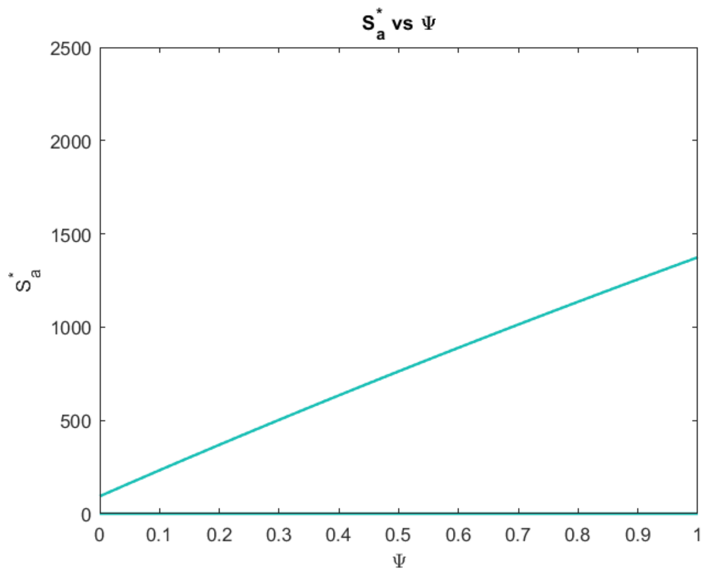

Figure 11: Parameter curve of S_a^* vs Ψ

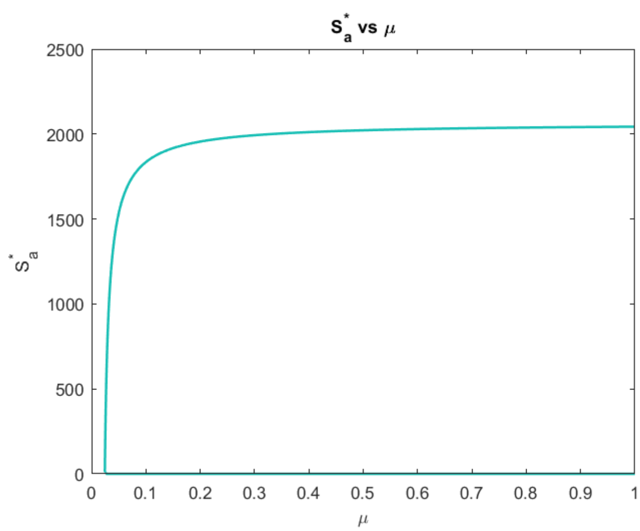

Figure 12: Parameter curve of S_a^* vs μ

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